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SUGGESTIONS TO TEACHERS.

NO text-book can wholly take the place of the living teacher, nor can *written* be a full substitute for *oral* instruction. Good books are only *aids*, and are made more especially for the pupil, and must lack much which would be invaluable to the teacher if it could be presented. Hence, it would be impracticable to attempt to crowd into a text-book all that should be in the hands of both teacher and pupil. Very much must be left for the teacher to supply. The following suggestions may prove of value:

Seek to cultivate in the pupil the habit of *self-reliance*, and of *thinking for himself*. Avoid doing the pupil's work, only assisting and suggesting enough to stimulate him, and enable him to overcome difficulties for himself.

Give many examples, both oral and written, besides those in the book, and require the pupil to construct problems and give their solution and explanation independently of rules. Be particular to have him state what is given and what is required, and to give clearly every step in the operation, since it is far more important to teach him *methods of thought*, and how to *reason*, rather than *what to do*.

A limited number of oral questions, but of considerable *variety*, have been given as *models;* but the teacher should add to their number, until the class fully understand and can promptly solve and analyze each kind of question, since *every oral exercise is preparatory to a written one.*

The models of analysis given for the solution of examples may be varied according to the nature of the question, and at the discretion of the teacher.

Dictation exercises should be freely used as far as they can be adapted to the subject.

CONTENTS.

NUMBERS.

PAGE
Definitions........................ 3
Notation and Numeration....... 4
Arabic Notation....... 5
A Scale.......................... 10
PAGE
Addition.......................... 15
Subtraction............... 27
Multiplication............... 39
Division........................... 56

PROPERTIES OF NUMBERS.

Factors and Divisors............ 81
Common Divisors.................. 85
Multiples........................ . 90
Cancellation....................... 94

COMMON FRACTIONS

Definitions, etc..................100
General Principles..............108
Addition 113
Subtraction........................115
Multiplication....................120
Division 130

DECIMAL FRACTIONS.

Notation and Numeration.......148
Principles......................153
Decimal Currency.... 156
Principles.......................157
Reduction 158
Addition 165
Subtraction........................167
Multiplication 170
Division 178
Accounts and Bills..............183

DENOMINATE NUMBERS.

Measures of Value... 189
Measures of Extension..........194
Measures of Capacity...........204
Measures of Weight.............207
Measures of Time.......... 211
Measures of Angles.. 213
Miscellaneous Measures.........215
Reduction....... 218
Measurements.....................240
Reduction of Denom. Fractions.226
Addition.......................231
Subtraction 233
Multiplication................236
Division 238

PERCENTAGE.

Definitions, etc.............246, 249
Profit and Loss..................258
Commission 263
Interest...........................266
Problems of Interest............275
Compound Interest...............279
Partial Payments.................280
Discount 283
Metric System......... 294
Roman Notation...................299

PREFACE.

HOW to save the time of both teacher and pupil, and at the same time enhance the mental discipline and practical knowledge of the latter, is an object, the attainment of which must not only be a source of economy, but an educational improvement of great importance and value.

At the present time there is an obvious demand that *one* of the books composing an arithmetical series should be adapted to that numerous class of pupils whose time and opportunities are too limited to permit their acquiring more than the elementary principles of arithmetic and the applications of these to the ordinary business transactions.

In the preparation of this treatise, the author has endeavored to supply such a book, by confining the treatment to a limited number of topics of the most useful and practical character, and by so thoroughly developing each, both in theory and applications, as to obviate entirely the necessity of studying these portions of the subject again in any larger book.

Scarcely too much importance can be given to the study of mental arithmetic in our elementary schools. There is no doubt, that when properly taught, it is one of the most effective means of intellectual training; and yet, although the mental exercises should always precede the written, this part of the subject is too generally neglected for want of time.

In this work, oral and written exercises have been thoroughly combined—the oral preceding and made preparatory to the written. Sufficient oral arithmetic has been introduced to answer the purpose of a separate book, thus ensuring attention to the subject and the saving of much valuable time.

Superiority is claimed for this treatise, in the *arrangement* and *treatment* of the subjects; in *clear* and *concise definitions;* in *brief* and *comprehensive rules;* in *new* and *improved methods* of operations and analyses; in the great *number* and *variety* of *well-graded, practical examples,* both *oral* and written; in the combination of new processes and methods with preceding ones as the work advances, which serves to *test* and *drill* the pupil, without the mechanical monotony which too often attends a single class of exercises; in the engraved *cuts*, designed especially to aid in developing and illustrating the subject taught; and last, but not least, in the *typography* and mechanical *finish* of the whole work.

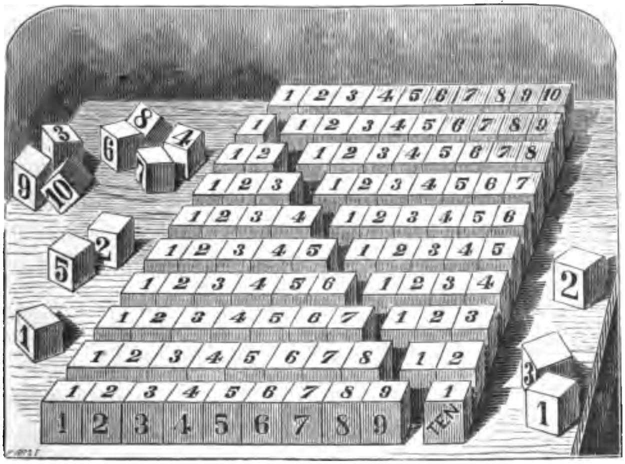
If the pupil, after finishing this book, desires to take up the remaining and higher subjects of arithmetic, usually taught in advanced classes, he will find them fully treated in the "SECOND PART" of the COMPLETE ARITHMETIC of the "*Shorter Course,*" and bound up in a separate volume.

In the said "SECOND PART," the higher subjects of Commercial Arithmetic, Proportion, Involution, Evolution, Progressions, and Mensuration, are developed and applied *inductively*, in the same manner as the subjects of this work, thus making it a very thorough and complete *sequel* to this book.

It is confidently believed that this work will not only meet the wants of *intermediate classes in graded schools*, but prove to be just the text-book needed in a large number of schools in rural districts, and in *evening schools* in cities, where the attendance is too limited to allow the use of a fuller and more extended treatise.

BROOKLYN, *December*, 1874. D. W. F.

INTERMEDIATE ARITHMETIC.



NUMBERS.

ARTICLE 1. 1. A single thing is a *unit,* or ***one.***
2. One unit and one unit are two units, or ***two.***
3. Two units and one unit are three units, or ***three.***
4. Three units and one unit are four units, or ***four.***
5. Four units and one unit are five units, or ***five.***
6. Five units and one unit are six units, or ***six.***
7. Six units and one unit are seven units, or ***seven.***
8. Seven units and one unit are eight units, or ***eight.***
9. Eight units and one unit are nine units, or ***nine.***
10. Nine units and one unit are ten units, or ***ten.***

11. How many are one block and one block?

12. How many are two blocks and one block?

13. How many are three blocks and one block?

14. How many are four blocks and one block? Four blocks and two blocks? Four blocks and four blocks?

15. How many are five and one? Five and two? Five and three? Five and four? Five and five?

16. How many are six and one? Six and two? Six and three? Six and four?

17. How many are seven and one? Seven and two?

18. How many are eight and one? Eight and two?

19. How many are nine and one? Five and four?

20. How many are four and two? Five and two? Six and two? Seven and two? Eight and two?

21. How many are five and three? Five and four?

22. How many are six and three? Six and four?

23. How many are seven and two? Seven and three?

24. If you have five pennies, and five more be given you, how many will you then have?

25. Six books and four books are how many?

26. Four pencils and three pencils are how many?

27. Three cents and five cents are how many?

28. Two blocks and eight blocks are how many?

29. Five balls and four balls are how many?

30. How many ones in two? In three? In four?

31. How many ones in five? In six? In seven?

32. How many ones in eight? In nine? In ten?

33. How many ones in two and one? In three and two?

34. How many ones in four and six? In six and four?

35. How many ones in seven and three? In three and seven?

DEFINITIONS.

2. ***A Unit*** is one, or a single thing: as *one* orange, *one* book, *one* day.

3. ***A Number*** is a unit, or collection of units; as *one* penny; *five* books; *six* days; *three; four.*

4. ***A Concrete Number*** is a number applied to a particular thing, or quantity: as one *ball;* four *boxes;* three *miles.*

5. ***An Abstract Number*** is a number not applied to any object: as *one, six, eight.*

6. ***Like Numbers*** are such as have the same kind of unit, and may be either concrete or abstract: as four *books* and six *books; seven* and *nine.*

7. ***Unlike Numbers*** are such as have not the same kind of unit: as five *books* and seven *balls;* three *days* and five *weeks.*

EXERCISES.

8. 1. How many units in one? In one cent? In three dollars? In five yards?

2. Six is a collection of how many units? Seven?

3. Is four books a concrete, or an abstract number? Two? Five? Four days? Six houses?

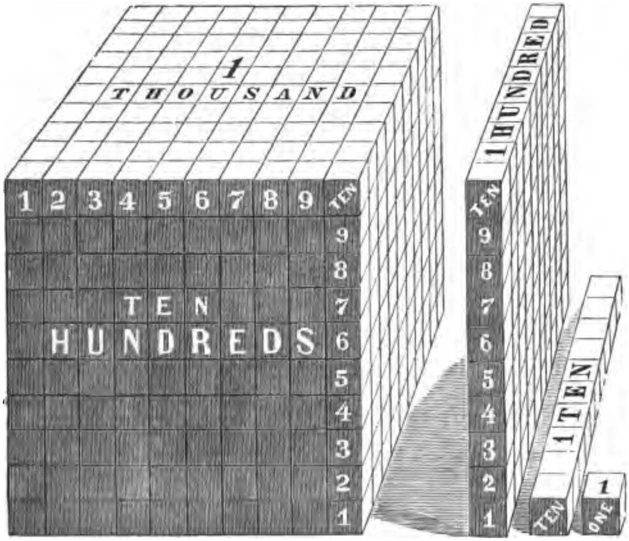
4. Why is seven rods a concrete number?

5. Why is nine an abstract number?

6. Name two numbers that are *like* numbers.

7. Name two numbers that are *unlike* numbers.

8. What kind of numbers are three acres and four acres? Five miles and seven years? Eight and nine?



NOTATION AND NUMERATION.

9. To illustrate the method of uniformity in representing numbers, objects are regarded as arranged in *groups* of *tens;* hence we have single things, or units; next, *groups* containing *ten* units, or *ten;* next, *groups* containing *ten* tens, or *one hundred;* and again, *groups* containing *ten* hundreds, or *one thousand,* etc.

10. This method of grouping is called the *Decimal System,* from the Latin word *decem,* which signifies *ten.*

11. The number of objects may be represented by *words* or by *characters.*

12. The characters may be either *figures,* or *letters.*

13. ***Notation*** is a method of *writing,* or of representing numbers by characters.

14. ***Numeration*** is a method of *reading* numbers represented by figures.

15. ***Figures*** are characters used to express numbers.

16. The method of expressing numbers by *figures* is called *Arabic,* because invented by the Arabs.

ARABIC NOTATION.

17. This method employs *ten* different characters or figures to express numbers, viz.:

Figures,	1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
Names,	One,	Two,	Three,	Four,	Five,	Six,	Seven,	Eight,	Nine,	Naught.

18. The first nine are called *significant* figures, because each has a value of its own; the last is called *Naught, Zero,* or *Cipher,* and has no value of its own.

19. The *first nine* numbers are each represented by a *single figure,* and are called *units* of the *first order.*

20. But since there is no single character to represent the number *ten,* by grouping ten *ones* together there is formed a unit of the *second order,* called *ten,* which is represented by writing the figure 1 in the second place with a cipher after it; thus, 10.

21. In the same manner are represented,

Two tens, or Twenty,	by 20	Six tens, or Sixty,	by 60
Three tens, or Thirty,	" 30	Seven tens, or Seventy,	" 70
Four tens, or Forty,	" 40	Eight tens, or Eighty,	" 80
Five tens, or Fifty,	" 50	Nine tens, or Ninety,	" 90

The suffix *ty* means *ten,* as three tens, or thirty, etc.

22. The numbers between *ten* and *twenty* are represented by writing 1 in the *second* place and the units in the *first* place. Thus,

Eleven	11	Fourteen	14	Seventeen	17
Twelve	12	Fifteen	15	Eighteen	18
Thirteen	13	Sixteen	16	Nineteen	19

The words thir-teen, four-teen, etc., mean *three* and *ten*, *four* and *ten*, etc.

23. In like manner the numbers between 20 and 30, are represented by writing the *tens* in the *second* place, and the *units* in the *first* place. Thus,

Twenty-one	21	Twenty-four	24	Twenty-seven	27
Twenty-two	22	Twenty-five	25	Twenty-eight	28
Twenty-three	23	Twenty-six	26	Twenty-nine	29

EXERCISES.

Copy and read the following:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
15	54	43	73	91	77
32	46	39	61	37	65
46	37	66	55	88	98
31	29	58	76	79	89

Write in figures and arrange in columns:

7. Three tens and six units, seven tens and five units, four tens and three units, and eight tens.

8. The numbers between forty and fifty.

9. The numbers between seventy-five and eighty-six.

10. The numbers between eighty-two and ninety-four.

11. Twelve, twenty-five, fifty-seven, thirty-four, sixty-two, forty-nine, seventeen.

12. Forty-six, sixty-eight, forty-nine, ninety.

24. The greatest number that can be represented by *two* figures is 99.

25. By grouping ten units of the *second* order or *ten tens* into a larger collection, there is formed a unit of the *third* order, called a *hundred*, which is represented by writing the figure 1 in the *third* place with two ciphers after it; thus, 100.

26. In like manner are represented

Two hundred	by 200	Six hundred	by 600
Three hundred	" 300	Seven hundred	" 700
Four hundred	" 400	Eight hundred	" 800
Five hundred	" 500	Nine hundred	" 900

27. The numbers from one hundred to *nine hundred ninety-nine* are represented by writing the *hundreds* in the *third* place, the *tens* in the *second* place, and the *units* in the *first* place. Hence

28. ***Orders of Units*** are denoted by the *position* of the figures used in expressing a number.

Thus, 426 represents 6 units of the *first order*, 2 units of the *second order*, or 2 *tens*, and 4 units of the *third order*, or 4 *hundreds*, and is read four hundred and twenty-six.

29. PRINCIPLES 1. *Ten units of any order in a number make one unit of the next higher order.*

2. *When any order of units in a number is vacant, fill the place with a cipher.*

EXERCISES.

30. Copy and read the following and name the number of *hundreds, tens,* and *units* in each.

(1.)	(2.)	(3.)	(4.)	(5.)
231	128	727	810	190
426	561	636	244	927
147	600	235	506	879
320	284	317	789	586

Write in figures and arrange in columns:

6. Seven hundred five tens two units, three hundred six tens eight units, five hundred seven units.

7. The numbers between two hundred and twenty-eight, and two hundred and thirty-four.

8. The numbers between four hundred and ninety-six, and five hundred and seven.

9. One hundred and sixty-five, three hundred and forty, seven hundred and eighty-one, and nine hundred and seventy-four.

31. The greatest number that can be expressed by *three* figures is 999.

32. By grouping ten units of the *third* order, or *ten hundreds* into a larger collection, there is formed a unit of the *fourth* order called a *thousand,* which is represented by writing the figure 1 in the *fourth* place with three ciphers after it; thus, 1000.

33. In like manner are represented:

Two thousand	by 2000	Six thousand	by 6000
Three thousand	" 3000	Seven thousand	" 7000
Four thousand	" 4000	Eight thousand	" 8000
Five thousand	" 5000	Nine thousand	" 9000

34. The numbers from one thousand to *nine thousand nine hundred and ninety-nine,* are represented by writing *thousands* in the *fourth* place, *hundreds* in the *third* place, *tens* in the *second* place, and *units* in the *first* place.

Thus, 3508 represents 8 units of the *first* order, 0 units of the *second* order, or no *tens,* 5 units of the *third* order, or 5 *hundreds,* and 3 units of the *fourth* order, or 3 *thousands,* and is read three thousand five hundred and eight.

EXERCISES.

35. Copy and read the following, naming the number of units of *each order.*

(1.)	(2.)	(3.)	(4.)	(5.)
3176	6308	8034	4600	2244
2042	4672	6245	9872	5000
5360	1908	7002	6789	9999
7032	5410	2876	3070	9876

Write in figures and read:

6. Three units of the fourth order, five units of the third order, two units of the second order, and one unit of the first order.

7. Six units of the fourth order, eight of the second order, and four of the first order.

8. Nine units of the fourth order, four units of the second order, and seven of the first order.

9. One thousand three hundred and forty-seven.

10. Five thousand six hundred and seventeen.

11. Four thousand nine hundred and ten.

12. Eight thousand and seventy-nine.

13. Nine thousand six hundred and twelve.

14. Six thousand six hundred and six.
15. Nine thousand nine hundred and nineteen.
16. Seven thousand and seventy-eight.
17. Five thousand two hundred and five.
18. Eight thousand eight hundred and ten.
19. Nine thousand three hundred and fifteen.

36. The greatest number that can be expressed by *four* figures is 9999.

37. In the same manner other *new* orders are formed to represent larger numbers, by grouping *ten* units of the *fourth* order to form the *fifth* order, or *tens* of thousands; and *ten* units of the *fifth* order to form the *sixth* order, or *hundreds* of thousands; and *ten* units of the *sixth* order to form the *seventh* order, or *millions,* etc.

Thus, 263045 represents 5 units of the *first* order, 4 units of the *second* order, or 4 *tens,* 0 units of the *third* order, or no *hundreds,* 3 units of the *fourth* order, or 3 *thousands,* 6 units of the *fifth* order, or 6 *tens* of thousands, and 2 units of the *sixth* order, or 2 *hundreds* of thousands, and is read two hundred sixty-three thousand and forty-five.

38. The *value* of any order of units is *increased tenfold* for every place it is moved from the *right* to the *left.*

39. The *value* of any order of units is *diminished tenfold* for every place it is moved from the *left* to the *right.*

40. ***A Scale*** in numbers, is a succession of units, increasing and decreasing according to a certain law, or rule. Scales are *uniform* or *varying.*

41. ***A Decimal Scale*** is one in which the law of increase and decrease is *uniformly* TEN. Hence, Arabio Notation has a *decimal scale.*

EXERCISES.

42. Copy and read the following:

(1.)	(2.)	(3.)	(4.)
12342	43720	167248	500962
67218	75400	603521	389000
50406	30074	826400	98762

Write in figures and read,

5. Five units of the 5th order, one of the 3d, eight of the 1st.

6. Three units of the 5th order, six of the 4th, four of the 3d, seven of the 1st.

7. Two units of the 6th order, four of the 5th, nine of the 4th, three of the 3d, five of the 1st.

8. Six units of the 7th order, four of the 6th, three of the 5th, eight of the 4th, five of the third, eight of the 2d, seven of the 1st.

9. Forty-seven thousand six hundred and forty.

10. One hundred forty-six thousand five hundred.

11. Ninety-five thousand three hundred and ten.

12. Four hundred seventy thousand two hundred.

13. Eight hundred five thousand one hundred and three.

14. Thirty-seven thousand five hundred and fourteen.

15. Six hundred twenty-five thousand nine hundred.

16. Seven hundred ten thousand four hundred.

17. Nine hundred thousand nine hundred and nine.

43. This method of numeration, groups the successive orders into *periods* of three figures each, which are commonly separated by commas, as in the following Table, in which the names of the *orders* and *periods* to the *fifteenth* place, are given.

NUMERATION TABLE.

PERIODS.	5th.			4th.			3d.			2d.			1st.		
NAME.	Trillions			Billions			Millions			Thousands			Units		
ORDERS OF UNITS.	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
NUMBER.		4	5,	3	7	0,	0	3	6,	4	0	8,	0	6	0

The number is read forty-five *trillion,* three hundred seventy *billion,* thirty-six *million,* four hundred and eight *thousand,* and sixty.

1. In *reading* numbers the name of the units period is omitted.
2. Every period except the highest must contain *three* figures.

44. Until the pupil can write and read numbers readily, he may be required to prepare and arrange on slate or paper, exercises similar to the following, the teacher carefully inspecting the work.

Bill.			Mill.			Thou.			Un.		
H.	T.	U.	H.	T.	U.	H.	T.	U.	H.	T.	U.
					2	0	0	2	0	0	2
				3	3	0	3	3	0	3	3
			4	4	4	4	4	4	4	4	4
		5	0	3	6	2	0	0	4	0	7

The first example is read, 2 millions, 2 thousand and 2.
The second is read, 33 million, 33 thousand and 33.
The third is read, 444 million, 444 thousand, 444.
The fourth is read, 5 billion, 36 million, 200 thousand, 407.

The diagram at first may be prepared for only two or three periods, and be gradually enlarged to five or six periods for greater numbers.

Each pupil may be allowed to dictate an example to be written and read by the whole class.

45. RULE FOR NOTATION.—*Begin at the left and write the hundreds, tens and units of each period in their proper order, filling all vacant places and periods with ciphers.*

46. RULE FOR NUMERATION.—I. *Begin at the right and separate the number into periods of three figures each.*

II. *Begin at the left and read each period as if it were units, giving its name.*

EXERCISES IN NOTATION AND NUMERATION.

Write the following numbers in figures :

1. Three units of the 8th order, five of the 7th, four of the 6th, three of the 5th, eight of the 4th, five of the 2d.

2. Three units of the 9th order, eight of the 7th, four of the 6th, six of the 5th, nine of the 1st.

3. Twenty-five units in the 2d *period*,, four hundred ninety-six in the 1st. *Ans.* 25,496.

4. Three hundred sixty-four units in the 3d period, seven hundred ten in the 2d, eight hundred thirty in the 1st.

5. Four hundred thirty-six units in the 4th period, twelve in the 3d, one hundred in the 2d, sixty-five in the 1st.

6. Eighty-one units in the 5th period, two hundred nineteen in the 4th, fifty-six in the 2d.

7. Write 25 million 25 thousand 5 hundred.

8. Write 317 million 204 thousand 506.

9. Write fifty million 50 thousand and fifty.
10. Write 10 million four hundred and fifty thousand.
11. Write 102 billion 27 million 46 thousand and sixty.

Copy, point off, and read the following:

12. 12364; 603802; 420600; 1047009; 760091.
13. 4600200; 2603180; 43061084; 70030760924.

QUESTIONS FOR REVIEW.

47. 1. Define a Unit. A Number. A Concrete Number. An Abstract Number. Like Numbers. Unlike Numbers.

2. What method has been adopted to establish uniformity in representing numbers? What is this system of grouping called? Why? How are numbers represented? By what characters?

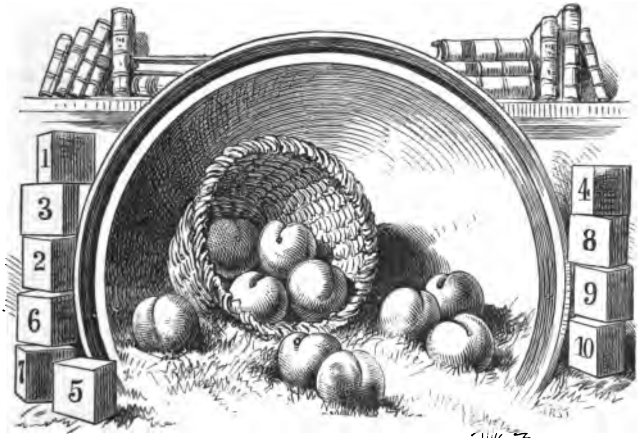
3. Define Notation. Numeration. A Figure. What is the method of representing numbers by figures called? Why? How many figures are employed to represent numbers? Name them. Which are called significant figures? Why?

4. How many numbers can be represented by a single figure? What are they called? How are units of the second order formed? What, called? The greatest number that can be expressed by two figures?

5. How are units of the third order formed? What, called? How are numbers from one hundred to nine hundred and ninety-nine expressed? How are orders of units denoted? Principles? The greatest number that can be represented by three figures.

6. How are units of the fourth order formed? What, called? How are other higher orders formed? How is the value of a figure affected by moving it one place to the left? To the right? What is a scale? How many kinds? What is a decimal scale? Give an example.

7. What are Periods? How separated? How many figures must they contain? Name five periods. Rule for Notation. Rule for Numeration.



ADDITION.

48. 1. How many units are 2 and 4? 3 and 4?

2. How many are 2 peaches and 3 peaches? 2 peaches and 4 peaches?

3. How many are 3 books and 3 books? 4 books and 6 books? 3 books and 7 books?

4. How many are 4 blocks and 6 blocks? 5 blocks and 5 blocks? 5 blocks and 4 blocks?

5. Can you add 6 books and 4 peaches? Why not?

6. Can you add 3 blocks and 7 blocks? Why?

7. How many are 5 cents and 8 cents? 6 boys and 7 boys? 7 hats and 5 hats?

8. How many are 3 tens and 4 tens? 5 tens and 3 tens?

9. How many are 3 hundreds and 4 hundreds? 4 hundreds and 5 hundreds?

10. How many units are 2 tens? 3 tens? 4 tens? 5 tens? 6 tens? 8 tens?

11. How many units are 3 tens and 4 units? 4 tens and 3 units? 5 tens and 4 units?

12. How many are 5 tens and 7 units? 4 tens and 9 units? 6 tens and 5 units?

13. How many are 5 and 9? 25 and 9? 45 and 9? 55 and 9?

14. How many are 7 and 6? 17 and 6? 37 and 6? 67 and 6?

15. A farmer sold 6 sheep to one man and 9 to another. How many did he sell to both?

16. If a coat cost 10 dollars and a pair of boots 8 dollars, how much will both cost?

17. Mary is 9 years old. How old will she be seven years hence? 9 years hence? 10 years hence?

18. George gave 5 pears to Martin, and 4 to Jane, and kept 6. How many had he at first?

19. How many are 9 dollars and 7 dollars? 7 dollars and 9 dollars? How many are 7 and 8? 8 and 7?

20. Is the result changed by adding the same numbers in a different order?

21. Count by 2's from one to 25.

OPERATION.—1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25.

Count

22. By 2's from 0 to 38.
23. By 3's from 0 to 30.
24. By 3's from 2 to 32.
25. By 4's from 0 to 40.
26. By 4's from 1 to 37.
27. By 4's from 3 to 43.
28. By 5's from 0 to 50.
29. By 5's from 2 to 52.
30. By 5's from 6 to 66.
31. By 6's from 0 to 60.
32. By 6's from 3 to 69.
33. By 6's from 4 to 70.

This exercise should be repeated until the pupil can add by 2's, 3's, 4's, 5's, and 6's with rapidity and accuracy.

ADDITION TABLE.

	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	13
2	3	4	5	6	7	8	9	10	11	12	13	14
3	4	5	6	7	8	9	10	11	12	13	14	15
4	5	6	7	8	9	10	11	12	13	14	15	16
5	6	7	8	9	10	11	12	13	14	15	16	17
6	7	8	9	10	11	12	13	14	15	16	17	18
7	8	9	10	11	12	13	14	15	16	17	18	19
8	9	10	11	12	13	14	15	16	17	18	19	20
9	10	11	12	13	14	15	16	17	18	19	20	21
10	11	12	13	14	15	16	17	18	19	20	21	22
11	12	13	14	15	16	17	18	19	20	21	22	23
12	13	14	15	16	17	18	19	20	21	22	23	24

DEFINITIONS.

49. ***Addition*** is the process of uniting two or more *like* numbers into *one* equivalent number.

50. ***The Sum,*** or ***Amount,*** is the number obtained by adding.

51. ***The Sign of Addition*** is +. It is read *plus,* and signifies *more;* thus, 5 + 6 is read, 5 *plus* 6, and means that 5 and 6 are to be *added.*

52. ***The Sign of Equality*** is =. It is read *equals,* or *equal to;* thus, 5 + 6 = 11, is read 5 *plus* 6 *equals* 11, and means that the *sum* of 5 and 6 is *equal to* 11. It may be read 5 + 6 *are* 11.

Read the following:

1. $8 + 4 + 2 = 14$.
2. $7 + 6 + 5 = 18$.
3. $12 + 6 = 8 + 10$.
4. $16 + 8 = 15 + 9$.

53. ***The Sign of Dollars*** is \$. It is read *dollars*. Thus, \$25 is read 25 dollars; \$7 is read 7 dollars.

Read the following expressions:

1. $\$10 + \$8 = \$18$.
2. $\$12 + \$11 = \$23$.
3. $\$15 + \$10 = \$16 + \$9$.
4. $\$16 + \$8 = \$10 + \$14$.

54. ***An Equation*** in Arithmetic, is an expression of *equality* between two numbers or sets of numbers.

EQUATION.

All that is written *before* the sign of equality is called the *first member* of the equation, and all that is written *after* it is called the *second member* of the equation.

The numbers in each member are called *terms* of the equation.

Thus, $6 + 4 = 10$ is an equation, and is read 6 *plus* 4 *equals* 10, and means that the sum of 6 and 4 is equal to 10. $6 + 4$ is the *first member* of the equation, and 10 is the *second member*; and 6, 4, and 10 are the *terms* of the equation.

Name the *members* and the *terms* of each of the following equations:

1. $7 + 10 = 17$
2. $8 + 12 = 20$
3. $15 + 6 = 21$
4. $30 = 20 + 10$
5. $9 + 6 = 10 + 5$
6. $12 + 8 = 16 + 4$

EXERCISES ON THE TABLE.

4 and 6 are how many?
6 and 7 are how many?
10 and 3 are how many?
8 and 10 are how many?
7 and 7 are how many?
12 and 8 are how many?
15 and 6 are how many?
7 and 5 are how many?
10 and 11 are how many?

7 and 8 are how many?
17 and 7 are how many?
16 and 8 are how many?
9 and 5 are how many?
8 and 11 are how many?
15 and 9 are how many?
9 and 15 are how many?
18 and 5 are how many?
15 and 9 are how many?

$4 + 3 + 2 = ?$
$3 + 5 + 1 = ?$
$6 + 4 + 2 = ?$
$1 + 9 + 0 = ?$
$10 + 4 + 3 = ?$
$7 + 5 + 6 = ?$
$8 + 2 + 5 = ?$

$7 + 6 + 10 = ?$
$12 + 10 + 2 = ?$
$5 + 6 + 7 = ?$
$6 + 7 + 5 = ?$
$7 + 5 + 6 = ?$
$10 + 10 + 4 = ?$
$14 + 4 + 4 = ?$

$9 + 2 + 10 = ?$
$8 + 9 + 7 = ?$
$2 + 12 + 7 = ?$
$14 + 4 + 6 = ?$
$13 + 5 + 2 = ?$
$8 + 8 + 8 = ?$
$6 + 9 + 8 = ?$

$9 + 0 + 6 = ?$
$12 + 4 + 4 = ?$
$4 + 12 + 4 = ?$
$6 + 9 + 3 = ?$
$12 + 7 + 6 = ?$
$14 + 5 + 7 = ?$
$9 + 10 + 8 = ?$
$20 + 12 + 9 = ?$

$15 + 3 + 6 = ?$
$3 + 15 + 4 = ?$
$2 + 11 + 10 = ?$
$16 + 0 + 10 = ?$
$7 + 20 + 5 = ?$
$4 + 15 + 9 = ?$
$18 + 6 + 7 = ?$
$7 + 23 + 10 = ?$

$8 + 9 + 6 = ?$
$9 + 6 + 8 = ?$
$10 + 7 + 5 = ?$
$21 + 7 + 6 = ?$
$9 + 13 + 10 = ?$
$18 + 10 + 3 = ?$
$24 + 6 + 8 = ?$
$11 + 19 + 12 = ?$

1. The expression "= ?" is read *equals how many*, or *what*.

2. The pupil should be drilled on exercises similar to the above until he is thoroughly familiar with the table.

ORAL EXERCISES.

55. 1. A man paid \$7 for a hat, \$5 for a vest, and \$6 for a pair of shoes. How much did he pay for all?

Analysis.—He paid the sum of \$7, \$5, and \$6, which is \$18. Hence he paid \$18 for all.

2. A farmer sold ten bushels of oats for \$8, and 7 bushels of corn for \$5. How much money did he receive?

3. What is the sum of 14 cents, 9 cents, and 6 cents?

4. How many are \$11 + \$4 + \$5 + \$8 + \$7?

5. Ella had 7 cents, Lillie gave her 5 more, and her father gave her 10 more. How many had she then?

6. What is the sum of 7 miles, 10 miles, and 6 miles?

Add

7. By 7's from 0 to 56.
8. By 7's from 3 to 59.
9. By 7's from 5 to 75.
10. By 8's from 0 to 80.
11. By 8's from 2 to 82.
12. By 8's from 6 to 86.

13. Add by 2's and 3's alternately from 1 to 31.

Operation.—1, 3, 6, 8, 11, 13, 16, 18, 21, 23, 26, 28, 31.

Add

14. By 2's and 3's alternately from 0 to 40.
15. By 3's and 4's alternately from 3 to 52.
16. By 3's and 5's alternately from 4 to 60.
17. By 4's and 5's alternately from 2 to 56.
18. By 2's, 3's and 4's successively from 0 to 54.
19. By 2's, 4's and 5's successively from 3 to 58.

56. Principles.—1. *Only like numbers and units of the same order can be added.*

2. *The sum and the numbers added must all be like numbers.*

WRITTEN EXERCISES.

57. When the sum of the units of each order is less than 10.

1. What is the sum of 321, 164 and 412.

OPERATION.

Hunds.	Tens.	Units.	
3	2	1	
1	6	4	
4	1	2	
8	9	7	Sum.

ANALYSIS.—Arranging the numbers so that units of the same order shall stand in the same column, begin with the lowest order of units, and add each column separately, and instead of saying 2 units and 4 units are 6 units, and 1 unit are 7 units, name the successive *results* only; thus, 2, 6, 7, the sum of the *units*, which write under the units column. Next, 1, 7, 9, the sum of the *tens*, which write under the tens column.

Lastly, 4, 5, 8, the sum of the *hundreds*, which write under the hundreds column. Hence the *sum* is 897.

Begin at the top and add the columns downward in the same manner. If the two results agree, the work is probably correct.

In the same manner, copy, add and prove:

(2.)	(3.)	(4.)	(5.)	(6.)
112	120	423	123	400
270	233	234	34	2164
304	614	140	702	1315

7. What is the sum of 203, 176, and 510?

8. What is the sum of 124, 403, and 271?

9. What is the sum of 1234, 5140, and 2405?

10. Add five thousand one hundred and forty, four hundred and five, and two thousand three hundred and fifty.

11. Add 3 thousand 46, 12 thousand 5 hundred and 11, and 20 thousand 2 hundred and 30.

12. A man paid $514 for a pair of horses, $240 for a carriage, and $123 for a harness. How much did he pay for all ?

13. A man traveled 461 miles by railroad, 310 miles by steamboat, and 125 miles by stage. What was the total number of miles traveled ?

14. A carpenter built a house for $2464, a barn for $320, and fences for $215. What was the cost of all ?

ORAL EXERCISES.

58. Add

1. By 9's from 0 to 81.
2. By 9's from 2 to 83.
3. By 9's from 5 to 104.
4. By 9's from 7 to 97.
5. By 10's from 1 to 101.
6. By 10's from 9 to 191.

Add rapidly the following

7. 3, 2, 5, 4 and 6.
8. 2, 1, 7, 5 and 4.
9. 5, 6, 2, 3 and 1.
10. 8, 4, 3, 2 and 6.
11. 2, 9, 4, 8 and 3.
12. 7, 2, 6, 9 and 4.
13. $3 + 4 + 5 + 3 + 4 + 5$, etc., till the sum $= 60$.
14. $5+6+5+6+5+6+5+6$, etc., till the sum $=77$.
15. $6+4+6+4+6+4+6+4$, etc., till the sum $=90$.
16. Count by 2's, 5's and 4's successively from 5 to 93.
17. Count by 4's, 5's and 6's successively from 9 to 99.
18. In an orchard are 35 apple trees and 24 pear trees. How many trees in the orchard ?

ANALYSIS.—The sum of 35 trees and 24 trees. 35 is 3 tens and 5 units, and 24 is 2 tens and 4 units; 3 tens and 2 tens are 5 tens, or 50; and 5 units and 4 units are 9 units, added to 50, make 59. Hence, there are 59 trees in the orchard.

19. James earned 47 cents one day, and 32 the next. How many cents did he earn in both days ?

20. What is the sum of 35 and 15? Of 28 and 42?

21. What is the sum of 46 and 27? Of 54 and 38?

22. A tailor bought two pieces of cloth, one containing 36 yards, and the other 29 yards. How many yards in both pieces?

23. A woman sold 15 pounds of butter at one time, 12 pounds at another, and 10 pounds at another. How many pounds did she sell in all?

24. A man bought a sleigh for $40, paid $20 for repairing it, and $9 for painting it. What was the entire cost?

25. A farmer sold 10 bushels of wheat for $16, 5 cords of wood for $12, and a ton of hay for $20. How much did he receive for all?

WRITTEN EXERCISES.

59. When the sum of the units of any order equals or exceeds 10.

1. What is the sum of 465, 362, 857, and 684.

OPERATION.

$$\begin{array}{r} 4\ 6\ 5 \\ 3\ 6\ 2 \\ 8\ 5\ 7 \\ 6\ 8\ 4 \\ \hline 2\ 3\ 6\ 8 \end{array} \text{ Sum.}$$

ANALYSIS.—Arrange the numbers as before, and beginning with the lowest order of units, add each column separately, pronouncing the *results* only; thus, 4, 11, 13, 18, the sum of the *units*, equal to 1 ten and 8 units. Write the 8 units under the units column, and reserve the 1 ten to add to the tens column.

Next, adding the 1 ten reserved, say, 1, 9, 14, 20, 26, the sum of the *tens*, equal to 2 hundreds and 6 tens. Write the 6 tens under the tens column, and reserve the 2 hundreds to add to the hundreds columns.

Lastly, adding the 2 hundreds reserved, say 2, 8, 16, 19, 23, the sum of the *hundreds*, equal to 2 thousands and 3 hundreds, which write in hundreds and thousands places. Hence, the *sum* is 2368.

In like manner add and prove:

(2.)	(3.)	(4.)	(5.)
876 feet.	407 pounds.	715 men.	$2342
325 "	390 "	306 "	760
422 "	678 "	478 "	1479
276 "	536 "	956 "	687

6. What is the sum of 370, 4032, 5600, and 287?
7. Find the amount of 1002, 564, 7030, 256, and 73.

Rule.—I. *Write the numbers so that figures of the same order stand in the same column.*

II. *Beginning at the right, add each column separately, and write the sum, if expressed by one figure, under the column added.*

III. *If the sum of any column consists of two or more figures, write the unit figure under that column, and add the remaining figure or figures to the next column.*

Proof.—*Beginning at the right, add each column downward. If the results agree, the work is probably correct.*

It is important that the pupil at once acquire the habit of performing all numerical operations on the slate or blackboard mentally, only writing *results*.

Write, add and prove

8. 627, 826, and 679.
9. 574, 786, and 895.
10. 1284, 96, and 706.
11. 102, 2406, and 500.
12. 3147, 7605, and 8297.
13. 6078, 9578, and 5809.
14. 7100, 873, and 36086.
15. 609, 8004, and 50326.

16. 24036+962+3172+801+96= ?
17. 1268+382+4008+144+75= ?
18. 76921+58346+87521+86434+8764341= ?
19. 19823+60587+75333+82177+5687274= ?
20. 17853+85471+92846+15698+87659+1932881= ?
21. 76543+87884+99772+88594+66777+8778896= ?
22. $3675+$40706+$2008+$540+$84+$60508= ?
23. 4703+60173+345+18040+20009+8090 feet= ?

24. A flour merchant sold 1728 barrels of flour one month, 2009 the next, and as many the third as in the other two. How many barrels did he sell in three months?

25. A father divided his estate among his four sons and two daughters, giving each son $3896, and each daughter $2675. What was the value of his estate?

26. A man engaging in trade, gained $450 the first year, $684 the second, and as much the third as he gained during the first and second. How much was his whole gain?

27. Bought three village lots for $12570, and sold them so as to gain $745 on each lot. For how much were they sold?

28. A has $3240, B has $5672, and C has $1000 more than A and B together. How many dollars have all?

29. The Old Testament contains 39 books, 929 chapters, 23214 verses, 592439 words, and 2728100 letters; the New Testament contains 37 books, 269 chapters, 7959 verses, 181153 words, and 838380 letters. What is the total number of each in the Bible?

Ans. 76 books, 1198 chapters, 31173 verses, 773592 words, and 3566480 letters.

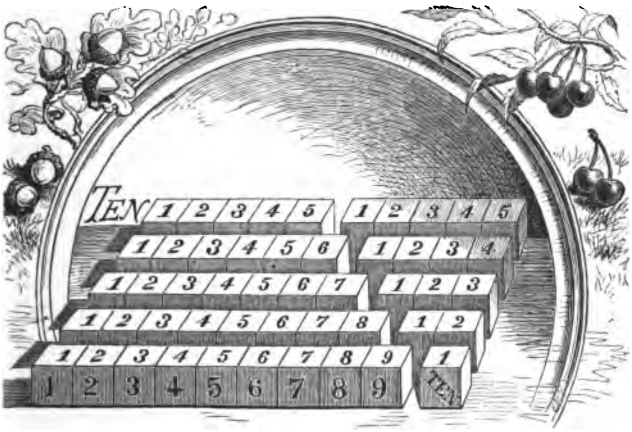
30. A man gave his property to his wife, two sons, and four daughters. To his daughters he gave $4370 each, to his sons $6045 each, and to his wife as much as he gave to one son and one daughter. What was the value of his entire property?

(31.)	(32.)	(33.)	(34.)
8786	78982	437987	3216677
5968	69771	566789	4569911
8789	68339	544321	6543344
9896	56234	891389	9576677
3674	95876	219721	1539900
1767	57412	625247	6662233
5989	73375	431321	4235566

(35.)	(36.)	(37.)	(38.)	(39.)
20992	38912	35495	639	543678
5248	9728	6457	91758	2766
1312	2432	94667	9347	68345
328	304	6463	81731	563875
41984	1216	34698	9342	7427
164	99	93965	35446	945956
656	77824	367	8237	756431
2624	152	29787	12845	54747
83968	608	9826	87677	3864

QUESTIONS FOR REVIEW.

60. Define Addition. Sum or Amount. Sign of Addition. Use and meaning. Sign of Equality. Use and meaning. Sign of dollars. An Equation. Members. Terms. Illustrate. Principles 1, 2. Rule. Proof.



SUBTRACTION.

61. 1. How many are 7 cents, less 5 cents? 18 boys, less 4 boys? 9 days less 5 days?

2. How many are 10 blocks less 3 blocks? 10 blocks less 4 blocks? 10 blocks less 5? 10 less 6?

3. Five from 10 leave how many? 7 from 10, how many? 8 from 10?

4. How many are 7 tens less 3 tens? 9 tens less 5 tens?

5. James had 12 marbles and lost 5. How many had he left?

6. Clarence had 15 cents and gave 6 to his sister. How many had he left?

7. Ella is 16 years old, and her brother 9 years younger. How old is her brother?

8. Bought a slate for 20 cents, and a book for 8 cents less. How much did the book cost?

SUBTRACTION TABLE.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	2	3	4	5	6	7	8	9	10	11	12	13
2	3	4	5	6	7	8	9	10	11	12	13	14
3	4	5	6	7	8	9	10	11	12	13	14	15
4	5	6	7	8	9	10	11	12	13	14	15	16
5	6	7	8	9	10	11	12	13	14	15	16	17
6	7	8	9	10	11	12	13	14	15	16	17	18
7	8	9	10	11	12	13	14	15	16	17	18	19
8	9	10	11	12	13	14	15	16	17	18	19	20
9	10	11	12	13	14	15	16	17	18	19	20	21
10	11	12	13	14	15	16	17	18	19	20	21	22
11	12	13	14	15	16	17	18	19	20	21	22	23
12	13	14	15	16	17	18	19	20	21	22	23	24

DEFINITIONS.

62. ***Subtraction*** is the process of finding the difference between two numbers of the same unit value.

63. ***The Difference,*** or ***Remainder,*** is the result obtained by subtracting.

64. ***The Minuend*** is the number from which another is to be subtracted.

65. ***The Subtrahend*** is the number to be subtracted.

66. ***The Sign of Subtraction*** is —. It is read *minus,* and signifies *less:* thus, 9 — 6, is read 9 *minus* 6, and means that 6 is to be *subtracted* from 9.

67. ***A Parenthesis*** ( ) is used to include within it such numbers as are to be considered together and subjected to the same operation. A ***Vinculum,*** or ***Bar*** ———, has the same signification. Thus, $15 + (12 - 7)$, or $15 + \overline{12 - 7}$, signifies that to 15 is to be added the difference between 12 and 7

EXERCISES ON THE TABLE.

4 from 7, how many ?
3 from 9, how many ?
5 from 8, how many ?
7 from 10, how many ?
6 from 12, how many ?
9 from 15, how many ?
10 from 20, how many ?
8 from 16, how many ?
12 from 18, how many ?

21 less 9, how many ?
18 less 7, how many ?
20 less 6, how many ?
19 less 4, how many ?
22 less 8, how many ?
14 less 5, how many ?
23 less 10, how many ?
17 less 11, how many ?
24 less 12, how many ?

$8 - 5 = ?$
$10 - 7 = ?$
$12 - 8 = ?$
$9 - 4 = ?$
$13 - 5 = ?$
$14 - 8 = ?$
$11 - 5 = ?$
$15 - 7 = ?$
$13 - 9 = ?$
$16 - 8 = ?$
$19 - 8 = ?$
$16 - 9 = ?$

$12 - 7 = ?$
$8 - 3 =$
$9 - 5 = ?$
$13 - 10 = ?$
$17 - 5 = ?$
$20 - 9 = ?$
$14 - 7 = ?$
$21 - 10 = ?$
$18 - 12 = ?$
$11 - 5 = ?$
$20 - 7 = ?$
$18 - 11 = ?$

$16 - 7 = ?$
$22 - 12 = ?$
$16 - 9 = ?$
$14 - 11 = ?$
$23 - 8 = ?$
$17 - 9 = ?$
$13 - 4 = ?$
$20 - 11 = ?$
$19 - 7 = ?$
$24 - 12 = ?$
$23 - 5 = ?$
$20 - 7 = ?$

ORAL EXERCISES.

68. 1. Subtract by 2's from 24 to 0.

Operation.—24, 22, 20, 18, 16, 14, 12, 10, 8, 6, 4, 2, 0.

In the same manner subtract

2. By 2's from 25 to 1.
3. By 3's from 30 to 0.
4. By 3's from 31 to 1.
5. By 4's from 40 to 0.
6. By 4's from 41 to 1.
7. By 5's from 50 to 0.
8. Count by 3's from 0 to 36, and back from 36 to 0.
9. Count by 4's from 4 to 48, and back from 48 to 4.
10. Count by 4's from 2 to 50, and back to 2.
11. Count by 5's from 3 to 63, and back to 3.
12. Count by 5's from 4 to 59, and back to 4.
13. Count by 6's from 0 to 72, and back to 0.
14. Count by 6's from 7 to 67, and back to 7.
15. Count by 6's from 5 to 65, and back to 5.

What is the second *member* of the following equations :

16. $6+4-3=$?
17. $9+5-6=$?
18. $7-5+6=$?
19. $7+5-6=$?
20. $8+7-4=$?
21. $10-4+8=$?
22. $6+9-7=$?
23. $10+6-8=$?
24. $11-4+9=$?

25. George had 14 marbles and gave 6 to James. How many had he left ?

Analysis.—The difference between 14 marbles and 6 marbles, which is 8 marbles. Hence, he had 8 marbles left.

26. A grocer bought a tub of butter for $17 and sold it for $12. How much did he lose ?

27. Charles is 15 years old, and his sister Mary is 9. What is the difference between their ages ?

28. If a man earn $25 a week, and spend $10, how much has he left ?

69. PRINCIPLES.—1. *Only like numbers and units of the same order can be subtracted, the one from the other.*

2. *The minuend must be equal to the sum of the remainder and subtrahend.*

WRITTEN EXERCISES.

70. When no figure of the subtrahend is greater than the corresponding figure of the minuend.

1. From 857 subtract 532.

OPERATION.	
Minuend	8 5 7
Subtrahend	5 3 2
Remainder	3 2 5

ANALYSIS.—Write the less number under the greater, so that units of the same order stand in the same column. Begin at the right and subtract each order of units, thus: 2 units from 7 units leave 5 units, which write under the units. 3 tens from 5 tens leave 2 tens, which write under the tens. 5 hundreds from 8 hundreds leave 3 hundreds, which write under the hundreds. Hence the remainder is 325.

PROOF.—The remainder 325 added to the subtrahend 532, equals 857 the minuend. Hence the work is correct.

Copy, subtract and prove:

	(2.)	(3.)	(4.)	(5.)
Minuend	349	695	753	876
Subtrahend	212	535	502	435

	(6.)	(7.)	(8.)	(9.)
From	546 yards.	438 cents.	637 men.	$764
Take	234 "	321 "	431 "	$423

	(10.)	(11.)	(12.)
From	2467 feet.	4825 pounds.	$6041
Take	1024 "	2103 "	$3011

In like manner subtract and prove:

13. 4431 from 7642.
14. 6430 from 8730.
15. 3764 from 6987.
16. 8431 from 9642.
17. 2361 from 6472.
18. 4076 from 5279.

19. A drover bought 3245 sheep, and sold twelve hundred of them. How many had he left?

20. A man sold a house and lot for $41763, and thereby gained $11521. How much did they cost him?

21. George Washington was born in 1732, and died in 1799. How old was he at his death?

22. Queen Victoria was born in 1819. What will be her age in 1879?

ORAL EXERCISES.

71. 1. Subtract by 6's from 54 to 1.

2. By 7's from 49 to 0.
3. By 7's from 64 to 1.
4. By 8's from 56 to 0.
5. By 8's from 65 to 1.
6. By 9's from 72 to 0.
7. By 9's from 73 to 1.
8. Count by 7's from 0 to 70 and back to 0.
9. Count by 8's from 0 to 80 and back to 0.
10. Count by 9's from 0 to 90 and back to 0.
11. Count by 10's from 0 to 100 and back to 0.
12. Count by 7's from 5 to 75 and back to 5.
13. Count by 8's from 7 to 95 and back to 7.
14. Count by 9's from 3 to 84 and back to 3.
15. Count by 10's from 4 to 104 and back to 4.

16. If the minuend is 18 and the subtrahend 7, what is the remainder?

17. If the difference of two numbers is 8, and the smaller number is 12, what is the greater number?

18. If the remainder is 9 and the subtrahend 11, what is the minuend?

19. Clarence had 8 cents, and Ella gave him 6 more: he then gave 5 cents for an orange. How many cents had he left?

ANALYSIS.—The difference between 5 cents and the sum of 8 cents and 6 cents, which is 9 cents. Hence he had 9 cents left.

20. A lady bought a shawl for $6 and a pair of gloves for $2, and gave in payment two $5 bills. How much change should she receive?

Find the missing *term* in the following equations:

21. $12 - 4 = 5 + ?$
22. $20 - 5 = 10 + ?$
23. $9 + 9 = ? + 2$
24. $24 - 8 = 6 + ?$
25. $15 - 5 = 10 - ?$
26. $18 - 6 = ? + 6$
27. $14 + 7 = 18 + ?$
28. $8 + 7 = 17 - ?$
29. $13 + 7 = ? - 4$
30. $22 - 10 = 8 + ?$

31. From a piece of cloth containing 52 yards, 27 yards were cut. How many yards remained?

ANALYSIS.—The difference between 52 yards and 27 yards. 27 is 2 tens and 7 units. 2 tens or 20 from 52 leaves 32, and 7 from 32 leaves 25. Hence 25 yards remained in the piece.

32. How many are 4 tens less 2 tens? 40 less 20?

33. From 4 tens 7 units, subtract 3 tens 5 units.

34. Sold a watch for $50 that cost $35. What was the gain?

35. From a cask containing 56 gallons, 24 gallons leaked out. How many gallons remained in the cask?

36. William having 50 cents gave twelve cents for a book and 15 cents for a slate. How many cents had he left?

The pupil should be drilled on exercises similar to the preceding until he can perform them with rapidity and correctness.

WRITTEN EXERCISES.

72. When any figure of the subtrahend is greater than the figure of the corresponding order of the minuend.

1. From 764 subtract 496.

OPERATION.

	6	15	14
Minuend	7	6	4
Subtrahend	4	9	6
Remainder	2	6	8

ANALYSIS.—Write the numbers as before. Begin at the right and subtract each order of units separately.

Since 6 units cannot be subtracted from 4 units, increase the 4 units by a unit from the next higher order, 1 ten or 10 units, which make 14 units. 6 units from 14 units leave 8 units, which write under the units.

Since 1 of the tens was united with the units, there are only 5 tens left. As 9 tens cannot be subtracted from 5 tens, increase the 5 tens by a unit from the next higher order, 1 hundred or 10 tens, which make 15 tens. 9 tens from 15 tens leave 6 tens, which write under the tens.

Since 1 of the hundreds was united with the tens, there are only 6 hundreds left. 4 hundreds from 6 hundreds leave 2 hundreds, which write under the hundreds. Hence the remainder is 268.

In like manner subtract and analyze :

2. 2452 from 6731.
3. 1863 from 3248.
4. 3462 from 7320.
5. 3842 from 4030.
6. 1040 from 5320.
7. 2765 from 4723.

RULE.—I. *Write the subtrahend under the minuend, placing units of the same order under each other.*

II. *Begin at the right and subtract successively the units of each order of the subtrahend, from the units of the corresponding order of the minuend, and write the result beneath.*

III. *If the units of any order of the subtrahend be greater than the units of the corresponding order of the minuend, increase the latter by* 10 *and subtract; then diminish by* 1 *the units of the next higher order in the minuend and proceed as before.*

PROOF.—*Add the remainder to the subtrahend, and if the sum is equal to the minuend, the work is correct.*

Instead of *diminishing* by 1 the units of the next higher order in the *minuend*, we may *increase* by 1 the units of the next higher order in the *subtrahend*.

Subtract

8. 3420 from 5146.
9. 4076 from 4897.
10. 9361 from 24864.
11. 7036 from 13560.

From

12. $40562, take $2576.
13. 9371 miles, take 7645 miles.
14. 10376 men, take 3046 men.
15. 60401 rods, take 5392 rods.

How many years from the date of each of the following events to the present year.

16. America was discovered in 1492.

17. American Independence was declared in 1776.

18. George Washington was born in 1732.

19. First steamboat was built by Robert Fulton in 1808.

20. Cotton was first planted in the United States in 1769.

21. Newspapers were first published in 1630.

22. Glass windows first used in England in 1180.

23. Printing was invented in 1441.

24. The subtrahend is 16034 and the remainder 30421. What is the minuend?

25. The difference is 40239, and the minuend is 206417. What is the subtrahend?

26. In 1870 Texas planted 900937 acres of cotton, and Louisiana planted 920700 acres. Which planted the more, and how much?

27. The Bible contains 31173 verses, of which 23214 compose the Old Testament. How many does the New Testament contain?

28. From 2 million, subtract 2 thousand.

29. From 101 thousand, subtract 56 hundred.

30. Alaska contains 577390 square miles, and Texas 274356. Which is the larger, and how much?

Find the difference between

31. 6731 and 3204.
32. 2567 and 4320.
33. 7346 and 7932.
34. 32760 and 9406.
35. 46710 and 42062.
36. 12501 and 24356.
37. 55006 and 61128.
38. 200316 and 18491.

39. In 1870, Mississippi raised 725000 bales of cotton, and Georgia 495000 bales. How many more bales did Mississippi raise than Georgia?

40. 941000 — 5007 = ?
41. 234100 — 9970 = ?
42. 78675 — 29486 = ?
43. 107356 — 18200 = ?
44. 865432 — 7520 = ?
45. 539864 — 29137 = ?

REVIEW.

ORAL EXAMPLES.

73. 1. The sum of two numbers is 40 and one of them is 12. What is the other?

2. The difference of two numbers is 15 and the greater is 27: what is the less?

3. The difference of two numbers is 22, and the less number is 25: what is the greater?

4. Charles bought a slate for 22 cents, and gave in payment 50 cents. How much change should he receive?

5. Mary gave 6 cents for a spool of thread, and 10 cents for some butons. How much change should she receive for a 25 cent stamp?

6. A man sold 14 sheep, then bought 10, and then had 16. How many had he at first?

7. Lillie had 45 cents, and gave 10 cents for a pencil, and 24 cents for a book. How much had she left?

8. A man having $40, paid $15 for a coat, $10 for a vest, and $9 for a pair of boots. How much had he left?

9. A farmer gave a cow and $15 in money for a wagon valued at $60. How much did he get for his cow?

10. $(28 - 12) + 16 = ?$
11. $40 - 18 = 12 + ?$
12. $32 + 10 = 50 - ?$
13. $57 - 25 = \ ? + ?$
14. $52 - 24 + 31 = ?$
15. $9 + (28 - 15) = 40 - ?$
16. $60 - (30 - 19) = 36 + ?$
17. $15 + 16 + 27 = \ ? + 20$

WRITTEN EXAMPLES.

74. 1. A man owing $1250, paid at one time $450, and at another $327. How much did he still owe?

2. A drover having 2370 sheep, sold 1575 and then bought 649 more. How many had he then?

3. What is the difference between 847 + 362, and 302 + 567?

4. What is the sum of 1240 and 376 increased by the difference between 982 and 327?

5. From the difference between 975 and 10272, subtract the sum of 392 and 276.

6. What two numbers equal 27842, if one of the numbers equal 13042 — 875?

7. What number increased by the difference between 7032 and 4100, will make 14062 + 458?

8. What is the result of 8472 + 602 + 72, diminished by 472 + 1020 + 125?

9. Mr. Jones having $12470, paid $4070 for a house, $927 for furniture, and $1000 for improvements. How much money had he left?

10. A gentleman possessing $47840, at his death gave to each of his two sons $18755, and the remainder to his daughter. How much did the daughter receive?

11. A grain dealer bought 5075 bushels of wheat at one time, and 12300 bushels at another. He sold 3763 bushels to one customer, and 4000 bushels to another. How many bushels had he left?

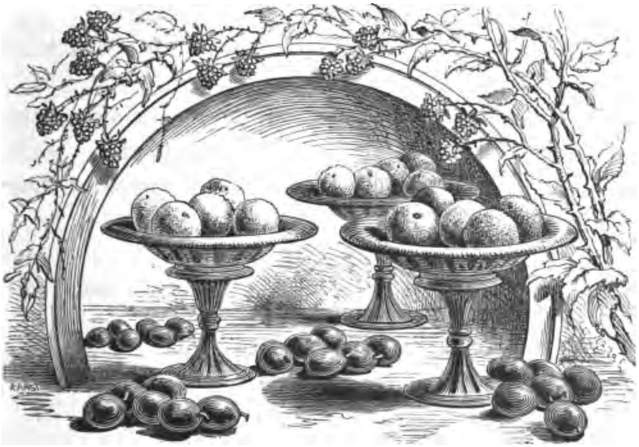
12. The sum of four numbers is 847621; the first is 44000, the second is 12375 less than the first; the third is 2400 more than the second: what is the fourth?

13. If a man's income is $10000 a year, and he pays $1500 for rent, $1250 for provisions, $375 for taxes, and $1463 for other expenses, how much will he have left?

14. A man bought a western farm for $22700; he expended $4375 for improvements, and $2862 for stock: he sold the whole for $32000. Did he gain or lose, and how much?

REVIEW QUESTIONS.

75. Define Subtraction. Difference, or Remainder. Minuend. Subtrahend. Sign of Subtraction. Show its meaning and use. Define Parenthesis—Vinculum—illustrate use. Principles of Subtraction 1, 2. Rule. Proof.



MULTIPLICATION.

76. 1. There are 5 oranges in 1 dish; how many are there in 3 dishes? 5 and 5 and 5 are how many? Three 5's, or 3 times 5 are how many?

2. If there are 3 berries in 1 cluster, how many berries in 5 clusters? 3 + 3 + 3 + 3 + 3 are how many? Five 3's, or 5 times 3 are how many? Count by 3's to 15? How many 3's are 15?

3. There are 6 plums in 1 group; how many plums in 4 groups? 6 + 6 + 6 + 6 equals how many? 4 times 6 are how many? Count by 6's to 24. How many 6's are 24.

4. There are 3 feet in 1 yard; how many feet in 3 yards? In 4 yards? In 5 yards? In 7 yards?

5. There are 7 days in 1 week; how many days in 4 weeks? 7 + 7 + 7 + 7 are how many? Four 7's, or 4 times 7 are how many?

6. What will 4 tons of coal cost, at \$6 a ton?

Analysis.—Since 1 ton cost \$6, 4 tons will cost 4 times \$6, or \$24. Hence 4 tons of coal will cost \$24.

7. If a man travel 4 miles an hour, how far will he travel in 5 hours? In 6 hours? In 7 hours?

8. At \$7 dollars a barrel, what will 3 barrels of flour cost? 5 barrels? 8 barrels?

9. How many are 6 times 7? 7 times 6?

10. How many are 8 + 8 + 8 + 8, or 4 times 8? 8 times 4? 3 times 8? 8 times 5?

11. How many are three 9's? Nine 3's? Seven 4's? Four 7's? Six 10's? Eight 8's?

Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

DEFINITIONS.

77. ***Multiplication*** is the process of taking one of two numbers as many times as there are units in the other. Or,

It is a short method of *adding equal* numbers.

78. ***The Multiplicand*** is the number to be multiplied.

79. ***The Multiplier*** is the number by which to multiply. It shows *how many times* the multiplicand is to be taken.

80. ***The Product*** is the result obtained by multiplying.

The multiplicand and multiplier are called the *factors* of the product.

81. ***The Sign of Multiplication*** is ×. It is read *times*, or *multiplied by;* thus, 8 × 6 is read 8 *multiplied by* 6, or 6 *times* 8, and means that 8 and 6 are to be multiplied together.

Since changing the order of the factors does not change the result, 8 × 6 may be read 6 *times* 8, or 8 *times* 6.

EXERCISES ON THE TABLE.

8 times 7 are how many?
5 times 4 are how many?
3 times 5 are how many?
3 times 4 are how many?
8 times 6 are how many?
7 times 8 are how many?
5 times 3 are how many?

2 times 4 are how many?
5 times 9 are how many?
9 times 8 are how many?
3 times 3 are how many?
2 times 3 are how many?
7 times 4 are how many?
6 times 8 are how many?

3 × 7 = ?
8 × 9 = ?
0 × 8 = ?
5 × 6 = ?
7 × 3 = ?
6 × 6 = ?
5 × 4 = ?
7 × 5 = ?
4 × 8 = ?

7 × 7 = ?
9 × 6 = ?
7 × 9 = ?
7 × 10 = ?
6 × 9 = ?
4 × 8 = ?
8 × 8 = ?
4 × 9 = ?
7 × 6 = ?

7 × 8 = ?
4 × 9 = ?
6 × 5 = ?
3 × 9 = ?
9 × 9 = ?
7 × 3 = ?
9 × 5 = ?
5 × 9 = ?
5 × 5 = ?

3 × 9
4 × 10
8 × 7
6 × 11
10 × 5
9 × 8
11 × 7
7 × 10
8 × 6
9 × 7

6 × 7
5 × 8
10 × 6
4 × 12
7 × 9
8 × 11
12 × 3
9 × 9
8 × 10
10 × 8

5 × 12
0 × 9
9 × 0
10 × 9
10 × 10
5 × 11
8 × 8
3 × 11
6 × 12
12 × 6

10 × 11
9 × 11
7 × 12
12 × 8
10 × 12
11 × 11
12 × 10
9 × 12
11 × 11
12 × 12

ORAL EXERCISES.

82. 1. 6 times 7 are 42. Which of these numbers is the multiplicand? The multiplier? The product? The factors?

2. How many are 5 times 6 yards? 4 times 7 books?

3. How many are 6 times $9? 7 times 8 pounds.

4. Multiply from 0 times 2, to 12 times 2.

Operation.—0 times 2 is 0, 1 times 2 is 2, 2 times 2 are 4, 3 times 2 are 6, 4 times 2 are 8, 5 times 2 are 10, and so on.

5. Multiply back from 12 times 2 to 0 times 2.

Operation.—12 times 2 are 24, 11 times 2 are 22, 10 times 2 are 20, 9 times 2 are 18, 8 times 2 are 16, and so on.

6. Multiply from 0 times 3 to 12 times 3, and back.
7. Multiply from 0 times 4 to 12 times 4, and reverse.
8. Multiply from 0 times 5 to 12 times 5, and reverse.
9. Multiply from 0 times 6 to 12 times 6, and reverse.
10. Multiply from 0 times 7 to 12 times 7, and reverse.
11. Multiply from 0 times 8 to 12 times 8, and reverse.
12. Multiply from 0 times 9 to 12 times 9, and reverse.
13. At $5 a cord, what will 8 cords of wood cost? 7 cords? 9 cords? 11 cords?
14. There are 7 days in 1 week: how many days in 5 weeks? 8 weeks? 7 weeks? 12 weeks?
15. How many are 7 times $6, plus $3?
16. How many are 5 times 9 pounds, plus 7 pounds?
17. How many are 8 times 7 feet, plus 6 feet?
18. How many are 4 times 12 rods, minus 10 rods?
19. How many are 7 times $10, minus $8?
20. George had 10 cents and James had 5 times as many. How many had both?
21. Henry gave 4 cents each for 5 oranges. How much change should he receive for 25 cents.

83. Principles.—1. *The multiplier is always regarded as an abstract number.*

2. *The multiplicand and product are like numbers, and may be either concrete or abstract.*

In examples containing concrete numbers, the concrete number is the *true multiplicand*, but when it is the smaller, it is often, for convenience, used abstractly, as the multiplier.

WRITTEN EXERCISES.

84. When the multiplier consists of but one figure.

1. How many are 4 times 63 ?

1ST OPERATION.

```
     63
     63
     63
     63
    ---
Sum 252
```

ANALYSIS.—By this operation the result is obtained by Addition.

First find the sum of four 3's, or 4 *times* 3 units, which are 12 units, equal to 1 ten and 2 units. Write the 2 units in units place, and reserve the 1 ten to be added to the sum of the tens.

Next the sum of four 6's, or 4 *times* 6 tens, are 24 tens, plus 1 ten reserved make 25 tens, or 2 hundreds and 5 tens, which write in hundreds and tens places. Hence the sum is 252.

2D OPERATION.

```
Multiplicand  63
Multiplier     4
             ---
Product      252
```

ANALYSIS.—By this operation, which is much shorter, the multiplicand 63 is written but once, and as it is to be taken 4 times, write the multiplier 4 under it, and commence at the right to multiply. 4 times 3 units are 12 units, or 1 ten and 2 units. Write the 2 units in units place and reserve the 1 ten to add to the product of the tens.

Next 4 times 6 tens are 24 tens, plus 1 ten reserved are 25 tens, or 2 hundreds and 5 tens, which write in the hundreds and tens places. Hence the *product* is 252, the same as the *sum* in the first operation.

Solve by both methods :

2. 3 times 74.
3. 4 times 55.
4. 5 times 134.
5. 6 times 268.
6. 7 times 406.
7. 8 times 325.
8. 6 times 503.
9. 9 times 232.

10. Multiply 4632 by 4; by 5; by 6; by 7.

11. Multiply 5034 by 6; by 7; by 5; by 8.

What is the product

12. Of $3252 by 7? by 4? by 8? by 6? by 5?

13. Of 12364 rods by 3? by 5? by 7? by 8?

14. Of 230460 pounds by 4? by 6? by 8? by 5? by 9?

15. What will be the cost of 231 tons of coal at $9 a ton?

Although $9 is the *true multiplicand*, for convenience we use it as the multiplier, and 231 as the multiplicand (83, Note), but the *product* is *dollars* because the true multiplicand is dollars.

16. What cost 2140 barrels of flour at $8 a barrel?

17. What cost 870 cords of wood at $5 a cord?

18. What cost 379 pounds of nails at 6 cents a pound?

19. At 9 cents a pound, what will 1872 pounds of sugar cost?

20. At $98 a month, how much can a man earn in 8 months?

21. In 1 mile are 5280 feet; how many feet in 9 miles?

22. What cost 8 building lots at $2075 each?

23. What cost 7 houses at $3560 each?

ORAL EXERCISES.

85. 1. Multiply from 0 times 10 to 12 times 10 and reverse.

2. Multiply from 0 times 11 to 12 times 11, and reverse.

3. Multiply from 0 times 12 to 12 times 12, and reverse.

4. What will 12 yards of ribbon cost at 8 cents a yard? At 9 cents? At 10 cents? At 12 cents?

5. At 10 cents a pound, what will 9 pounds of sugar cost? 10 pounds? 11 pounds? 12 pounds?

6. If 5 men can do a piece of work in 9 days, how many days will it take 1 man to do it?

Analysis.—It will take 1 man 5 times as many days as it will 5 men; 5 times 9 days are 45 days. Hence it will take 1 man 45 days.

7. If 8 men can build a wall in 10 days, in how many days can 1 man build it?

8. If two barrels of flour last 8 persons 3 months, how long will they last one person?

9. If 7 horses consume twelve bushels of oats in 5 days, how many horses would consume the same in 1 day?

10. If a man travel 23 miles in 1 day, how far can he travel in 4 days?

Analysis.—In 4 days he can travel 4 times 23 miles. 23 is equal to 2 tens and 3 units; 4 times 2 tens are 8 tens, and 4 times 3 units are 12 units, or 1 ten and 2 units, added to 8 tens, make 9 tens and 2 units, or 92. Hence he can travel 92 miles in 4 days.

11. What will be the cost of 7 bushels of apples at 25 cents a bushel?

12. At $35 each, what will be the cost of 5 cows?

13. What cost 8 pounds of coffee at 42 cents a pound?

14. In an orchard are 9 rows of trees, and 26 trees in each row; how many trees in the orchard?

15. At $28 a month, what will be the cost of 6 months board?

16. What cost 9 pounds of coffee, at 28 cents a pound?

17. If the cars run 25 miles an hour, how far will they run in 7 hours?

18. What cost 12 barrels of pork at $15 a barrel?

19. When eggs are 18 cents a dozen, what will be the cost of 5 dozen? Of 7 dozen? Of 10 dozen?

WRITTEN EXERCISES.

86. When the multiplier consists of two or more figures.

1. Multiply 736 by 45.

OPERATION.

Multiplicand	736		
Multiplier	45		
1st partial product	3680	=	5 times the multiplicand.
2d partial product	2944	=	40 times the multiplicand.
Entire product	33120	=	45 times the multiplicand.

ANALYSIS.—Write the multiplier under the multiplicand.

Since 45 is composed of 5 units and 4 tens, 45 times any number must be equal to 5 times the number, *plus* 4 tens, or 40 times the number. 5 times 736 are 3680, the first *partial* product.

4 tens times 6 units are 24 *tens*, or 2 hundreds and 4 tens. Write the 4 tens in tens place in the second *partial* product and reserve the 2 hundreds to add to the product of hundreds.

4 tens times 3 tens are 12 *hundreds*, plus 2 hundreds reserved are 14 hundreds, or 1 thousand and 4 hundreds. Write the 4 hundreds in hundreds place in the second *partial* product, and reserve the 1 thousand to add to the product of thousands.

4 tens times 7 hundreds are 28 *thousands*, plus 1 thousand reserved are 29 thousands, or 2 tens of thousands and 9 thousands, which write in tens of thousands and thousands places.

The sum of the *partial* products, is the *entire product* 33120.*

In multiplying, the pupil should learn to pronounce partial results only, as in Addition, performing the operations mentally.

In like manner multiply

2. 325 by 18.
3. 462 by 24.
4. 607 by 36.
5. 584 by 44.
6. 4762 by 28; by 37.
7. 3087 by 46; by 29.
8. 7624 by 65; by 72.
9. 5607 by 83; by 96.

RULE.—I. *Write the multiplier under the multiplicand with units of the same order under each other.*

When the multiplier consists of one figure:

II. *Begin at the right and multiply the units of each order of the multiplicand by the multiplier. Write in the product the units of each result, and reserve the tens to add to the next result.*

When the multiplier consists of more than one figure:

III. *Multiply the multiplicand by the units of each order of the multiplier successively, beginning at the right, and write the right-hand figure of each partial product under the order of the multiplier used.*

The sum of the partial products will be the required product.

PROOF.—*Review the work carefully, or multiply the multiplier by the multiplicand; if the results are the same, the work is probably correct.*

When there are ciphers between significant figures of the multiplier, multiply by the significant figures only, since the product of any number by 0 is 0.

10. Multiply 4372 by 25; by 48; by 67; by 128.
11. Multiply 3065 by 72; by 84; by 93; by 204.
12. Multiply 36204 by 414; by 306; by 555; by 107.

What is the product of

13. 47672 by 234.
14. 302076 by 603.
15. 73008 by 2036.
16. 430605 by 4005.
17. 290361 by 30406.
18. 216 times 2784 bushels.
19. 435 times 68470 feet.
20. 2406 times $3060724.
21. 8042 times 130065 rods.
22. 12094 times 98070 pounds.

23. Multiply 4070625 by 3006; by 12084; by 67125.

24. Multiply 29041300 by 864; by 18641; by 27608.

25. What will be the cost of building 279 miles of railroad at $27384 a mile?

26. A crop of cotton was put up in 340 bales, each bale containing 596 pounds. What was the weight of the entire crop?

27. What is the value of 108 building lots at $1896 each?

28. What cost 257 yoke of oxen at $175 a yoke?

29. What cost 428 horses at $284 each?

30. In 1 ream of paper are 480 sheets. How many sheets in 217 reams?

31. How much will it cost to build a line of telegraph from New York to Boston, the distance being 236 miles, at $1270 a mile?

32. If a cotton mill manufacture 658 yards of cloth in a day, how many yards can it make in 309 days?

33. $127 \times 48 \times 12 = ?$
34. $325 \times 25 \times 127 = ?$
35. $769 \times 204 \times 0 = ?$
36. $2070 \times 68 \times 102 = ?$
37. $3709 \times 432 \times 608 = ?$
38. $376 \times 206 \times 244 = ?$
39. $932 \times 95 \times 462 = ?$
40. $2008 \times 0 \times 187 = ?$
41. $7306 \times 306 \times 241 = ?$
42. $29087 \times 736 \times 802 = ?$

43. A merchant bought 29 pieces of cloth, each piece containing 47 yards, at $8 a yard. What was the cost of the whole?

44. What will be the cost of 45 sets of Cyclopedias, each set containing 16 volumes, at $7 a volume?

45. How many yards of sheeting in 57 bales, each bale containing 26 pieces, and each piece 44 yards?

46. An hotel-keeper at Saratoga has an average of 278 boarders for 9 weeks. If he receive $16 a week from each, what will be his total receipts?

47. In a cotton mill are 47 looms, and each loom can be made to weave 36 yards daily. At this rate, how many yards could be woven in 296 days?

48. A stock train consists of 18 cars, and each car contains 16 head of cattle, averaging in weight 794 pounds each. What is the total weight of the cattle?

49. Shipped 27 cases of books, each case containing 238 volumes, worth 96 cents a volume. What was their total value?

50. If it requires 126 tons of iron, worth $74 a ton, to build 1 mile of railroad, what will be the cost of sufficient iron to construct a road 237 miles in length?

51. A man's yearly income is $4500; he pays for rent $550, his other expenses amount to 5 times as much minus $200. How much does he save yearly?

87. To multiply by the factors of a number.

The ***Factors*** of a number, are the numbers which multiplied together will produce it. Thus, 4 and 7 are factors of 28; 2, 3, and 5 are factors of 30.

The pupil should be carefully taught to distinguish between *factors* and *parts* of a number. The *factors* are *multiplied,* but the *parts* are *added,* to produce a number.

Thus, 3 and 4, 2 and 6, 2, 2 and 3 are *factors* of 12; the *parts* of 12 are 8 and 4, 6 and 6, 7 and 5, etc.

88. PRINCIPLE.—*The product of any number of factors will be the same in whatever order they are multiplied.*

1. Multiply 128 by 36.

OPERATION.

$$36 = 6 \times 6, \quad \text{or } 9 \times 4, \quad \text{or } 12 \times 3.$$

128	128	128	128
36	6	9	12
768	768	1152	1536
384	6	4	3
4608	4608	4608	4608

It will be observed that the multiplicand, multiplied by the given multiplier, or by any *set of factors* into which it can be separated, produces the same result.

In like manner multiply

2. 3246 by 42 = 6 × 7.
3. 6074 by 56 = 7 × 8.
4. 3708 by 60 = 3 × 4 × 5.
5. 36706 by 72.
6. 50047 by 64.
7. 75034 by 108.

RULE.—I. *Separate the multiplier into two or more factors.*

II. *Multiply the multiplicand by one of the factors, the resulting product by another factor, and so continue until all the factors have been used. The last product will be the product sought.*

8. What will 75 horses cost, at $197 each ?

9. What will 60 acres of land cost, at $246 an acre ?

10. If the average weight of a bale of cotton be 506 pounds, what will be the total weight of 81 bales ?

11. If the receipts of a horse railroad average $807 a day, what will be the entire receipts for 96 days ?

89. When either the multiplicand or multiplier, or both, have ciphers on the right.

1. Multiply 325 by 100.

OPERATION.

$$\begin{array}{r} 325 \\ 100 \\ \hline 32500 \end{array}$$

ANALYSIS.—Since removing any order of units *one place* to the *left*, increases its value *ten times* (**38**), therefore annexing a cipher to a number multiplies it by 10. Annexing *two* ciphers multiplies it by 100, etc. Hence 325 × 100 = 32500, the product required.

2. Multiply 960 by 400.

OPERATION.

$$\begin{array}{r} 960 \\ 400 \\ \hline 384000 \end{array}$$

ANALYSIS.—960 = 96 × 10 and 400 = 4 × 100. First multiply together the factors 96 and 4, and then multiply their product, 384 by 10 × 100 or by 1000, by annexing *three* ciphers, which gives 384000, the required product.

What is the product

3. Of 436 by 10? by 100? by 1000? by 10000?

4. Of 2340 by 60? by 500? by 3000? by 2500?

RULE.—*To the product of the significant figures annex as many ciphers as there are ciphers on the right of both factors.*

Multiply

5. 73600 by 600.

6. 40500 by 250.

7. 731000 by 400.

8. 300700 by 20400.

9. 60300 by 15000.

10. 503700 by 40030.

11. At $9 a barrel, what will be the cost of 1000 barrels of flour?

12. At $160 an acre, what will 500 acres of land cost?

13. If there be 530 shingles in a bunch, how many shingles in 2700 bunches?

REVIEW.

ORAL EXAMPLES.

90. 1. What number multiplied by 6 is equal to the sum of 8, 4, 6, 9, and 3 ?

2. The sum of 10, 12, 8, 7, and 5 equals the product of 7 by what number ?

3. The sum of 24, 26, and 10 equals the product of what *two* numbers ? Of what *three* numbers ?

4. The product of what two factors is equal to 16 + 8 + 22 — 6 ?

5. The product of what three factors is equal to 44 — 32 + 18 ?

6. What is the sum of 2 times 3 multiplied by 4, and 5 times 4 multiplied by 3 ?

7. What is the difference between 9 times ten less 5, and 7 times 12 less 4 ?

8. What is the product of $15 + \overline{12 - 9}$ by 18 — 12 ?

9. Which is greater, 8 × 5 + 7, or 3 × 12 + 10 ?

10. How much more is 6 × 12 + 9, than 7 × 9 — 3 ?

11. How much less is 40 — (5 × 5) than 15 + 9 + 6 ?

12. James has 16 marbles, and Henry has 3 times as many less 10. How many have both ?

13. Bought 5 pounds of sugar at 12 cents a pound, and gave in payment a 1 dollar bill. How much change should be returned ?

14. A farmer sold a grocer 8 pounds of butter at 25 cents a pound, and received in payment 15 pounds of sugar at 10 cents a pound. How much was still due the farmer ?

15. A lady went shopping and bought 2 yards of silk at $2 a yard, 4 pairs of hose at half a dollar a pair, and a veil for $3, and gave in payment a 10 dollar bill. How much change should she receive?

16. If a boy earn $7 dollars a week, and spend $4 for board and washing, how much can he save in 12 weeks?

17. When lemons cost 6 cents, and oranges 5 cents each, how much will 7 lemons and 4 oranges cost?

18. Which will cost the more, and how much?

Find the required *term* in the following equations:

19. $4\times8+7-9=\ ?$
20. $5\times3\times0+8=\ ?$
21. $10-3+(5\times6)=\ ?$
22. $\overline{7\times6}+\overline{3\times6}=\ ?$
23. $(4\times10)-(4\times5)=?$
24. $8\times5+6+10=\ ?$
25. $50-(5\times0)+7=\ ?$
26. $\overline{8\times4}-\overline{6\times0}=\ ?$
27. $4\times10+3+\ ?\ =50$
28. $0\times7+\overline{8+\ ?}\ =40$
29. $12+4-8\times\ ?\ =32$
30. $9\times8+10-\ ?\ =75$
31. $48-20+(4\times\ ?)=40$
32. $61+7-(\ ?\times10)=48$

WRITTEN EXAMPLES.

91. 1. A flour merchant bought 240 barrels of flour for $1920, and sold the same at $10 a barrel. How much did he gain?

2. Bought 36 tubs of butter, each weighing 108 pounds; the tubs alone weigh 21 pounds each. What is the weight of the butter without the tubs?

3. From 227 × 87 × 15 subtract 406 × 327?

4. What sum must be added to 356 × 264 to make the amount 3447 less than 243042?

5. What is the difference between $26460-(487\times37)$ and $105\times246-12075$?

6. From $207300-(1236\times48)$ take $976\times98+10050$.

7. A man bought 45 acres of land at $38 an acre, and 76 acres at $47 an acre, and sold the whole at $45 an acre. Did he gain or lose, and how much?

8. A farmer exchanged 584 bushels of wheat at $2 a bushel, for 78 barrels of flour at $9 a barrel, and received the balance in money. How much money did he receive?

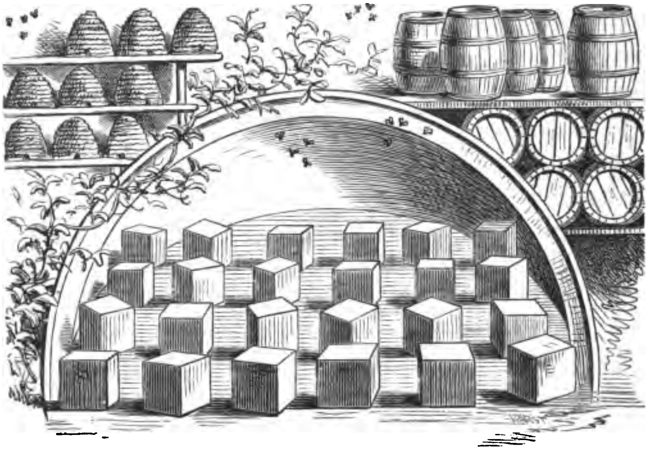
9. Two persons start from the same point and travel in opposite directions; one travels at the rate of 32 miles a day, the other at the rate of 39 miles a day. How far apart will they be in 14 days?

10. A man bought two farms; one of 157 acres at $26 an acre, and another of 138 acres at $34 an acre. He paid at one time $4725, and at another $1895; how much remained unpaid?

11. A planter sold 209 bales cotton at $76 a bale, and from the proceeds he bought 107 acres of land at $60 an acre, 18 mules at $75 each, and 4 span of horses at $218 a span. How much money had he left?

REVIEW QUESTIONS.

92. Define Multiplication. Multiplicand. Multiplier. Product. What are the multiplicand and multiplier called. Sign of multiplication? Show its use and meaning. What effect does changing the orders of the factors have upon the result? Principles 1, 2. Why do we often use the multiplicand for the multiplier? In examples containing concrete numbers, what is the true multiplicand? Rule. Proof. How are ciphers between significant figures of the multiplier treated? Define factors. Difference between factors and parts? Principle. Rule. How multiply by 10, 100, etc. How, when there are ciphers at the right of both factors. Rule.



DIVISION.

93. 1. How many times 2 horses are 8 horses?

2. How many times 3 hives are 9 hives?

3. How many times 4 bees are 12 bees? Are 16 bees?

4. How many times 5 barrels are 10 barrels?

5. How many shelves will be required to hold 9 bee-hives, if 3 hives be placed upon each shelf?

6. If 6 pears are put upon a plate, how many plates will be required for 18 pears?

7. How many times 7 cents in 21 cents? In 28 cents? In 35 cents?

8. From a pile of 24 blocks, how many groups of 4 blocks each can be made? Of 6 blocks each? Of 8 blocks each? Of 3 blocks each? Of 2 blocks each?

9. How many times 5 pounds are 30 pounds?

10. Thirty are how many times 6 ? How many times 5 ?

11. How many 4's in 28 ? 5's in 40 ? 6's in 42 ?

12. Distribute 20 cents equally among 4 boys. How many will each receive ?

13. Do you find how many times 4 boys are contained in 20 cents, or do you find *one* of the 4 *equal parts* of 20 ?

14. How do you find one of the 3 equal parts of a number ? One of the 5 equal parts ? One of the 6 equal parts ?

15. How *many times* 5 oranges are 30 oranges ? Is the result a concrete, or an abstract number ?

16. What is one of the 5 *equal parts* of 30 oranges ? Is the result a concrete, or an abstract number ?

DIVISION TABLE.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

DEFINITIONS.

94. ***Division*** is the process of finding *how many times* one number is contained in another; or of finding *one* of the *equal parts* of a number.

95. ***The Dividend*** is the number to be divided.

96. ***The Divisor*** is the number by which to divide.

97. ***The Quotient*** is the result of the division, and shows *how many times* the dividend contains the divisor.

The part of the dividend remaining when the division is not exact is called the *Remainder*, and must always be *less* than the divisor.

The division is said to be *exact* when there is no remainder.

98. ***The Sign of Division*** is ÷. It is read *divided by*. When placed between two numbers it shows that the one before it is to be divided by the one after it; thus, 36 ÷ 9 is read 36 *divided by* 9.

99. Division is also indicated by placing the dividend above the divisor with a line between them; thus, $\frac{24}{6}$ is is read 24 *divided by* 6.

EXERCISES ON THE TABLE.

6 in 36, how many times?
7 in 42, how many times?
9 in 81, how many times?
5 in 35, how many times?
8 in 72, how many times?
9 in 27, how many times?
6 in 54, how many times?

9 in 63, how many times?
6 in 12, how many times?
7 in 28, how many times?
7 in 49, how many times?
4 in 36, how many times?
8 in 64, how many times?
8 in 40, how many times?

$12 \div 4 = ?$
$35 \div 7 = ?$
$20 \div 5 = ?$
$24 \div 4 = ?$
$30 \div 6 = ?$
$36 \div 9 = ?$
$28 \div 7 = ?$

$32 \div 8 = ?$
$45 \div 5 = ?$
$42 \div 6 = ?$
$56 \div 8 = ?$
$63 \div 7 = ?$
$27 \div 3 = ?$
$42 \div 7 = ?$

$28 \div 4 = ?$
$36 \div 6 = ?$
$48 \div 8 = ?$
$45 \div 9 = ?$
$56 \div 7 = ?$
$54 \div 6 = ?$
$49 \div 7 = ?$

$63 \div 9 = ?$
$45 \div 5 = ?$
$70 \div 10 = ?$
$72 \div 8 = ?$
$72 \div 9 = ?$
$84 \div 7 = ?$
$55 \div 5 = ?$

$\frac{63}{7} = ?$
$\frac{56}{8} = ?$
$\frac{60}{10} = ?$
$\frac{88}{11} = ?$
$\frac{90}{9} = ?$
$\frac{48}{12} = ?$

$\frac{40}{8} = ?$
$\frac{81}{9} = ?$
$\frac{72}{12} = ?$
$\frac{96}{8} = ?$
$\frac{70}{7} = ?$
$\frac{84}{12} = ?$

$\frac{108}{12} = ?$
$\frac{96}{12} = ?$
$\frac{72}{6} = ?$
$\frac{120}{12} = ?$
$\frac{108}{9} = ?$
$\frac{77}{7} = ?$

$\frac{48}{12} = ?$
$\frac{99}{9} = ?$
$\frac{120}{10} = ?$
$\frac{88}{11} = ?$
$\frac{132}{11} = ?$
$\frac{144}{12} = ?$

100. Since one number is contained in another as many times as it can be *subtracted* from it, division may be regarded as a short method of performing several *subtractions* of a number.

Thus, $15 - 5 = 10$; $10 - 5 = 5$; $5 - 5 = 0$. We have performed *three* subtractions of 5; hence there are *three* 5's in 15, or 5 is contained in 15, 3 times.

101. Since one number is contained in another as many times as the one is a *factor* of the other, division may be regarded as the *reverse* of multiplication.

In multiplication *both factors* are given to find the *product;* in division *one factor* and the *product* (answering to the *dividend*) are given to find the *other factor,* which answers to the *quotient.*

Thus, $5 \times 3 = 15$, 5 being a factor of 15 *three* times; hence there are *three* 5's in 15, or 5 is contained in 15, 3 times.

102. The Object of Division is *two fold.*

First. To find *how many times* one number is contained in another of the *same kind.*

Ex. At 3 cents each, how many oranges can be bought for 12 cents?

Since 3 taken 4 times equals 12 ($3 \times 4 = 12$), it follows that 3 is contained in 12 *four times.*

Analysis.—As many oranges can be bought for 12 cents as 3 cents are contained times in 12 cents, which is 4 times; hence 4 oranges can be bought for 12 cents.

103. Second. To *separate* a given number into as many *equal parts* as there are units in another.

Ex. If 4 oranges cost 12 cents, what is the cost of 1 orange?

Since 3 taken 4 times equals 12, it follows that 3 is one of the four *equal parts* of 12 ($3 + 3 + 3 + 3 = 12$), and we say *one-fourth* of 12 is 3.

Analysis.—Since 4 oranges cost 12 cents, 1 orange costs *one-fourth* of 12 cents, which are 3 cents. Hence 1 orange costs 3 cents.

104. The *equal parts* into which a unit or whole thing is divided are sometimes called *fractions.*

The *names* of these equal parts of a unit vary according to the *number* of these parts; thus, *one-half* is one of *two* equal parts, *one-third* is one of *three* equal parts, *one-fourth* is one of *four* equal parts, into which the whole thing or number is divided.

So in like manner we have *fifths, sixths, sevenths, eighths, tenths, twelfths, twentieths,* etc.

105. These parts are expressed by writing the number denoting the *name* of the parts below a short horizontal line as a *divisor,* and the number denoting the *number* of parts taken or used, above the line as a *dividend;* thus,

$\frac{1}{3}$ signifies 1 *divided by* 3, and is read, *one-third.*
$\frac{3}{4}$ signifies 3 *divided by* 4, and is read, *three-fourths.*
$\frac{9}{20}$ signifies 9 *divided by* 20, and is read, *nine-twentieths.*

ORAL EXERCISES.

106. 1. Divide by 2, from 2 in 2 to 2 in 24.

OPERATION.—2 in 2 once; 2 in 4, 2 times; 2 in 6, 3 times; 2 in 8, 4 times; 2 in 10, 5 times, and so on to 2 in 24, 12 times.

In the same manner divide

2. By 3, from 3 in 3, to 3 in 36.
3. By 4, from 4 in 4, to 4 in 48.
4. By 5, from 5 in 5, to 5 in 60.
5. By 6, from 6 in 6, to 6 in 72.
6. By 7, from 7 in 7, to 7 in 84.
7. By 8, from 8 in 8, to 8 in 96.
8. By 9, from 9 in 9, to 9 in 108.
9. By 10, from 10 in 10, to 10 in 120.

The pupil may be required to *reverse* the above; thus, 2 in 24, 12 times; 2 in 22, 11 times; 2 in 20, 10 times; 2 in 18, 9 times, and so on to 2 in 2, once.

Also to *combine* the two: thus, 3 in 3, once; 3 in 6, 2 times, 2 in 6, 3 times; 3 in 12, 4 times, 4 in 12, 3 times, and so on to 3 in 36, 12 times, 12 in 36, 3 times.

10. If you separate a number into *two* equal parts, what is each part called? ***One-half,*** written $\frac{1}{2}$.

11. If you divide 8 peaches equally between 2 boys, how many do you give to each? What *part* of the whole do you give to each?

12. If you have 16 cents, and wish to give *one-half* of them to your sister, how many must you give her? What is *one-half* of 16?

13. What is one-half of 14? Of 18? Of 20? Of 24?

14. If you separate a number into *three* equal parts, what is each part called? ***One-third*** ($\frac{1}{3}$). Two parts? ***Two-thirds*** ($\frac{2}{3}$).

15. What is one-third of 9 cents? Of 15 days? Of 21 men? Of 24 pounds?

16. If you wish to distribute $12 among 3 families, what *part* will you give to each? How many dollars?

17. If a number be divided into *four* equal parts, what is each part called? ***One-fourth*** ($\frac{1}{4}$).

18. What are two parts called? ***Two-fourths*** ($\frac{2}{4}$). Three parts? ***Three-fourths*** ($\frac{3}{4}$).

19. What is one-fourth of 12? Of 16 rods? Of 24 miles? Of 40 pecks?

20. If 28 marbles be equally divided among 4 boys, what *part* of the whole will each receive? How many? How many marbles will 2 boys receive? 3 boys?

21. If a number be divided into *five* equal parts, what is each part called? ***One-fifth*** ($\frac{1}{5}$). Three parts? ***Three-fifths*** ($\frac{3}{5}$).

22. What is one-fifth of 15? 20? 25? 35? 40? 50?

23. If into *six* equal parts? ***One sixth*** ($\frac{1}{6}$).

24. What is one-sixth of 12? 18? 24? 36? 42? 48?

25. If into *seven* equal parts? ***One-seventh*** ($\frac{1}{7}$).

26. What is one-seventh of 14? 21? 35? 56? 63? 70?

27. If into *eight* equal parts? ***One-eighth*** ($\frac{1}{8}$).

28. What is one-eighth of 16? 24? 40? 48? 64? 72?

29. If into *nine* equal parts? ***One-ninth*** ($\frac{1}{9}$).

30. What is one-ninth of 18? 36? 45? 54? 72? 81?

31. If into *ten* equal parts? ***One-tenth*** ($\frac{1}{10}$).

32. What is one-tenth of 20? 40? 60? 80? 90? 100?

33. When lemons are 4 cents each, how many can be bought for 48 cents?

Analysis.—As many lemons as 4 cents are contained times in 48 cents, which are 12 times. Hence 12 lemons can be bought for 48 cents.

34. If a man travel 6 miles an hour, how long will it take him to travel 54 miles?

35. How many kegs containing 9 gallons each can be filled from a hogshead containing 63 gallons?

36. If a man drive 8 miles an hour, in what time will he drive 56 miles?

37. A farmer bought some sheep for $60, paying $5 a head. How many sheep did he buy?

38. At $9 a week, in what time will a man earn $36? $54? $72? $81? $108?

39. If 7 barrels of flour cost $63, what will 1 barrel cost?

Analysis.—Since 7 barrels cost $63, 1 barrel will cost 1 *seventh* of $63, or $9. Hence 1 barrel will cost $9.

40. If 6 barrels of flour cost $72, what will 1 barrel cost?

41. If a man travel 48 miles in 4 hours, how far does he travel in 1 hour?

42. What will be the cost of 1 ton of coal, if 8 tons cost $64?

43. If you divide 84 cents among 7 children, what part of the whole do you give to each? How many cents?

44. If a man can build 72 rods of fence in 8 days, how many rods can he build in 1 day?

45. If 9 dozens of eggs cost 108 cents, what is the cost of 1 dozen?

46. How many ploughs at $12 each can be bought for $84? For $108? For $120?

107. PRINCIPLES.—1. To find *how many times* one number is contained in another;

The divisor and dividend must be like numbers, and the quotient an abstract number.

2. To find one of the *equal parts* of a number;

The dividend and quotient must be like numbers, and the divisor an abstract number.

3. *The dividend is equal to the product of the divisor by the quotient, plus the remainder.*

WRITTEN EXERCISES.

108. When the divisor consists of one figure.

1. How many times is 6 contained in 972?

OPERATION.

```
Divisor. Dividend. Quotient.
     6 ) 9 7 2 ( 1 6 2
         6
         ---
         3 7
         3 6
         ---
           1 2
           1 2
           ---
```

ANALYSIS.—Write the divisor at the left and the quotient at the right of the dividend and begin at the left to divide.

6 is contained in 9 hundreds 1 hundred times and a remainder. Write the 1 hundred in the quotient. Multiplying the divisor 6 by the 1 hundred of the quotient produces 6 hundreds, which write under the hundreds of the dividend. Subtracting, there is a remainder of 3 hundreds, or 30 tens, to which annex the 7 tens of the dividend, making 37 tens.

6 is contained in 37 tens, 6 tens times and a remainder. Write the 6 tens in the quotient. Multiplying the divisor by the 6 tens of the quotient produces 36 tens, which write under the *partial* dividend 37 tens. Subtracting there is a remainder of 1 ten, or 10 units, to which unite the 2 units of the dividend, making 12 units.

6 is contained in 12 units, 2 times, which write in the quotient. Multiplying and subtracting as before, nothing remains. Hence the quotient is 162.

The solution of the preceding example may be abbreviated by what is termed *Short Division,* as follows:

OPERATION.

$6\,)\,\underline{9\;7\;2}$

Quotient $1\;6\;2$

ANALYSIS.—6 is contained in 9, 1 time and 3 remainder. 3 prefixed to 7 makes 37. 6 is contained in 37, 6 times and 1 remainder. 1 prefixed to 2 makes 12. 6 is contained in 12, 2 times. Hence the quotient is 162.

109. ***Short Division*** is the process of dividing by performing the multiplications and subtractions *mentally,* writing only the quotient, and is generally used when the divisor does not exceed 12.

In like manner divide and explain

(2.)	(3.)	(4.)	(5.)
$5\,)\,\underline{875}$	$4\,)\,\underline{6736}$	$6\,)\,\underline{9870}$	$7\,)\,\underline{8603}$

6. Divide 91624 by 8; 68240 by 5; 83524 by 7.
7. How many times is 8 contained in 2435?

OPERATION.

Divisor. Dividend.

$8\,)\,\underline{2\;4\;3\;5}$

Quotient $3\;0\;4\,\frac{3}{8}$

ANALYSIS.—Since 8 is not contained in 2 thousands any thousands times, unite the 2 thousands and 4 hundreds, making 24 hundreds. 8 is contained in 24 hundreds 3 hundreds times, which write in hundreds place in the quotient.

Next 8 is not contained in 3 tens any ten times, so write a cipher in tens place in the quotient, and unite the 3 tens and 5 units, making 35 units. 8 is contained in 35 units 4 times and 3 units remainder, which is written over the divisor and added as a part of the quotient. Hence the quotient is $304\frac{3}{8}$.

PROOF.—Multiplying the quotient 304 by the divisor 8, the product is 2432; and 2432 plus the remainder 3 equals the dividend 2435. (PRIN. 3.)

The remainder and dividend must always be *like numbers.*

In like manner,

8. Divide 24687 by 4; 30466 by 5; 42760 by 6.
9. Divide 72947 by 7; by 8; by 5; by 6; by 3.
10. Divide 213064 by 4; by 9; by 8; by 5; by 6.

Rule for Short Division.—I. *Write the divisor at the left of the dividend with a line between them.*

II. *Find how many times the divisor is contained in the least number of the left hand figures of the dividend that will contain it, and write the result beneath, for the first figure of the quotient.*

III. *If there be a remainder, regard it as prefixed to the next lower order in the dividend, and divide as before, and so continue to do till all the orders of the dividend have been used.*

IV. *If any partial dividend will not contain the divisor, write a cipher in the quotient and prefix the partial dividend to the next lower order and divide as before.*

V. *If there be at last a remainder, write it with the divisor beneath as a part of the quotient.*

Proof.—*Multiply the divisor by the whole number of the quotient and to the product add the remainder, if any. The result if correct will be equal to the dividend.*

11. Divide 184210 by 5; by 9; by 6; by 3.
12. Divide 870324 by 8; by 4; by 9; by 6.
13. Divide 203462 by 7; by 5; by 8; by 4.
14. Divide 360478 by 9; by 7; by 3; by 8.
15. Divide 403076 by 8; 120050 by 7; by 9.
16. Divide 3761201 by 9; 897063 by 6; by 7.

How many times is

17. 7 contained in 14623? in 4354? in 7245?
18. 9 contained in 43706? in 8406? in 9187?
19. 8 contained in 20371? in 5637? in 13476?
20. 6 contained in 84370? in 6708? in 23456?

What is

21. 1 fifth of $7420?
22. 1 sixth of 1234 pounds?
23. 1 eighth of 27304 days?
24. 1 ninth of 9126 feet?

What is

25. 1 sixth of $8760?
26. 1 third of 4731 men?
27. 1 seventh of 14672 miles?
28. 1 twelfth of 60132 acres?

29. How many tons of coal at $7 a ton, can be bought for $87605?

30. If 36314 bushels of grain be put into 6 bins of equal size, how many bushels must each bin contain?

31. A gentleman divided his estate, worth $42641, equally among his wife and 5 children. How much did each receive?

ORAL EXERCISES.

110. 1. The quotient of two numbers is 20 and the divisor 8: what is the dividend?

2. The dividend is 120, and the quotient 10: what is the divisor?

3. The quotient is 9, the remainder 7, and the divisor 12: what is the dividend?

4. James sold 6 melons for 10 cents each, and gave all the money but 12 cents for 4 books. How much did the books cost apiece?

5. Ella gave 4 ten-cent pieces for 3 yards of ribbon, and received 4 cents in change. How much did the ribbon cost a yard?

6. A man bought 4 pounds of meat, paid 50 cents and received 10 cents in change. How much a pound did he pay for the meat?

7. Mary bought 6 yards of cambric, paid 75 cents and received 9 cents in change. How much a yard was the cambric?

8. If 4 barrels of flour cost $36, what will 7 barrels cost?

Analysis.—One barrel will cost 1 fourth of $36, or $9; and 7 barrels will cost 7 times $9, or $63. Hence 7 barrels will cost $63.

9. What cost 12 yards of cloth, if 6 yards cost $24?

10. If a stage run 45 miles in 5 hours, how far will it run in 7 hours? in 9 hours? in 12 hours?

11. If 9 weeks board cost $72, what will 7 weeks cost?

12. What will 10 yards of ribbon cost, if 6 yards cost 72 cents?

13. How many barrels of apples at $3 a barrel, will pay for 4 tons of coal at $6 a ton?

Analysis.—At $6 a ton, 4 tons of coal will cost 4 times $6, or $24. It will require as many barrels of apples to pay for the coal as $3 is contained times in $24, which is 8 times. Hence 8 barrels of apples will pay for 4 tons of coal.

14. How long will it take a man to travel 60 miles, if he travel 48 miles in 12 hours?

15. Bought 4 cords of wood at $4 dollars a cord, and 2 tons of hay at $10 a ton. How many days labor at $3 a day will pay for both?

16. If 5 coats can be cut from 20 yards of cloth, how many can be cut from 32 yards?

17. If 15 yards of cloth cost $60, how much will 7 yards of the same cloth cost?

Find the second *member* of the following equations :

18. $\overline{36+9}+\overline{40-16}=?$
19. $25+\overline{84\div12}-\overline{0\times8}=?$
20. $\overline{75\div15}\times\overline{30-12}=?$
21. $16+12-9\times\overline{48\div16}=?$
22. $(12\times3)+(9\times2)\div6=?$
23. $(9\times10)-(5\times6)\div12=?$
24. $(8\times12)-(63\div7)=?$
25. $\overline{30\div6}\times0+5=?$
26. $\overline{72\div8}\times7-10=?$
27. $48-12\times(48\div12)=?$

WRITTEN EXERCISES.

111. When the divisor exceeds 12.

1. Divide 3749 by 18.

OPERATION.

Divisor. Dividend. Quotient.

$$\begin{array}{l} 18)3749(208\tfrac{5}{18} \\ \underline{36} \\ 149 \\ \underline{144} \\ 5 \text{ Rem.} \end{array}$$

ANALYSIS.—Since 18 is not contained in 3 thousands any thousands times, unite the 3 thousands with the 7 hundreds, making 37 hundreds.

18 is contained in 37 hundreds, 2 hundreds times and a remainder. Write the 2 hundreds in the quotient, and multiply the divisor 18 by this quotient figure 2 hundreds, and write the product 36 hundreds, or 3 thousands 6 hundreds under units of the same order in the dividend, and subtract, leaving a remainder of 1 hundred. To this 1 hundred annex the 4 tens of the dividend, making 14 tens. 18 is not contained in 14 tens any tens times, so write a cipher in the quotient and bring down the 9 units of the dividend, making 149 units.

18 is contained in 149 units, 8 times and a remainder. Write the 8 units in the quotient and multiply the divisor by this last quotient figure, and write the product 144 under units of the same order in the partial dividend and subtract, leaving a remainder of 5.

Write the remainder over the divisor as a part of the quotient. Hence the quotient is $208\frac{5}{18}$.

Proof is the same as in Short Division.

112. ***Long Division*** is the process of dividing when the steps of the solution *are all written.*

2. What is the quotient of 87564 divided by 38.

OPERATION.	PROOF
Divisor. Dividend. Quotient.	
38)87564($2304\frac{12}{38}$	2304 Quotient.
76	38 Divisor.
115	18432
114	6912
164	87552
152	12 Remainder.
12 Remainder.	87564 Dividend.

Rule for Long Division.—I. *Write the divisor at the left of the dividend as in short division.*

II. *Find how many times the divisor is contained in the least number of left-hand orders of the dividend that will contain it, and write the result at the right of the dividend for the first figure of the quotient.*

III. *Multiply the divisor by this quotient figure, subtract the product from this partial dividend, and to the remainder annex the next lower order of the dividend.*

IV. *Divide as before until all the orders of the dividend have been used.*

V. *If any partial dividend will not contain the divisor, write a cipher in the quotient and bring down the next order of the dividend, and proceed as before.*

VI. *If there be at last a remainder, write it with the divisor underneath as part of the quotient.*

Proof.—*The same as in short division.*

1. If any remainder be *equal to* or *greater* than the divisor, the quotient is *too small* and must be *increased.*

2. If the product of the divisor by the quotient figure be *greater* than the *partial* dividend, the quotient is *too large* and must be *diminished.*

The operation in Long Division consists of five principal steps, viz.:

1st. Write down the numbers. 3d. Multiply.
2d. Find how many times. 4th. Subtract.
5th. Bring down another figure.

How many times

3. Is 13 contained in 169 ? in 273 ? in 4550 ?
4. Is 14 contained in 196 ? in 1596 ? in 3304 ?
5. Is 15 contained in 2630 ? in 7465 ? in 18306 ?
6. Is 17 contained in 238 ? in 2465 ? in 50571 ?
7. Is 18 contained in 7362 ? in 10404 ? in 11052 ?

8. At $17 a barrel, how many barrels of molasses can be bought for $8024 ?

9. How many acres of land can be bought for $8370, at $18 an acre ?

10. How many barrels of pork at $22 a barrel, can be bought for $35354 ?

11. A man paid $170352 for 36 city lots. What was the average cost per lot ?

12. In 94185 yards of sheeting are how many pieces, each containing 45 yards ?

13. A planter raised 43240 pounds of cotton on 94 acres. What was the average number of pounds to the acre ?

14. Divide 35280 by 45 ; by 49 ; by 48 ; by 55.

15. Divide 1276704 by 36 ; by 33 ; by 39 ; by 42.

16. Divide 95762 by 114 ; by 152 ; by 171 ; by 204.

17. Divide 637420 by 125; by 142; by 265; by 184.

18. A drover received $26460 for 196 head of cattle. How much was their average value a head?

19. If 43776 pounds of cotton be packed in 96 bales, what is the average weight of each bale?

20. Paid $7500 for 125 acres of land. What was the cost an acre?

21. How long will it take a vessel to sail from New York to China, if the distance be 9028 miles, and the ship sail 148 miles a day?

Divide
22. 40278 by 75.
23. 765431 by 96.
24. 300428 by 324.
25. 1560305 by 562.
26. 2071491 by 703.
27. 43210040 by 1236.
28. 56030169 by 2004.

Divide
29. 802671 by 605.
30. 2403082 by 2431.
31. 61700504 by 3042.
32. 109426051 by 7315.
33. 237000675 by 63041.
34. 4280960342 by 15003.
35. 31082600578 by 407053.

113. When the divisor is 10, 100, 1000, etc.

1. Divide 1342 by 10.

OPERATION.

$1|0\overline{)134|2}$

134 . . 2 Rem.

Or 134$\frac{2}{10}$ Quotient.

ANALYSIS.—Since removing any order of units *one place* to the *right diminishes* its value *ten times* (**39**), hence by cutting off, or taking away the right-hand figure of a number, each of the remaining figures is removed one place to the right, and consequently the value of each is diminished *ten* times, or *divided by* 10.

For similar reasons cutting off *two* figures divides by 100; cutting off *three* figures divides by 1000, and so on.

What is the quotient

2. Of 4727 divided by 10? 100?
3. Of 3062 divided by 100? 1000?
4. Of 137426 divided by 1000? 10000?
5. Of 20310684 divided by 10000? 100000?

RULE.—*From the right hand of the dividend cut off as many figures as there are ciphers in the divisor. The remaining figures are the quotient, and those cut off, the remainder.*

6. Paid $94 for 10 thousand bricks. What was the cost a thousand?

7. How many watches at $100 each, can be bought for $6500?

8. Bought 1000 barrels of flour for $9750. What was the cost a barrel?

114. When the divisor has ciphers on the right.

1. Divide 36254 by 800.

OPERATION.

$$\begin{array}{r|l} 8|00)\underline{362} & \underline{54} \\ 45\ .\ .\ & 254 \text{ Rem.} \end{array}$$

Or $45\frac{254}{800}$ Quotient.

ANALYSIS.—Resolve 800 into the factors 100 and 8. First divide by 100 by cutting off the two right-hand figures of the dividend (**113**), and the result is a quotient of 362, and a remainder of 54.

Next divide 362 by 8, and the result is a quotient of 45, and a second remainder of 2 hundreds, which prefixed to the first remainder 54 units, gives a *true* remainder of 254. Hence the required quotient is $45\frac{254}{800}$.

In like manner divide

2. 10800 by 900.
3. 47620 by 420.
4. 378000 by 1200.
5. 270670 by 23000.

RULE.—I. *Cut off the ciphers from the right of the divisor, and as many figures from the right of the dividend.*

II. *Divide the remaining part of the dividend by the remaining part of the divisor.*

III. *Prefix the remainder, if any, to the figures cut off, and the result will be the true remainder.*

Divide
6. 146200 by 1400.
7. 720460 by 4000.
8. 925500 by 2500.

Divide
9. 783180 by 1800.
10. 1617200 by 75000.
11. 6342034 by 64000.

12. How many bales, each weighing 510 pounds, can be made of 76500 pounds of cotton?

13. How many horses at $260 each, can be bought for $19500?

REVIEW.

ORAL EXAMPLES.

115. 1. The sum of two numbers is 25, and one of the numbers is 9: what is the other?

2. The difference of two numbers is 12, and the smaller is 10: what is the larger?

3. The difference of two numbers is 9, and the larger is 30: what is the smaller?

4. The product of two numbers is 108, and one of the numbers is 12: what is the other?

5. The quotient of two numbers is 30 and the divisor 12: what is the dividend?

6. The dividend is 120 and the quotient 12: what is the divisor?

7. The quotient is 12, the remainder 6, and the divisor 10: what is the dividend?

8. A boy bought 12 oranges for 42 cents, and sold them so as to gain 18 cents. For how much a piece did he sell them?

9. How many pounds of rice at 6 cents a pound, will cost as much as 8 pounds of meat at 9 cents a pound?

How many

10. In each of 6 equal parts of $(26+18-\overline{7\times 2})$?

11. In each of 9 equal parts of $(\overline{8\times 10}-\overline{6+4})$?

12. In each of 10 equal parts of $(50-20+\overline{0\times 12})$?

13. If 3 men can do a piece of work in 8 days, in how many days can 4 men do the same work?

Analysis.—If 3 men can do the work in 8 days, it will take one man 3 times 8 days, or 24 days; and 4 men can do it in 1 *fourth* of 24 days or 6 days. Hence 4 men can do the work in 6 days.

14. If 6 men can build a wall in 12 days, how many men will be required to build it in 2 days? In 4 days?

15. If 16 bushels of wheat make 4 barrels of flour, how many bushels will make 8 barrels? 10 barrels?

16. If 5 men have provision enough for 12 days, how long would it serve 3 men? 10 men? 12 men?

17. A man earns $9 a week, and his son $5; how much will both earn in 3 weeks? In 5 weeks?

18. How much more will the father earn in 12 weeks than the son?

19. If two persons start from the same point and travel in the same direction, one at the rate of 7 miles an hour, and the other 4 miles, how far apart will they be in 6 hours? In 8 hours? In 10 hours? In 12 hours?

20. If a man earn \$5 while a boy earns \$2, how many dollars will the man earn while the boy is earning \$18 ?

21. How far will a man travel in 12 days at the rate of 36 miles in 3 days ?

22. John gave 40 cents for some peaches, at the rate of 5 for 10 cents. How many peaches did he buy ?

23. What is the value of 5 tons of hay, at the rate of \$48 for 4 tons ?

24. A tailor bought 11 yards of one kind of cloth for \$55, and 9 yards of another kind for \$36. What was the difference in the price per yard ?

25. Multiply 10 by 8, subtract 30, add 13, divide by 7, multiply by 6, and what is the result ?

26. What cost 28 quarts of milk, at 40 cents for 5 quarts ?

27. Divide 60 by 12, multiply the quotient by 9, to the product add 25, from the sum subtract 40, and what will remain ?

28. How many pairs of boots at \$8 a pair, will pay for 6 weeks board at \$7 a week, and \$22 borrowed money ?

29. From 25 subtract 10, add 17, divide by 8, multiply by 20, subtract 14, divide by 6, multiply by 4, add 18, and what will be the sum ?

Find the missing *term* in each of the following equations:

30. $10+4\div7+20=\,?$
31. $8\times9-12\div5=\,?$
32. $\overline{40-8\div4}\times7=\,?$
33. $(25+10\div7)\times9=\,?$
34. $72\div12+50-16=\,?$
35. $48+12-10\div\,?=5$
36. $108\div9-4\times\,?=72$
37. $14+7\div3\times\,?=84$
38. $(144\div12-3)\times11=\,?$
39. $24+20-4=80\div\,?$
40. $\overline{56\div7}\times8=70-\,?$
41. $43+6\div7\times0=\,?$
42. $120\div10\times5=\,?\times12$
43. $20\times2\div8=\,?\div6$

WRITTEN EXAMPLES.

116. 1. How many times is 246 contained in 3690?

2. How many times can 246 be subtracted from 3690?

3. How many times 246 will produce 3690?

4. If the divisor be 1246 and the dividend 33642, what is the quotient?

5. The factors of a number are $84+23$ and $427-259$. What is the number?

6. The product of two numbers is 14304, and one of the numbers is $2400 \div 25$. What is the other?

7. What number added to 1863 will amount to 3248?

8. Divide the product of 96 and 48 by their difference.

9. What is the difference between 106×127, and $15341 \div 29$?

10. What is the sum of $4527 - 1039$, 250×86, and $3025 + 76$?

11. The divisor is 16, and the quotient 12624. If the divisor be diminished 4 times, what will be the quotient?

12. Of what number is 306 the divisor and quotient?

13. What number besides 216 will divide 75168 without a remainder?

14. How many pounds of sugar worth 13 cents a pound, can be bought for 126 pounds of butter worth 28 cents a pound?

15. If a mechanic receive $1250 a year for labor, and his expenses are $765 a year, how much can he save in 5 years?

16. A farmer bought 36 acres of land at $28 an acre, and 27 acres at $35 an acre. What was the average price an acre?

17. How long can 60 men subsist on an amount of food that will last 1 man 7620 days?

18. How many tons of hay at $16 a ton must be given for 24 cows worth $32 a head?

19. How many casks, each holding 31 gallons, can be filled from 14 pipes of wine, each containing 124 gallons?

20. Bought 150 acres of land for $7650, and sold it for $63 an acre. How much was the whole gain?

21. What number divided by 95 will give a quotient of 75, and a remainder of 47?

22. How long would it take a train of cars going at an average rate of 40 miles an hour, to travel around the earth at the equator, the distance being 24898 miles?

23. If 100 barrels of flour cost $600, what will 350 barrels cost at the same rate?

What is the value of each of the following equations:

24. $(6300 \div 15) - (372 \div 3) = ?$

25. $(12460 \div 10) \times (475 - 377) = ?$

26. $(867 \times 97) \div (200 \times 5 \times 3) = ?$

27. $(6070 - 1200) + (4680 \div 15) = ?$

28. A farmer sold 16 cords of wood at $3 a cord, and 40 bushels of wheat at $2 a bushel. He received 15 yards of cloth at $4 a yard, and the remainder in money; how much money did he receive?

29. The quotient of one number divided by another is 56, the divisor is 375, and the remainder 124. What is the dividend?

30. A coal dealer bought 250 tons of coal, receiving 2240 pounds for a ton; he sold it at 2000 pounds for a ton. How many tons did he gain?

31. An estate was divided among 4 children; the first received $5640, the second $7384, the third $2380 less than the second, and the fourth $1236 more than the first. What was the value of the estate?

32. A planter sold 40 bales of cotton at $68 a bale. With the proceeds he bought 8 horses at $180 each, and with the remainder bought cows at $32 a head. How many cows did he purchase?

33. Bought 228 barrels of flour for $1482, and sold the same for $2058. What was the gain on each barrel?

Find the value of each of the following equations:

34. $384+(4621-872)+\overline{(3176\div 8)\times 12}=?$

35. $25\times 370-(62\times 8)+\overline{(335+206)\times 45}=?$

36. $16\times 3125-(127\times 0)+\overline{(972-122)\div 50}=?$

37. $(125\times 30)\div(25\times 5)\times(324-216)+\overline{264-(450\div 5)}=?$

117. The pupil should be required to illustrate the following problems by original examples:

1. Given, several numbers, to find their sum.

2. Given, the sum of several numbers and all of them but one, to find that one.

3. Given, two numbers, to find their difference.

4. Given, the minuend and subtrahend, to find the remainder.

5. Given, the minuend and remainder, to find the subtrahend.

6. Given, the subtrahend and remainder, to find the minuend.

7. Given, two or more numbers, to find their product.

8. Given, the multiplicand and multiplier, to find the product.

9. Given, the product and multiplicand, to find the multiplier.

10. Given, the product and multiplier, to find the multiplicand.

11. Given, the divisor and dividend, to find the quotient.

12. Given, the divisor and quotient, to find the dividend.

13. Given, the dividend and quotient, to find the divisor.

14. Given, the divisor, quotient, and remainder, to find the dividend.

15. Given, the dividend, quotient, and remainder, to find the divisor.

QUESTIONS FOR REVIEW.

118. Define Division. Dividend. Divisor. Quotient. Remainder. Sign of division. Its meaning and use? In what other way is division indicated? Show that division is a short method of performing subtraction. Of what is division the reverse?. In division, what corresponds to the factors of the product in multiplication? What to the product?

2. What is the object of division? First? Illustrate. Second? Illustrate. What are the equal parts into which a unit may be divided, sometimes called? How do the names of these parts vary? Illustrate. How are they expressed? Illustrate.

3. Principles of division 1, 2, 3. How do long and short division differ? When is short division generally used? Rule for short division. Proof. Rule for long division. Proof. Name the steps in performing long division.

4. How divide by 10, 100, 1000, etc.? Why? Rule. When there are ciphers at the right of the divisor? Rule.

PROPERTIES OF NUMBERS.

FACTORS AND DIVISORS.

119. 1. What two numbers other than itself and 1, multiplied together will produce 6? 8? 10? 12?

2. What two numbers multiplied together will produce 24? 16? 20? 25? 36?

3. What numbers other than itself and 1, will exactly divide 18? 20? 30? 32?

4. What numbers will exactly divide 36? 28? 40? 45? 56? 60? 75?

5. What three numbers multiplied together will produce 24? 27? 36? 50?

6. What are the two smallest numbers other than 1 that will exactly divide 15? 18? 21? 30? 55?

7. Which is the largest number other than itself that will exactly divide 27? 42? 35? 48?

8. Name the numbers between 1 and 20 that are the product of two or more numbers greater than 1.

9. Name some numbers that cannot be produced by multiplication except by multiplying the given number and 1 together.

10. Name some numbers that can be produced by multiplying together other numbers than the given number and 1.

11. Of what number are 3 and 4 the factors? 2, 3 and 5? 3, 2 and 7? 2, 4 and 5? 2, 2, 3 and 3?

12. Of what number are 3, 3, 2 and 5 the factors?

DEFINITIONS.

120. ***An Integer,*** or ***Integral Number,*** is a number representing whole units; thus, 5, 7, 12 horses, 16 pounds, are integral numbers.

Integers are either *even* or *odd, prime* or *composite.*

121. ***An Even Number*** is one that is exactly divisible by 2.

All numbers terminating with 0, 2, 4, 6, or 8 are *even.*

122. ***An Odd Number*** is one that is not exactly divisible by 2.

All numbers terminating with 1, 3, 5, 7, or 9 are *odd.*

123. ***A Prime Number*** is one that has no *integral* factors except unity and itself; thus, 2, 3, 5, 11, 17, etc., are prime numbers.

124. ***A Composite Number*** is one that has other *integral* factors besides unity and itself; thus, 15 and 24 are composite numbers, since $15 = 5 \times 3$, and $24 = 8 \times 3$, or 6×4, etc.

125. ***The Factors*** of a number are the numbers which multiplied together will produce that number (**87**); thus, 4 and 5 are factors of 20; 3, 4 and 7 of 84.

126. ***Factoring*** is the process of separating a number into its factors.

127. ***An Exact Divisor*** of a number is one that will divide that number without a remainder; thus, 6 is an exact divisor of 48; and 9 of 72.

1. The *exact divisors* of a number are also the *factors* of that number.

2. An exact divisor of a number is sometimes called the *measure* of that number.

3. When a number is a factor, or divisor of each of *two* or *more* numbers, it is called a *common* factor, divisor, or measure of those numbers.

128. Numbers are ***prime to each other*** when they have *no common integral factors* or *divisors;* thus, 9 and 16, 12 and 25, are prime to each other.

129. ***A Prime Factor*** is one that is a prime number (**123**).

The *prime factors* of a number are also the *prime divisors* of it.

FACTORING.

130. 1. Name all the even numbers from 0 to 30.

2. Name all the odd numbers from 0 to 30.

3. Name all the prime numbers from 0 to 30.

4. Name all the prime factors of 30.

5. Name all the composite factors of 30.

6. What are the prime numbers between 20 and 40?

7. What are the prime factors of 21? Of 27? Of 33?

8. Name the composite numbers between 10 and 36.

9. What are the composite factors of 36? 48? 60?

10. What prime factor is common to 20 and 30?

11. Name the only prime *even* number.

12. Name the composite numbers consisting of 1 figure.

13. Name some odd numbers that are prime.

14. Of what number are 3, 3 and 11 prime factors?

15. Of what number are 2, 2, 5 and 7 prime factors?

16. What factors are common to 16 and 24? To their *sum?* To their *difference?*

17. What prime factor is common to 15 and 3 times 15?

18. What two composite factors are common to 16 and 3 times 16?

19. What factors are common to the sum and difference of 18 and 30?

20. Name 5 odd numbers of which 3 is a common factor?

21. Name five even numbers of which 5 is a common factor

131. PRINCIPLES.—1. *Every prime factor of a number is an exact divisor of that number.*

2. *A factor of a number is a factor also of any product of that number.*

3. *A factor common to two or more numbers is a factor also of their sum and of their difference*

4. *Every composite number is equal to the product of its prime factors.*

WRITTEN EXERCISES.

132. To resolve a composite number into its prime factors.

1. What are the prime factors of 252?

OPERATION.

$$\begin{array}{r|r} 2 & 252 \\ \cline{2-2} 2 & 126 \\ \cline{2-2} 3 & 63 \\ \cline{2-2} 3 & 21 \\ \cline{2-2} & 7 \end{array}$$

PROOF.

$7 \times 3 \times 3 \times 2 \times 2 = 252$

ANALYSIS.—Since the given number is even, divide by the least prime factor 2, and the quotient, also by 2.

Next divide by the prime number 3 and 3 successively, obtaining 7 for the last quotient, which being prime the division can be carried no further. Hence the divisors 2, 2, 3, 3, and the last quotient 7 are all the prime factors of 252.

2. What are the prime factors of 126? Of 225? Of 154?

Rule.—*Divide the given number by any one of its prime factors, and the resulting quotient by another, and so continue to divide until the quotient is a prime number. The several divisors and the last quotient will be the prime factors.*

Proof.—*The product of all the prime factors will be the given number.* (Prin. 4.)

What are the prime factors

3. Of 84.
4. Of 200.
5. Of 392.
6. Of 440.
7. Of 625.
8. Of 504.
9. Of 704.
10. Of 539.
11. Of 945.
12. Of 1155.
13. Of 1728.
14. Of 2205.

COMMON DIVISORS.

133. 1. What numbers are exact divisors of 21? 36?

2. What number is an exact divisor both of 27 and 63?

3. What are the prime divisors of 25? 35? 49?

4. What are the composite divisors of 72? 80? 108?

5. What prime divisor is common to 28, 42 and 56?

6. What is a common measure of 22, 33 and 55?

7. What common divisors have 40 and 60?

8. What is the greatest divisor common to 15 and 45? To 26 and 39? To 18, 36 and 72?

9. Name four odd numbers of which 7 is a common divisor. Four even numbers.

10. Name three numbers of which 6 and 9 are common divisors.

11. Name four even numbers of which 9 is a common measure. Four odd numbers.

12. What divisor is common to 14 and 3 times 14?

13. What numbers will exactly measure 20 and 4 times 20? 25 and 3 times 25?

14. What two numbers will exactly measure 10 and 20? Their *sum?* Their *difference?* Their *product?*

15. What is the greatest common divisor of 30, 45, and 60? Of 18, 36, 45 and 72?

134. PRINCIPLES.—1. *The only exact divisors of a number are its prime factors, or the product of two or more of them.*

2. *An exact divisor divides any number of times its dividend.*

3. *A common divisor of two numbers will divide their sum, and also their difference.*

4. *The greatest common divisor of two or more numbers is the product of all their common prime factors.*

DEFINITIONS.

135. ***A Divisor,*** or ***Measure,*** of a number is a number that will exactly divide it.

136. ***A Common Divisor*** of two or more numbers is a number that will exactly divide each of them.

137. ***The Greatest Common Divisor*** of two or more numbers is the greatest number that will exactly divide each of them.

Numbers *prime to each other* have no common divisor (**128**).

A common divisor is sometimes called a *Common Measure;* and the greatest common divisor, the *Greatest Common Measure.*

WRITTEN EXERCISES.

138. When the numbers can be readily factored.

1. What is the greatest common divisor of 28, 42 and 70?

1st OPERATION.

$28 = 7 \times 2 \times 2$

$42 = 7 \times 2 \times 3$

$70 = 7 \times 2 \times 5$

Hence $7 \times 2 = 14$.

ANALYSIS.—By factoring the given numbers, it is found that the prime factors common to *all* of them, are 7 and 2. Hence $7 \times 2 = 14$, is the greatest common divisor of 28, 42 and 70. (PRIN. 4.)

2D OPERATION.

$$\begin{array}{r|rrr} 2 & 28 & 42 & 70 \\ \hline 7 & 14 & 21 & 35 \\ \hline & 2 & 3 & 5 \end{array}$$

ANALYSIS.—Since the given numbers are exactly divisible by 2, and the resulting quotients by 7, they are also divisible by 7 × 2 or 14. (PRIN. 1.)

If there were *other* factors of the greatest common divisor, then the quotients 2, 3 and 5 would be exactly divisible by them.

Find the greatest common divisor

2. Of 72 and 120.
3. Of 36, 72 and 126.
4. Of 21, 42 and 77.
5. Of 26, 52 and 91.

RULE.—*Separate the numbers into their prime factors, and find the product of all that are common.* Or,

I. *Write the numbers in a line, with a vertical line at the left, and divide by any prime factor common to all the numbers.*

II. *Divide the quotients in like manner, and so continue the division till a set of quotients is obtained that are prime to each other. The product of all the divisors will be the greatest common divisor.* (PRIN. 4.)

Find the greatest common divisor

6. Of 96 and 288.
7. Of 264 and 418.
8. Of 56, 72 and 92.
9. Of 45, 27 and 54.
10. Of 36, 48 and 216.
11. Of 105, 216 and 405.
12. Of 32, 40, 64 and 108.
13. Of 64, 72, 88 and 96.

14. James had 30 cents, George 36, and Willie 42; and they bought oranges at such a price as to allow each to expend all his money. What was the price of the oranges?

15. How long must the boards be to make an exact number of lengths of fence that shall enclose a lot 42 feet wide and 112 feet long?

139. When the numbers cannot be readily factored.

1. Find the greatest common divisor of 84 and 203.

OPERATION.

		203
84	2	168
70	2	35
14	2	28
14	2	7
0		

ANALYSIS.—Draw two vertical lines, and place the larger number on the right, and the smaller on the left, one line lower down. Divide the larger number 203, by the smaller, 84, and write the quotient 2 between the vertical lines, the product 168, under the greater number, and the remainder 35, below.

Next divide 84 by this remainder 35, writing the quotient 2 between the verticals, the product 70 on the left, and the remainder 14 below. Again divide the last divisor 35 by 14, and write the quotient, product and remainder in the same order as before. Finally, dividing the last divisor 14 by the remainder 7, there is no remainder. Hence 7, the last divisor, is the greatest common divisor of 203 and 84.

Find the greatest common divisor

2. Of 154 and 210.
3. Of 316 and 664.
4. Of 126 and 189.
5. Of 225 and 270.

RULE.—I. *Draw two vertical lines and write the two numbers, one on each side, the greater number one line above the less.*

II. *Divide the greater number by the less, writing the quotient between the verticals, the product under the dividend, and the remainder below.*

III. *Divide the less number by the remainder, the last divisor by the last remainder, and so on, till nothing remains. The last divisor will be the greatest common divisor sought.*

IV. *If more than two numbers are given, first find the greatest common divisor of the least two, and then of this divisor and a third number, and so on until all the numbers have been used; the last divisor will be the greatest common divisor of all the numbers.*

If the last divisor is 1, the numbers are *prime to each other*, and have no common divisor greater than 1.

Find the greatest common divisor

6. Of 432 and 648.
7. Of 135 and 315.
8. Of 182 and 364.
9. Of 756 and 1575.
10. Of 1008 and 1036.
11. Of 216, 360 and 432.
12. Of 84, 126 and 462.
13. Of 141, 799 and 940.

14. A farmer wishes to put 336 bushels of corn, and 812 bushels of wheat into the least number of bins possible that shall contain the same number of bushels without mixing the two kinds of grain. What number of bushels must each bin hold?

MULTIPLES.

140. 1. What numbers between 3 and 15 are exactly divisible by 3? By 4?

2. What numbers between 5 and 30 are exactly divisible by 4? By 5? By 6? By 8?

3. Name some numbers that are products of which 5 is a factor. Of which 6 is a factor.

4. What prime factors have 6 and 3 times 6? 8 and 2 times 8? 9 and 4 times 9?

5. Name some numbers that are exactly divisible by 3 and 5. 4 and 6. 3 and 7.

6. Name two numbers divisible by 8 that are also divisible by 10.

7. Name three numbers that are divisible by both 3 and 5?

8. By what three prime factors can 30 be divided?

9. Name three numbers that are an exact number of times 7?

10. Name two numbers that are an exact number of times 4 or 8?

11. Name some numbers of which 3 and 5 are factors.

12. What numbers from 6 to 30 have 7 as a factor?

13. What numbers from 4 to 36 have both 2 and 3 as factors?

14. What is the least number of which 3, 4 and 5 are factors?

15. What is the least number exactly divisible by 3, 4 and 5?

17. What is the least number exactly divisible by 3, 4 and 8?

DEFINITIONS.

141. ***A Multiple*** is a number exactly divisible by a given number; or, it is any product of which a given number is a factor; thus, 10, 15, 20, 25, 30, etc., are multiples of 5.

142. ***A Common Multiple*** is a number exactly divisible by two or more given numbers; thus, 18 is a common multiple of 2, 3, 6 and 9.

143. ***The Least Common Multiple*** is the least number exactly divisible by two or more given numbers; thus, 24 is the least common multiple of 3, 4, 6 and 8, since no number less than 24 will contain each of them.

144. PRINCIPLES.—1. *A multiple of a number must contain all the prime factors of that number.*

2. *The least common multiple of two or more numbers is the least number that will contain all the prime factors of each of those numbers.*

WRITTEN EXERCISES.

145. To find the least common multiple.

FIRST METHOD.

1. Find the least common multiple of 12, 18 and 42.

OPERATION.

$$12 = 2 \times 3 \times 2$$
$$18 = 2 \times 3 \times 3$$
$$42 = 2 \times 3 \times 7$$
$$2 \times 3 \times 7 \times 3 \times 2 = 252.$$

ANALYSIS.—The least common multiple cannot be less than the largest number 42, since it must contain 42, hence it must contain all the prime factors of 42, viz.: 2, 3 and 7. (PRIN. 1.)

But the least common multiple of 42 must also contain all the prime factors of each of the other numbers, and since the prime factors 2 and 3 are common also to 18 and 12, omit them and annex the remaining factors 3 and 2 to those of 42, and the series 2, 3, 7, 3 and 2 are all the prime factors of the given numbers, and $2 \times 3 \times 7 \times 3 \times 2 = 252$, is their least common multiple. (PRIN. 2.)

2. Find the least common multiple of 9, 12 and 36.

3. What is the least common multiple of 4, 16 and 32?

RULE.—I. *Separate the given numbers into their prime factors.*

II. *The product of all the prime factors of the largest number, and such prime factors of the other numbers as are not found in the largest number will be the least common multiple.*

Find the least common multiple

4. Of 14, 16 and 18.
5. Of 27, 36 and 44.
6. Of 16, 48 and 108.
7. Of 14, 42 and 63.
8. Of 12, 16, 18 and 20.
9. Of 10, 45, 75 and 90.

SECOND METHOD.

146. 1. Find the least common multiple of 9, 12 and 27.

OPERATION.

$$\begin{array}{r|rrr} 3 & 9 & 12 & 27 \\ \hline 3 & 3 & 4 & 9 \\ \hline & & 4 & 3 \end{array}$$

ANALYSIS.—Write the numbers in a horizontal line, with a vertical line at the left.

Since 3 is a prime factor of one or more of the given numbers, it must also be a factor of the least common multiple of these numbers. (PRIN. 2.) Hence divide by 3 and write the quotients in a line underneath. For a like reason, divide again by 3, and write the quotient and undivided number 4 in a line below, as before, omitting to write any quotient when it is 1.

Since there is no factor common to 4 and 3, they are prime to each other, and hence the divisors 3 and 3, with the numbers 4 and 3 in the last line are all the prime factors contained in the given numbers, and their product 108 is the least common multiple sought. (Prin. 2.)

If in any example any of the smaller numbers are exactly contained in the larger, the smaller may be omitted in finding the least common multiple, inasmuch as a number that will contain a given number will contain any factor of that number.

Thus, if required to find the least common multiple of 6, 12, 36, 84 and 108, omit all the numbers except 84 and 108, since the others are factors of these, and the least common multiple of 84 and 108 will be the least common multiple of all the numbers.

2. Find the least common multiple of 8, 20 and 25.
3. Find the least common multiple of 9, 18 and 42.
4. Find the least common multiple of 8, 12, 36 and 72.

Rule.—I. *Write the given numbers in a horizontal line, omitting such of the smaller as are factors of the larger.*

II. *Divide by any prime factor that will exactly divide two or more of the given numbers, and write the quotients and undivided numbers, if any, in a line beneath.*

III. *In like manner divide the quotients and undivided numbers until they are prime to each other. The product of the divisors and the numbers remaining as quotients and undivided numbers will be the least common multiple.*

Find the least common multiple

5. Of 72 and 48.
6. Of 52 and 78.
7. Of 144 and 180.
8. Of 324 and 360.
9. Of 225 and 375.
10. Of 12, 15 and 42.
11. Of 21, 35 and 56.
12. Of 16, 40 and 96.
13. Of 84, 100 and 224.
14. Of 16, 20, 48 and 72.

15. What is the least common multiple of 4, 16, 20, 48 and 72?

16. What is the least common multiple of 8, 12, 18, 24, 27 and 36?

17. What is the least number of marbles that can be divided equally among 12, 18, and 24 boys?

18. What is the least number of dollars that will purchase an exact number of cows at $30 each, mules at $55, or horses at $105 each?

19. What is the smallest sum of money for which I can purchase an exact number of books at $3, or $4, or $5, or $6 each?

20. What is the least number of acres in a farm that can be exactly divided into lots of 10 acres, 14 acres, 16 acres and 20 acres each?

CANCELLATION.

147. 1. Divide 48 by 12. 24 by 6.

2. Divide 1 third of 48 by 1 third of 12.

3. Divide 1 fourth of 48 by 1 fourth of 12.

4. Divide 1 third of 24 by 1 third of 6.

5. What factors are common to 48 and 12?

6. What factors are common to 24 and 6?

7. Rejecting the factors common to 48 and 12, what factors of each will remain?

8. What is the quotient of 16×3 divided by 4×3?

9. What is the quotient of 12×4 divided by 3×4?

10. What is the quotient of $2 \times 5 \times 3$ divided by 2×5?

11. What is the quotient of $8 \times 3 \times 5$ divided by $2 \times 3 \times 5$?

148. PRINCIPLES.—1. *Rejecting a factor from any number divides the number by that factor.*

2. *Rejecting an equal factor from both dividend and divisor does not change the quotient.*

149. ***Cancellation*** is the process of abbreviating operations in division by rejecting equal factors from both dividend and divisor.

WRITTEN EXERCISES.

1. What is the quotient of 9 times 14 divided by 6 times 7?

OPERATION.

$$\frac{9 \times 14}{6 \times 7} = \frac{3 \times \cancel{3} \times \cancel{2} \times \cancel{7}}{\cancel{2} \times \cancel{3} \times \cancel{7}} = \frac{3}{1} = 3$$

ANALYSIS.—Write the numbers as in division, the dividend above and the divisor below the line. Resolve the numbers into their factors, the dividend being composed of 3 × 3 × 2 × 7, and the divisor of 2 × 3 × 7. Cancelling equal factors from both dividend and divisor, which is the same as dividing by those factors, there is left the factor 3 in the dividend. Hence the quotient is 3.

2. Divide $12 \times 8 \times 6$ by $8 \times 4 \times 3$.

OPERATION.

$$\frac{\overset{3}{\cancel{12}} \times \cancel{8} \times \overset{2}{\cancel{6}}}{\cancel{8} \times \cancel{4} \times \cancel{3}} = \frac{3 \times 2}{1} = 6$$

ANALYSIS.—In this example first strike out the factor 8, common to the dividend and divisor. Next cancel the factor 4 from 4 and 12, leaving the factor 3 in the dividend. In like manner cancel the factor 3 from 3 and 6, leaving the factor 2 in the dividend. Hence the quotient is $3 \times 2 = 6$.

By many it is thought more convenient to write the factors of the dividend on the right of a *vertical* line, and the factors of the divisor on the left.

3. Divide the product of 25, 18, 4, and 3 by the product of 7, 6, 5 and 3.

OPERATION.

$$\frac{\overset{5}{\cancel{25}} \times \overset{3}{\cancel{18}} \times 4 \times \cancel{3}}{7 \times \cancel{6} \times \cancel{5} \times \cancel{3}} = \frac{5 \times 3 \times 4}{7} = \frac{60}{7} = 8\tfrac{4}{7}. \text{ Or, } \quad \begin{array}{r|l} 7 & \cancel{25}^{5} \\ \cancel{6} & \cancel{18}^{3} \\ \cancel{5} & .\,4 \\ \cancel{3} & \cancel{3} \\ \hline 7 & 60 \\ \cline{2-2} & 8\frac{4}{7} \end{array}$$

In like manner,

4. Divide 4 × 5 × 56 by 8 × 5 × 7.
5. Divide 5 × 7 × 13 by 5 × 7.
6. Divide 7 × 9 × 15 by 9 × 5.

RULE.—I. *Write the numbers comprising the dividend above a horizontal line and the numbers composing the divisor below it.*

II. *Cancel all the factors common to both dividend and divisor.*

III. *Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

1. When a factor is canceled, 1 is supposed to take its place.

2. *One* factor in the divisor will cancel only *one equal* factor in the dividend

3. If all the factors or numbers of the divisor be cancelled, the product of the remaining factors of the dividend will be the quotient.

7. Divide 8 × 7 × 81 by 27 × 28.
8. Divide 36 × 10 × 7 by 14 × 5 × 9.
9. Divide 21 × 8 × 40 by 4 × 7 × 20.
10. Divide 9 × 8 × 64 by 27 × 4 × 30.
11. Divide 44 × 6 × 120 by 11 × 8 × 60.
12. Divide 56 × 7 × 12 by 14 × 8 × 6.

13. Divide $33 \times 35 \times 28$ by $11 \times 15 \times 14$.

14. Divide $21 \times 11 \times 26$ by 14×13.

15. What is the quotient of $21 \times 8 \times 60 \times 8 \times 6$ divided by $7 \times 12 \times 3 \times 8 \times 3$?

16. Multiply 8 times 32 by 6 times 27 and divide the product by 9 times 96.

17. The product of the numbers 26, 11, and 21, is to be divided by the product of the numbers 14 and 13: what is the quotient?

18. Divide the product of 200 multiplied by 60, by the product of 50 multiplied by 48.

19. How many times is the product of the numbers 84, 15, 7 and 6 contained in the product of the numbers 48, 72, 28 and 5?

20. How many barrels of flour at $9 a barrel must be given for 27 yards of cloth at $4 a yard?

21. How many bushels of apples at 75 cents a bushel will pay for 85 pounds of coffee at 30 cents a pound?

22. A farmer exchanged 45 bushels of potatoes worth 40 cents a bushel for 18 pounds of tea. What was the tea worth a pound?

23. From 5 pieces of cloth, each piece containing 24 yards worth $3 a yard, how many suits can be made worth $18 a suit?

24. When hay is worth $18 a ton, how many tons must be given for 12 barrels of flour at $7 a barrel?

25. If 6 acres of land cost $180, what will 150 acres cost?

26. Gave 3 boxes of tea, each containing 40 pounds, for 8 tubs of butter, each containing 56 pounds, worth 15 cents a pound. How much was the tea worth a pound?

27. How many men will be required to do a piece of work in 12 days that 7 men can do in 84 days?

28. A farmer sold 6 loads of apples, each load containing 14 barrels, and each barrel 3 bushels, at 50 cents a bushel. He received in payment 9 barrels of sugar, each weighing 210 pounds. What was the sugar worth a pound?

29. How many gallons of molasses, worth 45 cents a gallon, must be given for 120 pounds of cheese, at 9 cents a pound?

30. How many bales of cloth, each containing 40 pieces, and each piece containing 36 yards, worth $3 a yard, must be given for 120 government bonds worth $108 each?

31. How many barrels of potatoes, each containing 3 bushels, worth 30 cents a bushel, will pay for 12 boxes of soap, each containing 51 pounds, worth 10 cents a pound?

QUESTIONS FOR REVIEW.

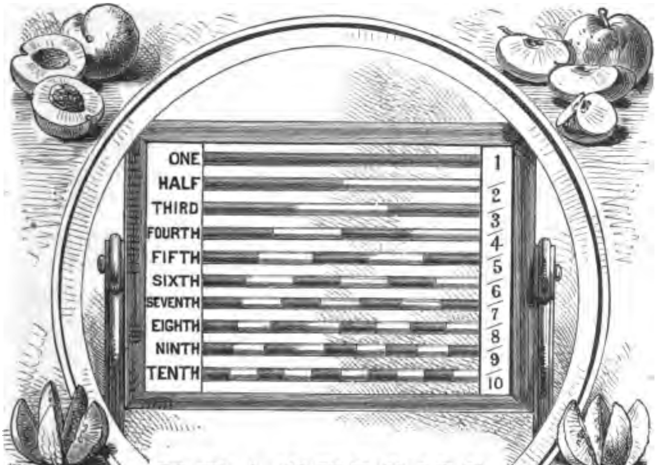
150. 1. Define an Integer. An Even Number. An Odd Number. A Prime Number. A Composite Number. Factors. Factoring. An Exact Divisor. What are the exact divisors of a number? Prime Factors. What are the prime factors of a number? When are numbers prime to each other?

2. Principles 1, 2, 3, 4. Rule for finding prime factors. Proof.

3. Define a Divisor. A Common Divisor. Greatest Common Divisor. What is the greatest common divisor sometimes called? Principles 1, 2, 3, 4. Rule.

4. Define a Multiple. Common Multiple. Least Common Multiple. Principles 1, 2. Rule for least common multiple by first method. By second method.

5. Define Cancellation. Principles 1, 2. Rule. When a factor is cancelled what is supposed to take its place? If all the factors of the divisor are cancelled, what will the quotient be?



FRACTIONS.

151. 1. If a unit, as a peach, be divided into 2 equal parts, what is the name of each part? *One-half.*

2. If an apple be divided into 3 equal parts, what is the name of each part? *One-third.*

3. If an orange be divided into 5 equal parts, what is 1 of its parts called? 2 of its parts? 3 of its parts? 4 of its parts?

4. How many *halves* in a unit? How many *thirds? Fourths? Fifths? Sixths? Sevenths? Eighths? Ninths?*

5. If a melon be cut into 4 equal pieces, what part of the whole melon is 1 piece? Are 2 pieces? 3 pieces?

6. What is 1 of 2 equal parts of a unit called? 2 of 3 equal parts? 3 of 4 equal parts?

7. What is meant by 1 fourth of anything? 2 fourths? 3 fourths? 2 thirds? 3 fifths?

8. What are 2 of 5 equal parts of a unit called? 3 of 7 equal parts? 5 of 12 equal parts?

9. What part of a whole melon are 3 of the 6 equal parts of it? 4 of the 5 equal parts?

10. Which are the *smaller,* halves or thirds? Halves or fourths? Thirds or fourths? Fifths or thirds?

11. Which are *greater,* fourths or thirds? Sixths or eighths? Sevenths or fifths? Ninths or twelfths?

152. PRINCIPLES.—1. *The LESS the NUMBER of equal parts into which a unit is divided, the GREATER is the VALUE of each part.*

2. *The GREATER the NUMBER of equal parts into which a unit is divided, the LESS is the VALUE of each part.*

DEFINITIONS.

153. ***A Fraction*** is one or more of the equal parts of a unit; thus; 1 *half,* 2 *thirds,* 3 *fifths,* are fractions.

154. ***A Fractional Unit*** is *one* of the equal parts into which any integer is divided; thus, 1 third, 1 fourth, etc., are fractional units of *thirds, fourths,* etc.

155. Fractions are divided into two classes, viz.: *Common Fractions* and *Decimal Fractions,* and are distinguished by their *forms.*

156. ***A Common Fraction*** is expressed in figures by two integers, one placed over the other with a line between them, thus:

One-half	is written	$\frac{1}{2}$	Two-thirds	is written	$\frac{2}{3}$
One-third	"	$\frac{1}{3}$	Three-fourths	"	$\frac{3}{4}$

Five-sixths is written	$\frac{5}{6}$	Nine-fourteenths is written	$\frac{9}{14}$
Three-eighths "	$\frac{3}{8}$	Eleven-twentieths "	$\frac{11}{20}$
Seven-eighths "	$\frac{7}{8}$	Twelve twenty-thirds "	$\frac{12}{23}$
Five-ninths "	$\frac{5}{9}$	Ten thirty-fifths "	$\frac{10}{35}$

157. ***The Denominator*** of a fraction is the number written *below* the line.

It shows into *how many* equal parts the unit is divided, and also indicates the *name* of these parts; thus, in the fraction $\frac{5}{6}$, 6 is the *denominator* and shows that the unit is divided into *six* equal parts, and named *sixths*.

158. ***The Numerator*** of a fraction is the number written *above* the line.

It shows the *number* of equal parts taken to form the fraction; thus, in $\frac{5}{6}$, 5 is the *numerator*, and shows that 5 of the 6 equal parts are taken, or expressed by the fraction.

159. ***The Terms*** of a fraction are its numerator and denominator; thus, 6 and 7 are the *terms* of the fraction $\frac{6}{7}$.

Express by figures,

1. Four-fifths. *Ans.* $\frac{4}{5}$.
2. Five-sevenths.
3. Seven-tenths.
4. Nine-fourteenths.
5. Twelve-twentieths.
6. Fifteen-seventeenths.
7. Fourteen-twenty-fourths.
8. Twenty-thirty-sixths.
9. Sixteen-fortieths.
10. Forty-seventy-fifths.

Write and read:

11. $\frac{7}{8}$; $\frac{9}{11}$; $\frac{7}{15}$; $\frac{4}{13}$.
12. $\frac{14}{27}$; $\frac{21}{28}$; $\frac{18}{42}$; $\frac{11}{30}$.
13. $\frac{8}{49}$; $\frac{29}{50}$; $\frac{16}{56}$; $\frac{36}{48}$.
14. $\frac{45}{80}$; $\frac{51}{96}$; $\frac{120}{144}$; $\frac{112}{316}$.

Which is greater, and why,

$\frac{1}{2}$ or $\frac{1}{3}$? $\frac{3}{5}$ or $\frac{4}{5}$? $\frac{5}{6}$ or $\frac{5}{7}$? $\frac{5}{4}$ or 1?
$\frac{1}{3}$ or $\frac{1}{4}$? $\frac{3}{4}$ or $\frac{3}{5}$? $\frac{6}{8}$ or $\frac{6}{7}$? 1 or $\frac{6}{6}$?

160. ***Common Fractions*** are classified as *Proper* and *Improper*.

161. ***A Proper Fraction*** is one whose numerator is less than its denominator, and its *value* is less than 1; thus, $\frac{3}{5}$, $\frac{5}{6}$, $\frac{9}{12}$ are proper fractions.

162. ***An Improper Fraction*** is one whose numerator equals or exceeds its denominator, and its *value* is 1, or more than 1; thus, $\frac{4}{4}$, $\frac{9}{5}$, $\frac{14}{9}$ are improper fractions.

163. ***A Mixed Number*** is an integer and a fraction united; thus, $3\frac{1}{2}$, $5\frac{3}{8}$, are equivalent to $3+\frac{1}{2}$, $5+\frac{3}{8}$.

164. Fractions may be considered as indicating division, the numerator corresponding to the dividend, and the denominator to the divisor (**99**). Hence

165. ***The Value*** of a fraction is the quotient of its numerator divided by its denominator; thus $\frac{9}{3} = 3$.

Analyze the fraction $\frac{5}{6}$.

ANALYSIS.—$\frac{5}{6}$ is a *common* fraction; 6 is the *denominator*, and shows that the unit is divided into 6 equal parts; $\frac{1}{6}$ is the *fractional unit*, since it is 1 of the 6 equal parts into which the unit is divided; 5 is the *numerator*, and shows that 5 of these equal parts are taken; 5 and 6 are the *terms* of the fraction; it is a *proper* fraction, since the numerator is less than the denominator; its *value* is less than 1; and it is read *five-sixths*.

In like manner, analyze

1. $\frac{4}{5}$.
2. $\frac{6}{7}$.
3. $\frac{8}{9}$.
4. $\frac{6}{5}$.
5. $\frac{7}{8}$.
6. $\frac{9}{7}$.
7. $\frac{11}{9}$.
8. $\frac{7}{10}$.
9. $\frac{18}{24}$.
10. $\frac{12}{20}$.

166. Since a fraction may indicate division, all changes in the *terms* of a fraction will affect the *value* of that fraction according to the laws of division, as shown in the following illustrations:

1. Multiply the numerator of the fraction $\frac{2}{3}$ by 2, and the result is $\frac{4}{3}$. The *value* of each of the fractional units remain the same, while their *number* is made 2 times as great. So $\frac{3^{\times 3}}{4} = \frac{9}{4}$.

2. Divide the numerator of the fraction $\frac{4}{5}$ by 2, and the result is $\frac{2}{5}$. The *value* of each of the fractional units remains the same, while their *number* is diminished $\frac{1}{2}$. So $\frac{6^{\div 2}}{7} = \frac{3}{7}$.

3. Multiply the denominator of the fraction $\frac{3}{5}$ by 2, and the result is $\frac{3}{10}$. The *value* of each of the fractional units is diminished $\frac{1}{2}$, while their *number* remains the same. So $\frac{2}{3_{\times 2}} = \frac{2}{6}$.

4. Divide the denominator of the fraction $\frac{5}{8}$ by 2, and the result is $\frac{5}{4}$. The *value* of each of the fractional units is made 2 times as great, while their *number* remains the same. So $\frac{5}{6_{\div 2}} = \frac{5}{3}$.

5. Multiply both terms of the fraction $\frac{4}{5}$ by 2, and the result is $\frac{8}{10}$. The *value* of each of the fractional units is diminished $\frac{1}{2}$, but their *number* is made 2 times as great. So $\frac{3 \times 3}{7 \times 3} = \frac{9}{21}$.

6. Divide both terms of the fraction $\frac{6}{8}$ by 2, and the result is $\frac{3}{4}$. The *value* of each of the fractional units is made 2 times as great, but their *number* is diminished $\frac{1}{2}$. So $\frac{8 \div 4}{16 \div 4} = \frac{2}{4}$.

167. From the preceding illustrations are deduced the following

GENERAL PRINCIPLES OF FRACTIONS.

I. A fraction is multiplied,
  1. *By multiplying the numerator;* Or,
  2. *By dividing the denominator.* (**166,** 1, 4.)

II. A fraction is divided,
  1. *By dividing the numerator;* Or,
  2. *By multiplying the denominator.* (**166,** 2, 3.)

III. The value of a fraction is not changed,
  1. *By multiplying both terms;* Or,
  2. *By dividing both terms* } *by the same number.* (**166,** 5, 6.)

168. These three principles may be embraced in one

GENERAL LAW.

A change in the NUMERATOR *produces a* LIKE *change in the value of the fraction; but a change in the* DENOMINATOR *produces an* OPPOSITE *change in the value of the fraction.*

REDUCTION.

169. ***Reduction of Fractions*** is the process of changing their *form* without changing their *value.*

CASE I.

170. To reduce fractions to higher terms.

1. If a melon be cut into 2 equal pieces, what part of the melon is each piece?

2. If each of these 2 pieces be cut into 2 equal pieces, what part of the whole melon is each piece?

3. One-*half* is equal to how many *fourths?*

ANALYSIS.—Since 1 is equal to 4 fourths, 1 half is equal to $\frac{1}{2}$ of 4 fourths, or 2 fourths.

4. If 1 half of a melon be cut into 3 equal pieces, what part of the melon is 1 piece? If into 4 equal pieces?

5. If 1 third of a cake be divided equally among 3 boys, what part of the whole cake will each boy have?

6. 1 third is equal to how many sixths? Ninths?

7. Name some equivalent fractions in which halves can be expressed. Thirds. Fourths. Fifths.

8. The number of *sixths* in a unit are how many times the number of *halves?* Of *thirds?*

9. The number of *twelfths* in a unit are how many times the number of *thirds?* Of *fourths?* Of *sixths?*

10. Multiply both terms of $\frac{2}{3}$ by 2 and give the result; by 3; by 4; by 5.

11. Name two equivalent fractions for $\frac{3}{4}$; $\frac{2}{3}$; $\frac{4}{5}$.

12. How many ninths are $\frac{1}{3}$? $\frac{2}{3}$? $\frac{4}{3}$?

171. A fraction is reduced to *Higher Terms,* when the numerator and denominator are expressed in larger numbers of the same comparative value.

172. PRINCIPLE.—*Multiplying both terms of a fraction by the same number does not change the value of the fraction* (**167,** III, 1).

WRITTEN EXERCISES.

1. Reduce $\frac{5}{8}$ to sixteenths.

OPERATION.

$16 \div 8 = 2$

$\frac{5 \times 2}{8 \times 2} = \frac{10}{16}$

ANALYSIS.—Since the given denominator 8 is contained in the required denominator 2 times, multiply both terms of the fraction by 2. (PRIN.) Hence $\frac{5}{8} = \frac{10}{16}$.

2. Change $\frac{5}{6}$ to a fraction whose denominator is 30.

3. Change $\frac{7}{9}$ to a fraction having 45 for its denominator.

4. Reduce $\frac{3}{8}$ to a fraction having 40 for its denominator.

RULE.—*Divide the required denominator by the denominator of the given fraction, and multiply the terms of the fraction by the quotient.*

Reduce	Change
5. $\frac{4}{5}$ to thirty-fifths.	8. $\frac{11}{15}$ to seventy-fifths.
6. $\frac{5}{7}$ to sixty-thirds.	9. $\frac{15}{18}$ to 108ths.
7. $\frac{7}{12}$ to eighty-fourths.	10. $\frac{21}{32}$ to 256ths.

CASE II.

173. To reduce fractions to lower terms.

1. How many *half* melons in 2 fourths of a melon?

2. How many *fifths* are there in 8 tenths?

ANALYSIS.—Since 1 fifth is equal to 2 tenths, 8 tenths are equal to as many fifths as 2 tenths are contained times in 8 tenths, which is 4 times. Hence there are 4 fifths in 8 tenths.

3. How many *fourths* are equal to $\frac{9}{12}$? $\frac{12}{16}$? $\frac{10}{20}$?

4. Reduce $\frac{9}{15}$ to fifths; $\frac{15}{24}$ to eighths.

5. What are the factors common to the terms of $\frac{12}{18}$? $\frac{20}{30}$?

6. Change $\frac{15}{30}$ to a fraction having lower terms; $\frac{18}{27}$; $\frac{16}{32}$.

7. Change $\frac{12}{36}$ to a fraction having a denominator $\frac{1}{3}$ as great; $\frac{1}{4}$ as great; $\frac{1}{2}$ as great.

8. In what lower terms can $\frac{40}{60}$ be expressed?

9. Express $\frac{12}{28}$ in its *lowest* terms; $\frac{18}{24}$; $\frac{24}{36}$; $\frac{20}{40}$.

A fraction is reduced to its *Lowest Terms,* when its numerator and denominator are *prime to each other.*

174. PRINCIPLE.—*Dividing both terms of a fraction by the same number does not change its value* (**167,** III, 2).

WRITTEN EXERCISES.

1. Reduce $\frac{36}{48}$ to its lowest terms.

OPERATION.

$$\tfrac{36}{48} \div \tfrac{4}{4} = \tfrac{9}{12}; \quad \tfrac{9}{12} \div \tfrac{3}{3} = \tfrac{3}{4}$$

Or,

$$\tfrac{36}{48} \div \tfrac{12}{12} = \tfrac{3}{4}$$

ANALYSIS.—Dividing both terms of the given fraction $\frac{36}{48}$ by 4 the result is $\frac{9}{12}$. (PRIN.) Again dividing both terms of $\frac{9}{12}$ by 3 the result is $\frac{3}{4}$. Since the terms of $\frac{3}{4}$ are *prime to each other,* the lowest terms of $\frac{36}{48}$ are $\frac{3}{4}$.

The same result may be obtained more directly by dividing the terms of the given fraction by their greatest common divisor 12.

2. Reduce $\frac{60}{72}$ to its lowest terms.

3. Reduce $\frac{72}{120}$ to its lowest terms.

4. Reduce $\frac{48}{64}$ to its lowest terms.

Rule.—*Reject all factors common to the terms of the given fraction.* Or,

Divide the terms of the given fraction by their greatest common divisor.

Reduce to their lowest terms,

5. $\frac{96}{100}$.
6. $\frac{63}{81}$.
7. $\frac{72}{108}$.
8. $\frac{144}{156}$.
9. $\frac{45}{315}$.
10. $\frac{98}{112}$.
11. $\frac{32}{256}$; $\frac{126}{162}$.
12. $\frac{324}{486}$; $\frac{292}{657}$.
13. $\frac{420}{2310}$; $\frac{5400}{7200}$.

14. Express in the simplest form 1344 divided by 1536.

Case III.

175. To reduce an integer or a mixed number to an improper fraction.

1. How many *thirds* in 4?

Analysis.—Since in 1 there are 3 thirds, in 4 there are 4 times 3 thirds, or 12 thirds. Hence in 4 there are $\frac{12}{3}$.

2. In 2 pears how many fourths of a pear? In 3 pears?

3. In $1 how many fifths of a dollar? Sixths? Eighths? Tenths? Twelfths?

4. In $5\frac{3}{4}$ yards, how many fourths of a yard?

Analysis.—Since in 1 yard there are 4 fourths, in 5 yards there are 5 times 4 fourths, or 20 fourths; 20 fourths and 3 fourths are 23 fourths. Hence in $5\frac{3}{4}$ yards there are $\frac{23}{4}$ yards.

5. How many sixths in $\$2\frac{5}{6}$? In $5\frac{1}{6}$ rods? In $7\frac{2}{6}$ acres?

6. Among how many boys can $6\frac{3}{4}$ quarts of walnuts be distributed, if each boy receive $\frac{1}{4}$ of a quart?

WRITTEN EXERCISES.

1. Reduce 7 to sixths.

OPERATION.

6 sixths

$$\begin{array}{l} 6 \text{ sixths} \\ \underline{\quad 7} \\ 42 \text{ sixths} = \frac{42}{6} \end{array}$$

ANALYSIS.—Since 1 is equal to 6 sixths, 7 is equal to 7 times 6 sixths, or 42 sixths. Hence $7 = \frac{42}{6}$.

2. Reduce $8\frac{4}{5}$ to fifths.

OPERATION.

$$\begin{array}{l} 8\frac{4}{5} \\ 5 \\ 40 \text{ fifths, and} \\ \frac{40}{5} + \frac{4}{5} = \frac{44}{5} \end{array}$$

ANALYSIS.—Since 1 is equal to 5 fifths, 8 is equal to 8 times 5 fifths, or 40 fifths, to which add 4 fifths, and the result is 44 fifths.

An *integer* is reduced to a fractional *form* by writing 1 under it for a denominator; thus, $7 = \frac{7}{1}$; $15 = \frac{15}{1}$.

3. Change 44 bushels to *fourths* of a bushel.

4. Change $15\frac{2}{3}$ years to *thirds* of a year.

5. Change $29\frac{5}{7}$ weeks to *sevenths* of a week.

RULE.—*Multiply the integer by the required denominator, and if there be a fraction, add its numerator to this product, and under the result write the denominator.*

Reduce

6. 48 to eighths.
7. $156\frac{5}{6}$ to sixths.
8. 124 to ninths.

Change

9. $95\frac{3}{10}$ to tenths.
10. 72 to fourteenths.
11. $216\frac{9}{25}$ to twenty-fifths.

Reduce to improper fractions,

12. $75\frac{9}{13}$.
13. $224\frac{5}{18}$.
14. $307\frac{18}{25}$.
15. $176\frac{14}{90}$.
16. $97\frac{7}{120}$.
17. $86\frac{5}{17}$.
18. $450\frac{12}{20}$.
19. $261\frac{7}{80}$.
20. $176\frac{27}{45}$.
21. $304\frac{72}{115}$.
22. $1227\frac{13}{152}$.
23. $2475\frac{24}{30}$.
24. $1306\frac{9}{75}$.
25. $2500\frac{3}{64}$.
26. $4006\frac{121}{240}$.

CASE IV.

176. To reduce an improper fraction to an integer, or mixed number.

1. In $\frac{9}{4}$, how many units?

ANALYSIS.—Since 4 fourths equal 1, 9 fourths equal as many times 1, as 4 fourths are contained times in 9 fourths, which is $2\frac{1}{4}$ times. Hence $\frac{9}{4} = 2\frac{1}{4}$.

2. How many times 1 are $\frac{13}{3}$? $\frac{9}{7}$? $\frac{25}{8}$? $\frac{36}{12}$?
3. How many times 1 are $\frac{42}{7}$? $\frac{56}{11}$? $\frac{63}{12}$?
4. How many yards are $\frac{12}{3}$ of a yard? $\frac{20}{3}$? $\frac{35}{8}$?
5. How many dollars are $\frac{16}{4}$ of a dollar? $\frac{45}{20}$?
6. If a man buy $\frac{30}{8}$ of a yard of cloth, how many yards does he buy? If $\frac{75}{12}$? If $\frac{108}{10}$?

WRITTEN EXERCISES.

1. Reduce $\frac{147}{9}$ to a mixed number.

OPERATION.

$\frac{147}{9} = 147 \div 9 = 16\frac{1}{3}$

ANALYSIS.—Since 9 ninths equal 1, 147 ninths equal as many times 1 as 9 ninths are contained times in 147 ninths, which is $16\frac{3}{9}$ times. Hence $\frac{147}{9} = 16\frac{1}{3}$.

2. In $\frac{96}{8}$, how many units?
3. In $\frac{107}{16}$ of a pound, how many pounds?
4. Change $\frac{162}{7}$ to a mixed number.

RULE.—*Divide the numerator by the denominator.*

Reduce to integers or mixed numbers.

5. $\frac{651}{7}$.
6. $\frac{247}{12}$.
7. $\frac{476}{17}$.
8. $\frac{1245}{22}$.
9. $\frac{1345}{24}$.
10. $\frac{4407}{113}$.
11. $\frac{4483}{35}$.
12. $\frac{5919}{59}$.
13. $\frac{2184}{42}$.
14. $\frac{7672}{112}$.
15. $\frac{17007}{33}$.
16. $\frac{32460}{48}$.

Case V.

177. To reduce fractions to a common denominator.

1. How many *fourths* in 1? In $\frac{1}{2}$?
2. How many *sixths* in 1? In $\frac{1}{2}$? In $\frac{1}{3}$?
3. How many *eighths* in 1? In $\frac{1}{2}$? In $\frac{1}{4}$? In $\frac{3}{4}$?
4. How many *ninths* in 1? In $\frac{1}{3}$? In $\frac{2}{3}$?
5. Express $\frac{1}{2}$ and $\frac{1}{3}$ each as twelfths.
6. Express $\frac{1}{3}$ and $\frac{1}{5}$ each as fifteenths.
7. Change $\frac{2}{3}$ and $\frac{3}{4}$ to fractions having the same denominator.
8. Change $\frac{1}{2}$ and $\frac{5}{6}$ to fractions having the same denominator.
9. Multiply both terms of $\frac{4}{5}$ by 3 and explain why the value of the fraction is not changed.
10. What is a multiple of 2? Of 3? Of 4? Of 5?
11. What is a common multiple of 2 and 3? Of 4 and 5?
12. What is the least common multiple of 2, 5 and 10?
13. What is a common multiple of the denominators of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$?
14. What is the least common multiple of the denominators of $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$?
15. Find a common multiple of 3, 4 and 6.
16. Name two common multiples of 4 and 8?
17. What is the least common multiple of 2, 4 and 6?
18. Change $\frac{1}{2}$ and $\frac{1}{3}$ to fractions having the same fractional unit.
19. Change $\frac{2}{3}$ and $\frac{3}{4}$ to fractions having the same denominator.

20. Name four fractions that can be reduced to 24ths.

21. Reduce $\frac{5}{6}$ and $\frac{3}{8}$ to equivalent fractions having 24 for their denominator.

22. Reduce $\frac{1}{3}$ and $\frac{2}{9}$ each to 18ths. To 27ths.

178. Principles.—1. *A Common Denominator of two or more fractions is a common multiple of their denominators.*

2. *The Least Common Denominator of two or more fractions is the least common multiple of their denominators.*

WRITTEN EXERCISES.

1. Reduce $\frac{3}{4}$ and $\frac{5}{6}$ to equivalent fractions having a common denominator.

OPERATION.

$$4 \times 6 = 24$$

$$\frac{3 \times 6}{4 \times 6} = \frac{18}{24}$$

$$\frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$

Analysis.—The product of the denominators 4 and 6 is 24, a common denominator.

But since the value of the fractions must not be changed, each numerator must be multiplied by the same number that its denominator is multiplied by (**167,** III, 1).

Hence multiplying the terms of $\frac{3}{4}$ by 6, the result is $\frac{18}{24}$, and of $\frac{5}{6}$ by 4, the result is $\frac{20}{24}$. Or,

The numerators may be found by taking such part of the common denominator 24 as the given fraction is of 1; thus, $\frac{3}{4}$ of 24 is 18, and $\frac{5}{6}$ of 24 is 20; hence $\frac{3}{4} = \frac{18}{24}$, and $\frac{5}{6} = \frac{20}{24}$.

Reduce to equivalent fractions having a common denominator,

2. $\frac{2}{3}$ and $\frac{6}{7}$.
3. $\frac{3}{8}$ and $\frac{5}{12}$.
4. $\frac{1}{6}$ and $\frac{7}{10}$.
5. $\frac{10}{16}$ and $\frac{11}{24}$.
6. $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{5}{6}$.
7. $\frac{1}{3}$, $\frac{3}{5}$ and $\frac{4}{7}$.
8. $\frac{4}{5}$, $\frac{1}{6}$ and $\frac{5}{8}$.
9. $\frac{11}{12}$, $\frac{3}{14}$ and $\frac{5}{16}$.
10. $\frac{3}{2}$, $\frac{3}{4}$ and $\frac{7}{9}$.
11. $\frac{7}{10}$, $\frac{5}{4}$ and $\frac{5}{6}$.
12. $\frac{7}{12}$, $\frac{1}{2}$, $\frac{4}{5}$ and $\frac{2}{3}$.
13. $\frac{2}{3}$, $\frac{1}{2}$, $\frac{5}{7}$ and $\frac{8}{9}$.

14. Change $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{9}$ to equivalent fractions having the least common denominator.

OPERATION.

$$\begin{array}{r|rrr} 3 & 3 & 4 & 9 \\ \hline & 1 & 4 & 3 \end{array}$$

$$3 \times 4 \times 3 = 36$$
$$36 \div 3 \times 2 = 24$$
$$36 \div 4 \times 3 = 27$$
$$36 \div 9 \times 5 = 20$$
$$\tfrac{2}{3} = \tfrac{24}{36};\ \tfrac{3}{4} = \tfrac{27}{36};\ \tfrac{5}{9} = \tfrac{20}{36}.$$

Analysis.—The least common multiple of the given denominators is 36, which is their least common denominator. (Prin. 2.) $\frac{1}{3}$ of 36 is 12, and $\frac{2}{3}$ of 36 is 2 times 12, or 24; that is, $36 \div 3 \times 2 = 24$, the numerator. Hence $\frac{2}{3}$ equals $\frac{24}{36}$, etc.

15. Change $\frac{3}{4}$, $\frac{5}{8}$ and $\frac{9}{16}$ to equivalent fractions having the least common denominator.

16. Change $\frac{4}{5}$, $\frac{3}{10}$ and $\frac{7}{8}$ to equivalent fractions having the least common denominator.

Rule.—1. To find a common denominator of two or more fractions.

Multiply the terms of each fraction by the denominators of all the other fractions.

2. To find their *least* common denominator.

I. *Find the least common multiple of the denominators of the given fractions for their least common denominator.*

II. *Divide this common denominator by each of the given denominators, and multiply each numerator by the corresponding quotient. The products will be the new numerators.*

Reduce to their least common denominator,

17. $\frac{1}{7}$, $\frac{5}{4}$ and $\frac{12}{24}$.
18. $\frac{3}{4}$, $\frac{7}{10}$ and $\frac{4}{5}$.
19. $\frac{5}{9}$, $\frac{6}{7}$ and $\frac{2}{3}$.
20. $\frac{3}{2}$, $\frac{3}{4}$ and $\frac{7}{9}$.
21. $\frac{5}{7}$, $\frac{3}{14}$ and $\frac{10}{21}$.
22. $\frac{12}{20}$, $\frac{7}{15}$ and $\frac{2}{3}$.
23. $\frac{2}{3}$, $\frac{4}{13}$, $\frac{25}{26}$ and $\frac{4}{39}$.
24. $\frac{5}{6}$, $\frac{7}{15}$, $\frac{5}{24}$ and $\frac{37}{60}$.
25. $\frac{7}{8}$, $\frac{5}{12}$, $\frac{13}{16}$ and $\frac{29}{96}$.

ADDITION.

179. 1. What is the sum of 1 fifth and 3 fifths? Of 2 sixths and 3 sixths? Of 1 eighth, 3 eighths and 2 eighths?

2. If Lillie have 3 fourths of an orange, Ella 2 fourths, and Clarence 1 fourth, how many fourths have all?

3. How many sevenths are $\frac{1}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, and $\frac{5}{7}$?

4. What is the sum of $\frac{4}{6}$, $\frac{5}{6}$ and $\frac{1}{6}$? Of $\frac{3}{5}$, $\frac{4}{5}$ and $\frac{2}{5}$?

5. How many times 1 are $\frac{4}{8}+\frac{5}{8}+\frac{7}{8}$? $\frac{4}{9}+\frac{5}{9}+\frac{6}{9}$?

6. A boy paid $\$\frac{1}{5}$ for a slate, $\$\frac{2}{5}$ for a reader, and $\$\frac{4}{5}$ for an arithmetic. How much did he pay for all?

7. How do you add fractions having a common denominator?

8. Mary paid $\$\frac{2}{3}$ for some ribbon, and $\$\frac{5}{6}$ for a pair of gloves: how much did she pay for both?

> ANALYSIS.—She paid the sum of $\$\frac{2}{3}$ and $\$\frac{5}{6}$. $\frac{2}{3}$ equal $\frac{8}{12}$, and $\frac{5}{6}$ equal $\frac{10}{12}$; $\frac{8}{12}$ and $\frac{10}{12}$ are $\frac{18}{12}$, equal to $1\frac{6}{12}$, or $1\frac{1}{2}$. Hence she paid $\$1\frac{1}{2}$ for both.

9. A farmer sold $\frac{1}{3}$ of his grain to one man, $\frac{1}{4}$ to another, and $\frac{1}{6}$ to another: what part of all his grain did he sell?

10. How do you add fractions having different denominators?

11. What is the sum of $\frac{2}{3}$ and $\frac{4}{5}$? Of $\frac{5}{7}$ and $\frac{1}{2}$? Of $\frac{5}{6}$ and $\frac{3}{8}$?

12. How much is $\frac{1}{5}+\frac{3}{4}+\frac{7}{10}$? $\frac{1}{2}+\frac{3}{4}+\frac{5}{8}$? $\frac{1}{2}+\frac{5}{6}+\frac{4}{9}$?

13. Find the sum of $\frac{5}{8}$, $\frac{1}{4}$ and $\frac{3}{10}$? Of $\frac{1}{2}$, $\frac{3}{8}$ and $\frac{5}{16}$?

14. How many twenty-fourths are $\frac{7}{12}+\frac{3}{8}+\frac{1}{6}$? $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}$?

15. How many thirty-sixths are $\frac{5}{18}+\frac{4}{9}$? $\frac{2}{3}+\frac{1}{4}+\frac{7}{12}$?

16. How many fortieths are $\frac{1}{4}+\frac{3}{8}+\frac{7}{10}$? $\frac{9}{20}+\frac{4}{5}$?

17. Sold $\frac{3}{5}$ of an acre of land to one man, $\frac{2}{3}$ to another, and $\frac{8}{15}$ to another: how much was sold to all?

18. A teamster drew $\frac{7}{8}$ of a ton of coal at one time, and $\frac{9}{10}$ at another: how much coal did he draw at both times?

19. Charles saved \$$\frac{5}{6}$ one week, \$$\frac{4}{5}$ the next, and \$$\frac{9}{10}$ the next: how much did he save in three weeks?

180. PRINCIPLE.—*Fractions can be added only when they have a common denominator, and when they express fractional units of the same kind.*

WRITTEN EXERCISES.

1. Find the sum of $\frac{5}{6}$ and $\frac{3}{8}$.

OPERATION.

$$\frac{5}{6}+\frac{3}{8}=\frac{20}{24}+\frac{9}{24}=\frac{29}{24}=1\frac{5}{24}$$

ANALYSIS.—Since the given fractions have not a common denominator, reduce them to equivalent fractions having 24 for their least common denominator. Then add their numerators and write the sum over the common denominator. Hence $\frac{5}{6}+\frac{3}{8}=1\frac{5}{24}$.

2. Find the sum of $\frac{2}{5}$, $\frac{5}{6}$ and $\frac{3}{10}$. *Ans.* $1\frac{8}{15}$.

3. Find the sum of $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{7}{9}$.

4. Find the sum of $\frac{7}{12}$, $\frac{5}{8}$ and $\frac{3}{4}$.

5. What is the sum of $6\frac{1}{2}$, $7\frac{3}{4}$ and $3\frac{5}{6}$?

OPERATION.

$$\begin{aligned} 6\tfrac{1}{2} &= 6\tfrac{6}{12} \\ 7\tfrac{3}{4} &= 7\tfrac{9}{12} \\ 3\tfrac{5}{6} &= \underline{3\tfrac{10}{12}} \\ 16+\tfrac{25}{12} &= 18\tfrac{1}{12} \end{aligned}$$

ANALYSIS.—The sum of the fractions is $\frac{25}{12}=2\frac{1}{12}$, which added to the sum of the integers 16, gives $18\frac{1}{12}$, the required result.

6. What is the sum of $8\frac{3}{4}$, $22\frac{4}{5}$ and 7?

7. What is the sum of $11\frac{7}{8}$, $16\frac{2}{3}$ and $4\frac{1}{2}$?

8. What is the sum of 25, $41\frac{1}{3}$, and $\frac{5}{9}$.

RULE.—I. *Reduce the given fractions to equivalent fractions having the least common denominator, then add their numerators, and write the sum over the common denominator.*

II. *When there are mixed numbers or integers, add the fractions and integers separately, and then add the results.*

1. If the mixed numbers are small, they may be reduced to improper fractions, and then added after the usual method.

2. The solution of an example in fractions should not be considered *complete* as long as the *result* contains an improper fraction, or one that can be reduced to lower terms.

Find the sum of

9. $\frac{3}{8}$, $\frac{3}{4}$ and $\frac{9}{10}$.
10. $\frac{6}{7}$, $\frac{2}{9}$ and $\frac{17}{21}$.
11. $\frac{5}{7}$, $\frac{11}{14}$ and $\frac{31}{42}$.
12. $\frac{4}{5}$, $\frac{9}{20}$ and $1\frac{5}{8}$.
13. $1\frac{1}{2}$, 2, $\frac{2}{3}$ and $3\frac{3}{4}$.
14. $\frac{7}{8}$, $15\frac{1}{4}$, $4\frac{1}{2}$ and $25\frac{1}{3}$.
15. $\frac{3}{4}$, $\frac{7}{16}$, $\frac{5}{6}$ and $4\frac{5}{12}$.
16. $30\frac{4}{5}$, $4\frac{9}{10}$, $10\frac{5}{6}$ and $57\frac{1}{3}$.

17. What is the sum of \$$14\frac{1}{5}$, \$$39\frac{7}{10}$, \$$16\frac{3}{8}$ and \$$25\frac{1}{4}$?

18. What is the sum of \$$128\frac{1}{4}$, \$$43\frac{7}{8}$, \$$435\frac{1}{7}$, \$15 and \$$\frac{3}{4}$?

19. Three men bought a horse. The first paid \$$41\frac{5}{8}$, the second paid \$$52\frac{11}{12}$, and the third paid as much as the other two. What was the cost of the horse ?

SUBTRACTION.

181. 1. What is the difference between 6 sevenths and 2 sevenths? 5 ninths and 7 ninths? 11 twelfths and 5 twelfths?

2. James had 5 sixths of a pine-apple, and gave 2 sixths to his sister : what part had he left ?

3. If from 1 yard of cloth 3 eighths be cut, how many eighths will remain ?

4. What is the difference between $\frac{4}{7}$ and $\frac{6}{7}$? $\frac{5}{9}$ and $\frac{8}{9}$?

5. How much is $\frac{9}{14}$ less $\frac{5}{14}$? $\frac{12}{20}$ less $\frac{7}{20}$? $\frac{16}{25}$ less $\frac{9}{25}$?

6. How do you subtract fractions having a common denominator?

7. John had $\$\frac{4}{5}$ and spent $\$\frac{1}{3}$ for a kite. How much had he left?

Analysis.—He had left the difference between $\$\frac{4}{5}$ and $\$\frac{1}{3}$. $\frac{4}{5}$ equal $\frac{12}{15}$ and $\frac{1}{3}$ equals $\frac{5}{15}$; $\frac{12}{15}$ less $\frac{5}{15}$ leaves $\frac{7}{15}$. Hence he had $\$\frac{7}{15}$ left.

8. What is the difference between $\frac{9}{10}$ and $\frac{4}{5}$? $\frac{2}{3}$ and $\frac{8}{9}$?

9. How do you subtract fractions having different denominators?

10. Subtract $\frac{1}{2}$ from $\frac{3}{4}$; $\frac{2}{3}$ from $\frac{5}{6}$; $\frac{3}{4}$ from $\frac{7}{8}$; $\frac{2}{5}$ from $\frac{5}{6}$.

11. If a grocer buys tea at $\$\frac{5}{8}$ a pound and sells it at $\$\frac{3}{4}$, does he gain or lose, and how much?

182. Principle.—*Fractions can be subtracted only when they have a common denominator, and when they express fractional units of the same kind.*

WRITTEN EXERCISES.

1. Subtract $\frac{3}{7}$ from $\frac{2}{3}$.

OPERATION.

$$\frac{2}{3} - \frac{3}{7} = \frac{14}{21} - \frac{9}{21} = \frac{5}{21}$$

Analysis.—Since the given fractions have not a common denominator, reduce them to equivalent fractions having 21 for their least common denominator. Then subtract the less numerator from the greater and write the difference over the common denominator. Hence $\frac{2}{3}-\frac{3}{7}=\frac{5}{21}$, the required difference.

2. Subtract $\frac{3}{4}$ from $\frac{5}{6}$.
3. Subtract $\frac{5}{8}$ from $\frac{11}{12}$.
4. Subtract $\frac{3}{8}$ from $\frac{6}{7}$.
5. Subtract $\frac{5}{12}$ from $\frac{4}{5}$.

6. From $16\frac{1}{3}$ take $7\frac{3}{4}$.

OPERATION.

$$\begin{array}{rcrr} 16\frac{1}{3} & = & 16\frac{4}{12} = & 15\frac{16}{12} \\ 7\frac{3}{4} & = & 7\frac{9}{12} & \underline{7\frac{9}{12}} \\ & & & 8\frac{7}{12} \end{array}$$

ANALYSIS.—First reduce the fractional parts of the given numbers to their least common denominator 12. Since $\frac{9}{12}$ cannot be subtracted from $\frac{4}{12}$, take 1 or $\frac{12}{12}$ from 16, leaving 15, and add it to the $\frac{4}{12}$, making $\frac{16}{12}$. Then subtracting the fractions and integers separately we have $15\frac{16}{12} - 7\frac{9}{12} = 8\frac{7}{12}$, the required result.

7. From 24 subtract $16\frac{5}{9}$.

OPERATION.

$$\begin{array}{rcr} 24 & = & 23\frac{9}{9} \\ & & \underline{16\frac{5}{9}} \\ & & 7\frac{4}{9} \end{array}$$

ANALYSIS.—Take 1 from 24, and reduce it to *ninths*, then 24 will equal $23\frac{9}{9}$, from which subtract $16\frac{5}{9}$, and the difference, $7\frac{4}{9}$ will be the required result.

8. From $25\frac{3}{4}$ take $18\frac{1}{2}$.
9. From $47\frac{1}{6}$ take $39\frac{1}{8}$.
10. From 328 take $314\frac{6}{7}$.
11. From $44\frac{2}{3}$ take $29\frac{3}{10}$.
12. From $276\frac{3}{5}$ take $40\frac{6}{7}$.
13. From $429\frac{14}{25}$ take 216.

RULE.—I. *Reduce the fractions when necessary to equivalent fractions having the least common denominator, and write the difference between the numerators over the common denominator.*

II. *When there are mixed numbers or integers, subtract the fractional and integral parts separately, and unite the results.*

If the mixed numbers are small, they may be reduced to improper fractions, and subtracted according to the usual method.

Subtract

14. $1\frac{5}{14}$ from $\frac{13}{7}$.
15. $3\frac{15}{18}$ from $6\frac{5}{12}$.
16. $1\frac{1}{3}$ from $\frac{15}{11}$.
17. $2\frac{8}{15}$ from $5\frac{1}{3}$.

Find the difference between

18. $\frac{7}{8}$ and $\frac{7}{9}$.
19. $\frac{7}{20}$ and $\frac{5}{8}$.
20. $\frac{7}{12}$ and $\frac{21}{60}$.
21. $\frac{9}{14}$ and $\frac{16}{63}$.
22. $\frac{1}{15}$ and $\frac{5}{21}$.
23. $\frac{5}{7}$ and $\frac{4}{11}$.
24. $18\frac{1}{3}$ and $\frac{15}{16}$.
25. $27\frac{7}{8}$ and $16\frac{5}{9}$.
26. $48\frac{7}{25}$ and 32.
27. $126\frac{1}{2}$ and $75\frac{6}{7}$.
28. 97 and $13\frac{7}{20}$.
29. $327\frac{4}{11}$ and 450.
30. $106\frac{9}{33}$ and $214\frac{7}{11}$.
31. 200 and $306\frac{5}{17}$.
32. $421\frac{1}{9}$ and $245\frac{1}{5}$.

33. What number added to $12\frac{6}{7}$ will make $18\frac{1}{4}$?

34. A grocer sold tea at $\$\frac{15}{16}$ a pound that cost him $\$\frac{14}{20}$. How much was his gain?

35. From a cask of wine containing $60\frac{1}{2}$ gallons, $21\frac{5}{8}$ gallons were drawn. How much remained?

ADDITION AND SUBTRACTION.

ORAL EXAMPLES.

183. 1. How much less than 1 is $\frac{1}{6}+\frac{1}{3}$? $\frac{1}{4}+\frac{2}{5}$? $\frac{1}{2}+\frac{3}{7}$?

2. How much greater than 1 is $\frac{5}{6}+\frac{1}{2}$? $\frac{3}{5}+\frac{2}{3}$? $\frac{4}{5}+\frac{3}{4}$?

3. What is the difference between 3 and $1\frac{1}{2}$? 4 and $2\frac{4}{5}$?

4. Henry paid $\frac{1}{4}$ of all his money for a knife, $\frac{1}{8}$ for a ball, and $\frac{1}{6}$ for a necktie: what part of his money had he left?

5. A seedsman bought $2\frac{1}{2}$ bushels of clover seed, and then sold $\frac{3}{8}$ of a bushel to one man, and $\frac{3}{4}$ to another: how much had he left?

6. A real estate broker sold from a piece of land at different times $\frac{1}{3}$, $\frac{3}{8}$ and $\frac{1}{4}$ of it: how much remained unsold?

7. What is the difference between $\frac{1}{2}+\frac{4}{5}$ and $\frac{1}{4}+\frac{7}{10}$?

8. How much greater than $2\frac{7}{8}$ is $1\frac{5}{6}+3\frac{1}{4}$?

9. From the sum of $\frac{3}{5}$ and $\frac{5}{6}$ take their difference.

10. Paid $\$6\frac{3}{8}$ for a ton of coal, and $\$2\frac{1}{6}$ for a load of wood: how much change must be returned for $10?

Find the second member of the following equations :

11. $\frac{3}{4}+\frac{2}{5}-\frac{1}{2}=?$
12. $\frac{7}{8}-\frac{1}{6}+\frac{1}{4}=?$
13. $\frac{9}{10}+2-1\frac{1}{2}=?$
14. $4\frac{1}{7}+\frac{9}{14}-3\frac{2}{7}=?$
15. $12-\frac{7}{12}+7\frac{1}{4}=?$
16. $(\frac{5}{6}+\frac{2}{3})-(\frac{1}{2}+\frac{1}{4})=?$
17. $(2\frac{1}{5}-\frac{3}{10})+4\frac{3}{4}=?$
18. $8\frac{4}{9}-(6\frac{2}{3}-\frac{2}{9})=?$
19. $\frac{9}{16}+5\frac{1}{2}+3\frac{1}{4}=?$
20. $12\frac{13}{20}+(5\frac{4}{5}-2\frac{1}{2})=?$

WRITTEN EXAMPLES.

184. 1. From the sum of $\frac{6}{7}$, $\frac{5}{8}$ and $\frac{3}{4}$ take $\frac{13}{14}$.

2. From the sum of $\frac{9}{16}$ and $\frac{7}{8}$ take the difference of $1\frac{1}{4}$ and $\frac{5}{16}$.

3. A man owning $\frac{19}{27}$ of a cotton-mill, sold $\frac{1}{3}$ of the whole to one man and $\frac{2}{9}$ to another : what part had he left ?

4. To what must $\frac{2}{5}$ be added that the sum may be $\frac{3}{4}$?

5. What number must be added to the sum of $1\frac{7}{20}$ and $2\frac{2}{3}$ to make $36\frac{1}{4}$?

6. From the sum of $14\frac{1}{2}$ and $9\frac{1}{8}$ take the difference of $25\frac{3}{4}$ and $16\frac{1}{3}$?

7. From the sum of $\frac{7}{9}$ and $\frac{17}{21}$ take the difference of $\frac{4}{7}$ and $\frac{2}{3}$.

8. The sum of two numbers is $14\frac{7}{24}$, and the less is $3\frac{1}{8}$: what is the greater ?

9. The sum of two fractions is $\frac{37}{48}$, and the greater is $\frac{5}{8}$: what is the less ?

10. A clerk earned \$$75\frac{3}{4}$ one month, and paid for board \$$28\frac{7}{8}$, for washing \$$7\frac{1}{2}$, and other expenses \$$27\frac{7}{16}$: how much had he left ?

11. A man bequeathed $\frac{5}{18}$ of his property to his wife, $\frac{7}{12}$ to his son, and the remainder to his daughter : what part did the daughter receive ?

12. The difference of two numbers is $7\frac{1}{5}$, and the less is $20\frac{3}{4}$: what is the greater ?

13. The sum of two numbers is $26\frac{15}{32}$, and the greater is $15\frac{5}{8}$: what is the less ?

14. A lady having $50, paid $$12\frac{3}{8}$ for a bonnet, $$17\frac{5}{8}$ for a shawl, and $$3\frac{7}{10}$ for a veil : how much money had she left ?

What is the value of

15. $\frac{1}{7}+\frac{3}{4}+\frac{5}{8}-\frac{9}{14}$.
16. $4\frac{5}{6}+2\frac{3}{8}-5\frac{1}{4}$.
17. $18-5\frac{1}{4}+3\frac{1}{5}-\frac{7}{10}$.
18. $8\frac{4}{7}+\overline{12\frac{3}{5}-11\frac{1}{7}}+\frac{19}{35}$.
19. $\frac{11}{16}+1\frac{1}{8}+5\frac{1}{2}-2\frac{5}{32}$.
20. $28\frac{1}{4}+36\frac{5}{9}-(44-13\frac{1}{6})$.
21. $86\frac{9}{40}-(50\frac{1}{8}+15\frac{1}{5}-6\frac{1}{2})$.
22. $8\frac{1}{2}+\frac{3}{4}+5\frac{1}{6}-7\frac{8}{18}$.

MULTIPLICATION.

Case I.

185. To multiply a fraction by an integer.

1. What part of a yard is 2 times 1 third of a yard ?
2. What part of a dollar is 3 times 1 fourth of a dollar?
3. How many times 1 is 5 times $\frac{2}{3}$? 4 times $\frac{3}{5}$?
4. At $$\frac{3}{8}$ a pound, what will 4 pounds of sugar cost ?

Analysis.—Four pounds will cost 4 times $$\frac{3}{8}$, or $$\frac{12}{8}$, equal to $$1\frac{1}{2}$. Hence 4 pounds of sugar will cost $$1\frac{1}{2}$.

5. At $$\frac{2}{3}$ a bushel, what will 3 bushels of apples cost ? 5 bushels ? 6 bushels ? 7 bushels ? 10 bushels ?
6. If 1 horse eat $\frac{3}{4}$ of a bushel of oats in a day, how much will 4 horses eat ? 6 horses ? 8 horses ?
7. How many times 1 are 4 times $\frac{5}{7}$? 3 times $\frac{4}{5}$?
8. What cost 9 baskets of peaches, at $$\frac{4}{5}$ a basket ?
9. What cost 5 pounds of tea, at $$\frac{9}{10}$ a pound ?

10. If a basket hold $\frac{5}{6}$ of a bushel, how much will 4 baskets hold? 6 baskets? 7 baskets?

11. How much is 4 times $\frac{3}{4}$ of a mile? 7 times $\frac{5}{6}$ of a yard? 10 times $\frac{6}{7}$ of a week?

12. At \$6$\frac{2}{3}$ a ton, what will 5 tons of coal cost?

ANALYSIS.—Five tons of coal will cost 5 times \$6$\frac{2}{3}$; 5 times \$$\frac{2}{3}$ are $\frac{10}{3}$, equal to 3$\frac{1}{3}$; 5 times \$6 are \$30, and \$3$\frac{1}{3}$ added make \$33$\frac{1}{3}$. Hence 5 tons of coal will cost \$33$\frac{1}{3}$.

13. What cost 7 oranges at 4$\frac{1}{2}$ cents each?

14. How much is 4 times 5$\frac{2}{3}$? 6 times 7$\frac{3}{8}$? 5 times 12$\frac{4}{7}$?

15. How much is $\frac{4}{5} \times 6$? $\frac{5}{9} \times 4$? $2\frac{3}{4} \times 8$? $5\frac{7}{12} \times 4$?

16. Multiply $\frac{7}{12}$ by 3; by 4; by 7; by 6; by 9.

17. Multiply 6$\frac{4}{5}$ by 4; by 5; by 6; by 7; by 12.

186. PRINCIPLE.—*A fraction is multiplied by multiplying its numerator, or by dividing its denominator* **(167, I).**

WRITTEN EXERCISES.

1. Multiply $\frac{11}{12}$ by 6.

OPERATION.

$$\frac{11}{12} \times 6 = \frac{11 \times 6}{12} = \frac{66}{12} = 5\frac{1}{2}$$

$$\text{Or, } \frac{11}{12} \times 6 = \frac{11}{12 \div 6} = \frac{11}{2} = 5\frac{1}{2}$$

Or,

$$\begin{array}{r|l} {}^{2}\cancel{1}\cancel{2} & 11 \\ & \cancel{6} \\ \hline 2 & 11 \\ \cline{2-2} & 5\frac{1}{2} \end{array}$$

ANALYSIS.—To multiply $\frac{11}{12}$ by 6, either multiply the numerator of the fraction by 6, which gives $\frac{66}{12}$, equal to 5$\frac{1}{2}$. Or, divide the denominator of the fraction by 6, which gives $\frac{11}{2}$, equal to 5$\frac{1}{2}$, the same result. (PRIN.)

By using the *vertical* line and *cancellation* both operations are combined, and *shortened.*

In the first operation it is obvious that the *number* of parts or fractional units is increased, while their *size* or value remains the same. In the second operation the *size* is increased, while their *number* remains unchanged.

In like manner, multiply

2. $\frac{9}{24}$ by 4.
3. $\frac{12}{35}$ by 7.
4. $\frac{16}{45}$ by 12.
5. $\frac{7}{96}$ by 16.
6. $\frac{11}{64}$ by 25.
7. $\frac{25}{120}$ by 20.

8. Multiply $9\frac{4}{5}$ by 7.

OPERATION.

$$\begin{array}{rcl} 9\frac{4}{5} & & \\ 7 & & \\ \hline 5\frac{3}{5} & = & \frac{4}{5} \times 7 \\ 63 & = & 9 \times 7 \\ \hline 68\frac{3}{5} & = & 9\frac{4}{5} \times 7 \end{array}$$

Or,

$$9\frac{4}{5} = \frac{49}{5}$$

$$\frac{49}{5} \times 7 = \frac{343}{5} = 68\frac{3}{5}$$

Analysis. — To multiply $9\frac{4}{5}$ by 7, either multiply the fraction $\frac{4}{5}$ and the integer 9 separately, and add their products, which gives $68\frac{3}{5}$, the required product. Or, reduce the mixed number to an improper fraction, $\frac{49}{5}$, and multiply as in the first example.

Find the product

9. Of $6\frac{3}{4}$ by 8.
10. Of $19\frac{4}{5}$ by 7.
11. Of $23\frac{1}{4}$ by 20.
12. Of $44\frac{3}{8}$ by 9.
13. Of $24\frac{6}{7}$ by 12.
14. Of $142\frac{3}{10}$ by 11.

RULE.—*Multiply the numerator or divide the denominator by the integer.*

When the multiplicand is a *mixed* number.

Multiply the fraction and integer separately and add the products. Or,

Reduce the mixed number to an improper fraction, and then multiply.

15. Multiply $\frac{9}{21}$ by 7; $\frac{28}{36}$ by 9; $\frac{18}{42}$ by 14; $\frac{45}{63}$ by 21.

16. Multiply $112\frac{7}{8}$ by 8; $296\frac{3}{4}$ by 11; $156\frac{4}{5}$ by 12; $264\frac{1}{7}$ by 20.

17. At \$$\frac{5}{8}$ a bushel, what will 34 bushels of pears cost?

18. At \$$7\frac{3}{4}$ a barrel, what will 46 barrels of flour cost?

19. At \$$\frac{11}{12}$ each, what will 36 grass hooks cost?

20. If a man travel $25\frac{7}{10}$ miles in 1 day, how far can he travel in 5 days? In 10 days? In 16 days?

21. At $\$75\frac{7}{8}$ each, what will be the cost of 25 horses? 42? 62? 70?

22. At $\$125\frac{4}{5}$ an acre, what cost 126 acres of land?

CASE II.

187. To multiply an integer by a fraction.

1. What is $\frac{1}{2}$ of 6 yards? $\frac{1}{3}$ of 12 apples?
2. What is $\frac{1}{4}$ of $\$8$? $\frac{1}{5}$ of 20 men?
3. Multiplying by $\frac{1}{2}$ is the same as dividing by what?
4. Multiplying by $\frac{1}{4}$ is the same as dividing by what?

When a *fractional part* of an integer or fraction is to be taken, the word "*of*," and not "*times*," should be used.

5. What is $\frac{1}{6}$ of 5 miles?

ANALYSIS.—$\frac{1}{6}$ of 5 miles is 5 times $\frac{1}{6}$ of 1 mile, or $\frac{5}{6}$ of 1 mile.

6. How much is $\frac{1}{5}$ of 4 acres? $\frac{1}{7}$ of three weeks? $\frac{1}{8}$ of 5 tons? $\frac{1}{6}$ of 11 pounds?

7. At $\$5$ a yard, what will $\frac{3}{8}$ of a yard of cloth cost?

ANALYSIS.—Since 1 yard cost $\$5$, $\frac{3}{8}$ of a yard will cost $\frac{3}{8}$ of $\$5$, or 3 times $\frac{1}{8}$ of $\$5$. $\frac{1}{8}$ of $\$5$ is $\$\frac{5}{8}$, and 3 times $\$\frac{5}{8}$ are $\$\frac{15}{8}$, or $\$1\frac{7}{8}$. Hence $\frac{3}{8}$ of a yard of cloth will cost $\$1\frac{7}{8}$.

8. At $\$7$ a gross, what will $\frac{7}{8}$ of a gross of bottles cost?

9. At $\$8$ a ton, what will $\frac{2}{3}$ of a ton of coal cost? $\frac{3}{5}$ of a ton? $\frac{6}{7}$ of a ton?

10. What is $\frac{1}{8}$ of 16? $\frac{3}{4}$ of 16? $\frac{3}{8}$ of 24? $\frac{5}{6}$ of 30?

11. What is $\frac{3}{7}$ of 25? $\frac{4}{5}$ of 36? $\frac{5}{6}$ of 44? $\frac{4}{9}$ of 65?

12. Which is greater, $5 \times \frac{1}{4}$, or $\frac{1}{4}$ of 5? $4 \times \frac{3}{5}$, or $\frac{3}{5}$ of 4?

13. How much is $6 \times \frac{3}{4}$? $8 \times \frac{4}{5}$? $\frac{6}{7} \times 9$? $\frac{5}{9} \times 8$? $15 \times \frac{3}{8}$?

14. If an acre of land produce 40 bushels of corn, how much will $\frac{3}{4}$ of an acre produce? $\frac{5}{8}$? $\frac{4}{5}$? $\frac{5}{9}$? $\frac{7}{10}$?

15. At $7 a bushel, what will $4\frac{3}{4}$ bushels of clover seed cost?

Analysis.—Since 1 bushel cost $7, $4\frac{3}{4}$ bushels will cost $4\frac{3}{4}$ times $7. 4 times $7 are $28, and $\frac{3}{4}$ of $7 is 3 times $\frac{1}{4}$ of $7, or $$5\frac{1}{4}$, which added to $28, make $$33\frac{1}{4}$. Hence $4\frac{3}{4}$ bushels of clover seed will cost $$33\frac{1}{4}$.

16. If a man earn $4 a day, how much will he earn in $6\frac{2}{3}$ days? In $5\frac{1}{6}$ days? In $8\frac{3}{4}$ days?

17. If a barrel of flour last a family 6 weeks, how long will $5\frac{3}{4}$ barrels last them? $4\frac{3}{8}$ barrels? $7\frac{4}{5}$ barrels?

How much is	What is the product of
18. $\frac{2}{5}$ of 4 tons?	23. $2\frac{1}{4}$ times 12 cents?
19. $\frac{3}{7}$ of 10 weeks?	24. $5\frac{2}{3}$ times $7?
20. $\frac{7}{8}$ of 12 rods?	25. $4\frac{1}{8}$ times 9 feet?
21. $\frac{5}{9}$ of 27 yards?	26. $7\frac{3}{5}$ times 8 barrels?
22. $\frac{5}{6}$ of 40 acres?	27. $9\frac{3}{7}$ times 10 cords?

188. Principle.—*To multiply an integer by a fraction is to take such a part of the integer as the fraction is of a unit.*

WRITTEN EXERCISES.

1. Multiply 48 by $\frac{3}{8}$.

OPERATION.

$$48 \times \frac{3}{8} = 48 \div 8 \times 3 = 18$$

$$\text{Or, } 48 \times \frac{3}{8} = \frac{{}^{6}\cancel{48} \times 3}{\cancel{8}} = 18$$

$$\begin{array}{r|l} & \cancel{48}^{6} \\ \cancel{8} & 3 \\ \hline & 18 \end{array}$$

Analysis.—To multiply 48 by $\frac{3}{8}$ is to find $\frac{3}{8}$ of 48. $\frac{3}{8}$ of 48 is 3 times $\frac{1}{8}$ of 48; $\frac{1}{8}$ of 48 is 6, and 3 times 6 are 18, the required product. Or,

Since $\frac{3}{8}$ is $\frac{1}{8}$ of 3, $\frac{3}{8}$ of 48 is $\frac{1}{8}$ of 3 times 48, or $\frac{48 \times 3}{8} = 18$.

What is the product of

2. 27 by $\frac{5}{9}$. | 4. 84 by $\frac{9}{14}$. | 6. 296 by $\frac{6}{7}$.
3. 108 by $\frac{7}{18}$. | 5. 156 by $\frac{3}{25}$. | 7. 420 by $\frac{5}{18}$.

1. A fraction is *multiplied* by a number equal to its *denominator* by *cancelling* the denominator. Thus $\frac{5}{7} \times 7 = 5$.

2. Cancelling a *factor* of the denominator *multiplies* the fraction by that factor. Thus, $\frac{7}{12} \times 3 = \frac{7}{4}$.

8. Multiply 26 by $5\frac{3}{4}$.

OPERATION.

$$\begin{array}{r l} 26 & \\ \underline{5\frac{3}{4}} & \\ 19\frac{1}{2} & = \frac{3}{4} \text{ of } 26 \\ \underline{130} & \\ 149\frac{1}{2} & \end{array}$$

Or, $5\frac{3}{4} = \frac{23}{4}$; and $\frac{26 \times 23}{4} = 149\frac{1}{2}$

ANALYSIS.—$5\frac{3}{4}$ times 26 is 5 times 26, or 130; and $\frac{3}{4}$ of 26 is $19\frac{1}{2}$, which added to 130 makes $149\frac{1}{2}$, the required product. Or,

$5\frac{3}{4}$ equals $\frac{23}{4}$, and 26 multiplied by $\frac{23}{4}$ equals $\frac{26 \times 23}{4} = 149\frac{1}{2}$.

What is the product of

9. 28 by $5\frac{2}{3}$. | 11. 98 by $6\frac{5}{7}$. | 13. 140 by $4\frac{7}{10}$.
10. 36 by $7\frac{3}{4}$. | 12. 126 by $8\frac{1}{9}$. | 14. 228 by $9\frac{5}{12}$.

RULE.—I. *Multiply the integer by the numerator, and divide the product by the denominator.*

When the multiplier is a *mixed* number.

II. *Multiply by the integer and fraction separately, and add the products.* Or,

Reduce the mixed number to an improper fraction and then multiply.

Multiply

15. 139 by $\frac{9}{13}$. | 18. 216 by $\frac{10}{13}$. | 21. 796 by $9\frac{1}{8}$.
16. 290 by $\frac{5}{6}$. | 19. 525 by $\frac{6}{27}$. | 22. 2112 by $10\frac{3}{5}$.
17. 426 by $\frac{21}{25}$. | 20. 610 by $\frac{3}{44}$. | 23. 1321 by $30\frac{6}{7}$.

24. Find $\frac{3}{4}$ of 196 ; of 440 ; of 672 ; of 261 ; of 900.

25. Multiply 224 by $6\frac{5}{8}$; by $5\frac{2}{9}$; by $8\frac{3}{7}$; by $14\frac{7}{12}$; by $24\frac{8}{31}$.

26. If an acre of land be worth $240, what is $\frac{7}{15}$ of it worth?

27. If a man travel 36 miles a day, how far can he travel in $16\frac{5}{6}$ days?

28. If a hogshead of sugar be worth $112, what is $\frac{7}{8}$ of it worth?

29. At $12 a bushel, what is $\frac{9}{16}$ of a bushel of clover seed worth?

30. At 84 cents a pound, what will $9\frac{5}{8}$ pounds of tea cost?

31. At 96 cents a gallon, what will $14\frac{7}{16}$ gallons of syrup cost?

CASE III.

189. To multiply a fraction by a fraction.

1. If $\frac{1}{2}$ of a melon be cut into 2 equal pieces, what part of the whole melon is 1 piece? If into 3 equal pieces? If into 4 equal pieces?

2. What part of 1 is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{4}$ of $\frac{1}{2}$?

3. A boy having $\frac{1}{3}$ of a pine-apple gave $\frac{1}{2}$ of it to his sister: what part of the pine-apple did he give her?

ANALYSIS.—Since $\frac{1}{3}$ is equal to $\frac{2}{6}$, $\frac{1}{2}$ of $\frac{1}{3}$ is equal to $\frac{1}{2}$ of $\frac{2}{6}$, or $\frac{1}{6}$. Hence he gave his sister $\frac{1}{6}$ of the pine-apple.

4. Taking $\frac{1}{2}$ of a fraction is the same as dividing it by what integer?

5. Taking $\frac{1}{3}$, $\frac{1}{4}$ or $\frac{1}{5}$ of a fraction is the same as dividing by what integers?

6. What is $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{4}$ of $\frac{1}{2}$? $\frac{1}{4}$ of $\frac{1}{3}$? $\frac{1}{2}$ of $\frac{1}{4}$?

7. What is $\frac{1}{2}$ of $\frac{3}{4}$? $\frac{1}{3}$ of $\frac{5}{6}$? $\frac{1}{5}$ of $\frac{2}{3}$? $\frac{1}{4}$ of $\frac{4}{5}$?

8. Which is greater, $\frac{1}{3}$ of $\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{4}$ of $\frac{1}{3}$, or $\frac{1}{6}$?

9. If I own $\frac{5}{8}$ of an acre of land, and sell $\frac{1}{2}$ of it, what part of an acre do I sell?

10. A man owning $\frac{4}{5}$ of a yacht sold $\frac{2}{3}$ of his share: what part of the whole yacht did he sell?

Analysis.—He sold $\frac{2}{3}$ of $\frac{4}{5}$ of the yacht, or 2 times $\frac{1}{3}$ of $\frac{4}{5}$ of it; $\frac{1}{3}$ of $\frac{4}{5}$ is $\frac{4}{15}$, and $\frac{2}{3}$ of $\frac{4}{5}$ is 2 times $\frac{4}{15}$, or $\frac{8}{15}$. Hence he sold $\frac{8}{15}$ of the yacht.

11. What is $\frac{1}{3}$ of $\frac{4}{7}$? $\frac{2}{3}$ of $\frac{4}{7}$? $\frac{1}{4}$ of $\frac{5}{6}$? $\frac{3}{4}$ of $\frac{5}{6}$? $\frac{1}{6}$ of $\frac{3}{5}$? $\frac{5}{6}$ of $\frac{3}{5}$? $\frac{3}{4}$ of $\frac{6}{7}$? $\frac{4}{9}$ of $\frac{3}{10}$?

12. How much is $\frac{3}{8} \times \frac{2}{3}$? $\frac{6}{7} \times \frac{3}{4}$? $\frac{7}{10} \times \frac{2}{5}$?

13. Multiply $\frac{8}{9}$ by $\frac{1}{4}$; by $\frac{3}{4}$; by $\frac{4}{5}$; by $\frac{1}{6}$; by $\frac{5}{6}$; by $\frac{1}{8}$; by $\frac{3}{8}$; by $\frac{5}{8}$.

14. Hattie, having $\frac{7}{8}$ of a yard of silk, gave her sister $\frac{2}{3}$ of it: what part of a yard did she give her?

15. James, having \$$\frac{8}{9}$, gave $\frac{3}{4}$ of it for a knife: what part of a dollar did he pay for the knife?

16. If a yard of ribbon cost \$$\frac{2}{3}$, what cost $\frac{1}{4}$ of a yard?

17. If a pound of coffee cost \$$\frac{3}{8}$, what cost $\frac{2}{3}$ of a pound?

18. A man owning $\frac{3}{8}$ of a ship, sold $\frac{1}{4}$ of his share. What part of the whole vessel did he sell, and what part did he retain?

19. How much money has James, if $\frac{5}{6}$ of \$18 is $\frac{3}{5}$ of of his money?

20. Mary used $\frac{1}{3}$ of $\frac{5}{8}$ of a yard of silk to trim her bonnet. What part of a yard was left?

21. A pair of pants cost \$10, and $\frac{4}{5}$ of their cost was $\frac{2}{3}$ the cost of a coat. What was the cost of the coat?

22. A man owning $\frac{3}{4}$ of a farm, sold $\frac{2}{3}$ of his share to his brother. What part of the farm did each own?

WRITTEN EXERCISES.

1. Multiply $\frac{15}{16}$ by $\frac{8}{9}$.

OPERATION.

$$\frac{15}{16} \times \frac{8}{9} = \frac{120}{144} = \frac{5}{6}$$

$$\text{Or, } \frac{{}^{5}\cancel{15}}{{}_{2}\cancel{16}} \times \frac{\cancel{8}}{\cancel{9}_{3}} = \frac{5}{6}$$

ANALYSIS.—To multiply $\frac{15}{16}$ by $\frac{8}{9}$ is to find $\frac{8}{9}$ of $\frac{15}{16}$, which is 8 times $\frac{1}{9}$ of $\frac{15}{16}$; $\frac{1}{9}$ of $\frac{15}{16}$ is $\frac{15}{16 \times 9}$; and $\frac{8}{9}$ of $\frac{15}{16}$ is 8 times $\frac{15}{16 \times 9}$, or $\frac{15 \times 8}{16 \times 9} = \frac{120}{144} = \frac{5}{6}$, the required product. Or, shortening the operation by cancelling equal factors in the numerators and denominators, we obtain $\frac{5}{6}$, the same result.

Multiply

2. $\frac{7}{15}$ by $\frac{4}{9}$.
3. $\frac{11}{12}$ by $\frac{9}{14}$.
4. $\frac{9}{16}$ by $\frac{18}{27}$.
5. $\frac{10}{14}$ by $\frac{7}{30}$.
6. $\frac{15}{21}$ by $\frac{17}{25}$.
7. $\frac{18}{49}$ by $\frac{7}{9}$.

8. Find the product of $\frac{5}{6}$, $4\frac{1}{2}$ and 2.

OPERATION.

$$\frac{5}{6} \times \frac{9}{2} \times \frac{2}{1} = \frac{15}{2} = 7\frac{1}{2}$$

Or,

$$\begin{array}{r|l} {}^{2}\cancel{6} & 5 \\ \cancel{2} & \cancel{9}^{3} \\ 1 & \cancel{2} \\ \hline 2 & 15 \\ \cline{2-2} & 7\frac{1}{2} \end{array}$$

ANALYSIS.—Before multiplying, change the mixed number $4\frac{1}{2}$ to an improper fraction, and the integer 2 to the *form* of a fraction by writing 1 for a denominator; then multiply as in the first example.

What is the product of

9. $\frac{3}{4} \times 4\frac{2}{3}$?
10. $1\frac{5}{7} \times \frac{21}{18}$?
11. $3\frac{1}{2} \times 2\frac{3}{4}$?
12. $10\frac{1}{8} \times 5\frac{1}{3}$?
13. $2\frac{5}{8} \times \frac{4}{5} \times 5$?
14. $\frac{5}{9} \times 3\frac{3}{4} \times 1\frac{4}{5}$?

RULE.—I. *Reduce all integers and mixed numbers to improper fractions.*

II. *Find the product of the numerators for a new numerator, and of the denominators for a new denominator.*

1. Use cancellation when convenient, both to *shorten* the operation, and to give the answer in its *lowest terms.*

2. Fractions with the word *of* between them are sometimes called *Compound Fractions*. The word *of* is equivalent to the sign (×) of multiplication. Thus $\frac{3}{4}$ of $\frac{5}{6}$ is equivalent to $\frac{5}{6} \times \frac{3}{4}$; $\frac{2}{3}$ of $4\frac{1}{2}$, to $4\frac{1}{2} \times \frac{2}{3}$.

15. When peaches are worth \$$\frac{7}{8}$ a basket, what will $\frac{3}{4}$ of $\frac{4}{7}$ of a basket cost?

16. At \$$\frac{5}{8}$ a gallon, what will be the cost of $\frac{4}{5}$ of 3 gallons of molasses?

What is the product

17. Of $\frac{3}{4}$, $\frac{2}{3}$ and $\frac{4}{5}$?
18. Of $\frac{2}{9}$, $\frac{1}{2}$ and $\frac{5}{8}$?
19. Of $\frac{2}{7}$, $\frac{7}{9}$ and $\frac{5}{10}$?
20. Of $\frac{5}{6}$, $\frac{8}{15}$ and $\frac{3}{7}$?
21. Of $2\frac{3}{7}$, $\frac{5}{11}$ and $\frac{22}{25}$?
22. Of $5\frac{1}{2}$, $\frac{7}{11}$ and $\frac{6}{7}$?
23. Of $12\frac{3}{5}$, $\frac{2}{6}$ and 7?
24. Of $\frac{3}{4}$, $\frac{2}{5}$, $1\frac{2}{3}$ and $\frac{10}{11}$?

Find the value of

25. $\frac{1}{3}$ of $\frac{2}{5}$ of $2\frac{1}{2}$.
26. $\frac{5}{9}$ of $\frac{3}{11}$ of 4.
27. $\frac{2}{7}$ of $\frac{8}{13}$ of $3\frac{1}{4}$.
28. $\frac{3}{4}$ of $\frac{6}{7} \times \frac{4}{9}$ of $\frac{1}{6}$.
29. $\frac{4}{5}$ of $3 \times \frac{7}{8}$ of $2\frac{1}{2}$.
30. $\frac{3}{5}$ of $\frac{1}{6} \times \frac{2}{3}$ of 10.
31. $\frac{5}{12}$ of $\frac{4}{5} \times \frac{1}{4}$ of $3\frac{3}{7}$.
32. $\frac{9}{10} \times 2\frac{1}{2}$ of $\frac{5}{9} \times 1\frac{1}{3}$.
33. $\frac{6}{7}$ of $\frac{7}{11} \times 5\frac{1}{2}$ times $\frac{5}{6}$.
34. $2\frac{1}{7}$ times $14 \times 1\frac{1}{3}$ times $2\frac{1}{4}$.

Find the cost of the following:

35. $\frac{7}{9}$ of a yard of flannel, at \$$\frac{3}{8}$ a yard.
36. $6\frac{1}{4}$ pounds of rice at $6\frac{1}{2}$ cents a pound.
37. $8\frac{3}{4}$ bushels of apples at $37\frac{1}{2}$ cents a bushel.
38. $21\frac{4}{5}$ yards of calico at \$$\frac{1}{8}$ a yard.
39. 15 tons of coal at \$$7\frac{5}{8}$ a ton.
40. $9\frac{3}{5}$ pounds of coffee, at $28\frac{1}{2}$ cents a pound.
41. 56 pounds of cotton, at $18\frac{3}{4}$ cents a pound.
42. $\frac{2}{3}$ of $5\frac{1}{2}$ pounds of wool, at 47 cents a pound.
43. $\frac{4}{5}$ of 9 pounds of honey, at $\frac{1}{2}$ of $18\frac{3}{4}$ cents a pound.
44. $5\frac{1}{4}$ quarts of chestnuts, at $12\frac{1}{2}$ cents a quart.

DIVISION.

CASE I.

190. To divide a fraction by an integer.

1. If $\frac{2}{3}$ of a dollar be equally divided between 2 boys, what part of a dollar will each have?

2. What is the quotient of $\frac{2}{3}$ divided by 2?

3. $\frac{1}{2}$ of $\frac{2}{3}$ is what part of 1?

4. Dividing by 2 is the same as multiplying by what fraction?

5. Dividing by 3 and 4 is the same as multiplying by what fractions? $\frac{1}{3}$ of $\frac{3}{4}$ is what part of 1?

6. If 3 pounds of butter cost \$$\frac{2}{3}$, what will 1 pound cost?

ANALYSIS.—Since 1 pound is $\frac{1}{3}$ of 3 pounds, 1 pound will cost $\frac{1}{3}$ of \$$\frac{2}{3}$, or \$$\frac{2}{9}$. Hence, 1 pound will cost \$$\frac{2}{9}$.

7. What is the quotient of $\frac{2}{3}$ divided by 3?

8. How much is $\frac{1}{2}$ of $\frac{4}{5}$? $\frac{4}{5}$ divided by 2?

9. How much is $\frac{1}{6}$ of $\frac{12}{13}$? $\frac{12}{13}$ divided by 6?

10. How much is $\frac{1}{4}$ of $\frac{16}{20}$? $\frac{16}{20}$ divided by 4?

11. What is the quotient of $\frac{18}{25}$ divided by 2? 3? 6? 9?

12. What is the quotient of $\frac{12}{17}$ divided by 2? 3? 4? 6?

13. How much is $\frac{1}{6}$ of $\frac{4}{5}$? $\frac{4}{5} \div 6$? $\frac{1}{4}$ of $\frac{8}{9}$? $\frac{8}{9} \div 4$?

14. How much is $\frac{5}{8} \times \frac{1}{3}$? $\frac{5}{8} \div 3$? $\frac{15}{16} \times \frac{1}{5}$? $\frac{15}{16} \div 5$?

15. How much is $\frac{3}{8} \div 4$? $\frac{5}{7} \div 5$? $\frac{7}{9} \div 6$? $\frac{9}{10} \div 7$? $\frac{10}{7} \div 8$?

16. If 1 man can do $\frac{5}{8}$ of a piece of work in 1 day, in what time can 4 men do it? 3 men? 5 men?

17. If 5 men can build $\frac{6}{7}$ of a wall in a day, what part of it can 1 man build? 2 men? 3 men? 4 men?

18. If 3 lemons cost $\frac{4}{5}$ of a dime, what cost 1 lemon?

19. If 4 slates cost \$$\frac{3}{5}$, what will 1 slate cost?

20. Divide $\frac{7}{8}$ of a barrel of flour equally among 4 poor families; what part of a barrel will each have?

21. John sold $\frac{2}{3}$ of 6 quarts of chestnuts for \$$\frac{2}{5}$, what was that a quart?

191. PRINCIPLE.—*A fraction is divided by an integer by dividing its numerator, or by multiplying its denominator*(**167**, II).

Always divide the numerator when it is exactly divisible by the divisor, otherwise multiply the denominator.

WRITTEN EXERCISES.

1. Divide $\frac{14}{15}$ by 7.

OPERATION.

$$\tfrac{14}{15} \div 7 = \tfrac{14 \div 7}{15} = \tfrac{2}{15}$$

Or, $$\tfrac{14}{15} \div 7 = \tfrac{14}{15 \times 7} = \tfrac{14}{105} = \tfrac{2}{15}$$

Or, $$\frac{14}{15} \div 7 = \frac{\cancel{14}^{2}}{15 \times \cancel{7}} = \frac{2}{15}$$

ANALYSIS.—First to divide $\frac{14}{15}$ by 7, divide the numerator of the fraction by 7, which gives $\frac{2}{15}$, the required quotient. Or,

Multiply the denominator of the fraction by 7, which gives $\frac{14}{105}$, equal to $\frac{2}{15}$, the same result (PRIN.). The second operation may be shortened by cancellation.

In the first operation it is obvious that the *number* of parts, or fractional units, is diminished while their *size* or value remains the same; in the second operation the *number* of the parts remains unchanged while their *size* is diminished.

Divide

2. $\frac{18}{7}$ by 6.
3. $\frac{4}{5}$ by 7.
4. $\frac{56}{37}$ by 8.
5. $\frac{7}{25}$ by 5.
6. $\frac{120}{165}$ by 15.
7. $\frac{37}{50}$ by 18.

8. Divide $17\frac{2}{3}$ by 4.

OPERATION.

$$17\tfrac{2}{3}\div4=\frac{53}{3\times4}=\frac{53}{12}=4\tfrac{5}{12}$$

Or,

$$\begin{array}{r} 4\,\underline{)\,17\tfrac{2}{3}} \\ 4\tfrac{5}{12} \end{array}$$

Analysis.—Reduce the mixed number to an improper fraction, then divide as in Ex. 1. Or,

Divide as in simple numbers; 4 is contained in $17\frac{2}{3}$, 4 times and a remainder of $1\frac{2}{3}$; $1\frac{2}{3}$ equal $\frac{5}{3}$, and $\frac{5}{3}$ divided by 4 gives $\frac{5}{12}$, which added to the partial quotient 4, gives $4\frac{5}{12}$, the required quotient.

What is the quotient of

9. $3\frac{4}{5}$ divided by 4?
10. $12\frac{3}{8}$ divided by 5?
11. $25\frac{4}{7}$ divided by 6?
12. $36\frac{3}{10}$ divided by 7.
13. $85\frac{2}{3}$ divided by 12.
14. $147\frac{3}{4}$ divided by 29.

Rule.—I. *Divide the numerator, or multiply the denominator by the integer.*

II. When the dividend is a *mixed* number.

Reduce the mixed number to an improper fraction, and then divide. Or,

Divide the integer and the fractional part separately and unite the quotients.

Divide.

15. $\frac{36}{43}$ by 18.
16. $\frac{27}{12}$ by 10.
17. $\frac{2}{30}$ by 14.
18. $\frac{42}{97}$ by 21.
19. $73\frac{3}{4}$ by 10.
20. $125\frac{5}{7}$ by 20.
21. $236\frac{1}{8}$ by 16.
22. $328\frac{2}{5}$ by 18.
23. $\frac{1}{4}$ of 30 by 12.
24. $\frac{4}{5}$ of 142 by 15.
25. $\frac{5}{6}$ of 8 by 20.
26. $\frac{5}{10}$ of 75 by 11.

27. If a man dig $29\frac{5}{16}$ rods of ditch in 14 days, how much does he dig in 1 day?

28. If 8 tons of coal cost $\$45\frac{7}{8}$, what is the cost of 1 ton?

29. If $\frac{3}{4}$ of 12 yards of silk cost $\$39\frac{3}{8}$, what is the cost of 1 yard?

CASE II.

192. To divide an integer by a fraction.

1. How many halves of a mile in 1 mile? In 2 miles?

2. How many $\frac{1}{2}$ yards in 1 yard? In 3 yards?

3. 1 is how many times $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$?

4. What is the quotient of 1 divided by $\frac{1}{2}$? by $\frac{1}{3}$? by $\frac{1}{5}$? by $\frac{1}{6}$? by $\frac{1}{8}$? by $\frac{1}{9}$? by $\frac{1}{12}$?

5. When potatoes are $\$\frac{2}{3}$ a bushel, how many bushels can be bought for $\$5$?

ANALYSIS.—As many bushels as $\$\frac{2}{3}$ is contained times in $\$5$; $\$5$ is equal to $\$\frac{15}{3}$, and 2 thirds is contained in 15 thirds $7\frac{1}{2}$ times. Hence $7\frac{1}{2}$ bushels can be bought for $\$5$.

6. 1 is how many times $\frac{2}{3}$? $\frac{3}{4}$? $\frac{4}{5}$? $\frac{5}{6}$? $\frac{6}{7}$? $\frac{7}{8}$? $\frac{9}{10}$?

7. 2 are how many times $\frac{1}{8}$? $\frac{3}{8}$? $\frac{2}{5}$? $\frac{4}{5}$? $\frac{2}{7}$? $\frac{5}{7}$? $\frac{5}{8}$? $\frac{4}{9}$?

8. If a horse eat $\frac{3}{7}$ of a bushel of oats in a day, how long will 4 bushels last him? 5 bushels? 6 bushels?

9. In how many days can a boy earn $\$6$, if he earn $\$\frac{3}{4}$ a day? If $\$\frac{5}{6}$? If $\$\frac{7}{8}$?

10. If a train of cars run $\frac{2}{5}$ of a mile a minute, how long will it be in running 10 miles? 12 miles?

11. How many times is $\frac{4}{5}$ contained in 10? $\frac{5}{8}$ in 9?

12. If a man spend $\$\frac{1}{3}$ a day for cigars, in what time will he spend $\$3$? $\$4$? $\$5$? $\$7$? $\$8$? $\$10$?

13. Divide 7 by $\frac{1}{2}$; 8 by $\frac{2}{3}$; 12 by $\frac{1}{4}$; 9 by $\frac{3}{4}$; 10 by $\frac{3}{5}$.

14. What is the value of $5 \div \frac{1}{7}$? $6 \div \frac{1}{8}$? $7 \div \frac{4}{5}$? $5 \div \frac{2}{9}$?

15. How many times is $\frac{5}{6}$ contained in 3? In 8?

16. At $\$\frac{3}{5}$ a bushel, how many bushels of turnips can be bought for $\$4$? $\$6$? $\$7$? $\$8$? $\$10$?

17. How much butter at $\$\frac{3}{8}$ a pound can be bought for $\$2$? $\$4$? $\$6$? $\$9$?

WRITTEN EXERCISES.

1. Divide 15 by $\frac{3}{4}$.

OPERATION.

$$15 = \tfrac{60}{4};\ \tfrac{60}{4} \div \tfrac{3}{4} = 20$$

$$\text{Or,}\ 15 \div \frac{3}{4} = \frac{{}^{5}\cancel{15} \times 4}{\cancel{3}} = 20$$

Analysis.—Reduce 15 to fourths, which is equal to $\frac{60}{4}$; then 3 fourths is contained in 60 fourths 20 times, the quotient required. Or, Since 15 divided by $\frac{1}{4}$ is 4 times 15, 15 divided by 3 fourths is $\frac{1}{3}$ of 4 times 15, or 20, the same result.

Divide

2. 9 by $\frac{2}{3}$.	4. 21 by $\frac{7}{12}$.	6. 60 by $\frac{9}{15}$.
3. 14 by $\frac{4}{7}$.	5. 44 by $\frac{6}{7}$.	7. 72 by $\frac{16}{17}$.

8. What is the quotient of 20 divided by $4\frac{4}{5}$?

OPERATION.

$$4\tfrac{4}{5} = \tfrac{24}{5}$$

$$20 \div \frac{24}{5} = \frac{\cancel{20}^{5} \times 5}{\cancel{24}_{6}} = \frac{25}{6} = 4\tfrac{1}{6}$$

Analysis.—Reduce the mixed number to an improper fraction, and then divide as in example first.

What is the quotient of

9. $16 \div 2\frac{1}{4}$?	12. $112 \div 6\frac{2}{5}$?	15. $63 \div 8\frac{5}{9}$.
10. $42 \div 3\frac{1}{2}$?	13. $28 \div 4\frac{2}{3}$?	16. $175 \div 7\frac{3}{10}$.
11. $52 \div 6\frac{1}{4}$?	14. $84 \div 5\frac{7}{12}$?	17. $250 \div 9\frac{1}{5}$.

Rule.—I. *Reduce the dividend to a fraction having the same fractional unit as the divisor, then divide the numerator of the dividend by the numerator of the divisor.* Or,

II. *Multiply the dividend by the denominator of the fraction and divide the product by the numerator.*

When the divisor is a mixed number reduce to an improper fraction and then divide.

Divide

18. 56 by $\frac{6}{11}$.
19. 84 by $\frac{7}{9}$.
20. 163 by $\frac{7}{13}$.
21. 120 by $\frac{6}{11}$.
22. 72 by $\frac{11}{18}$.
23. 90 by $\frac{15}{21}$.
24. 300 by $\frac{7}{8}$.
25. 155 by $\frac{9}{25}$.
26. 168 by $3\frac{5}{7}$.
27. 240 by $\frac{5}{6}$ of $1\frac{1}{2}$.
28. 189 by $\frac{3}{4}$ of $\frac{4}{5}$.
29. 106 by $6\frac{4}{9}$.

30. If $27 be divided among some laborers, giving them $\$\frac{9}{10}$ each, how many laborers will receive a share?

31. At $\$3\frac{2}{5}$ a cord, how many cords of wood can be bought for $40? For $150?

32. How many bushels of pears at $\$\frac{5}{6}$ a bushel can be bought for $18? For $39?

33. If a man walk $3\frac{2}{3}$ miles an hour, how many hours will he require to walk 48 miles?

34. A man whose daily wages were $\$3\frac{1}{8}$, at the end of a month received $75; how many days had he worked?

35. When corn is $\frac{3}{4}$ of $2 a bushel, how many bushels can be bought for $46?

36. How many acres of wild land can be bought for $125, at $\$1\frac{7}{10}$ an acre?

37. A bricklayer received $75 for labor at $\$3\frac{1}{2}$ a day: how many days did he work?

38. Paid $86 for $5\frac{3}{8}$ barrels of sugar: what was the cost of 1 barrel?

39. At $\$6\frac{2}{3}$ a ton, how many tons of coal can be bought for $160? For $248?

40. At $\$\frac{11}{12}$ a yard, how many yards of cloth can be bought for $9? For $24? For $64?

41. At $\$\frac{3}{8}$ a pound, how many pounds of butter will $110 buy?

42. How many baskets of peaches can be bought for $\$183\frac{1}{3}$, at $\$1\frac{5}{6}$ a basket?

CASE III.

193. To divide a fraction by a fraction.

1. How many times is 2 fifths of a dime contained in 4 fifths of a dime? $\frac{3}{8}$ of a pound in $\frac{6}{8}$ of a pound?

2. How many times is $\frac{2}{7}$ contained in $\frac{6}{7}$? $\frac{3}{8}$ in $\frac{6}{8}$? $\frac{4}{9}$ in $\frac{8}{9}$?

3. How many books at $\$\frac{3}{10}$ each can be bought for $\$\frac{6}{10}$? For $\$\frac{9}{10}$? For $\$\frac{12}{10}$?

4. If a family use $\frac{2}{9}$ of a barrel of flour in a week, how long will $\frac{4}{9}$ of a barrel last them? $\frac{8}{9}$? $\frac{12}{9}$? $\frac{16}{9}$?

5. How do you divide one fraction by another when they have a common denominator?

6. How much is $\frac{7}{5} \div \frac{3}{5}$? $\frac{9}{10} \div \frac{4}{10}$? $\frac{12}{15} \div \frac{6}{15}$? $\frac{15}{20} \div \frac{7}{20}$?

7. At $\$\frac{2}{5}$ a yard, how many yards of ribbon can be bought for $\$\frac{3}{4}$?

ANALYSIS.—As many yards as $\$\frac{2}{5}$ is contained times in $\$\frac{3}{4}$; $\$\frac{2}{5}$ equals $\$\frac{8}{20}$, and $\$\frac{3}{4}$ equals $\$\frac{15}{20}$; 8 twentieths is contained in 15 twentieths $1\frac{7}{8}$. Hence $1\frac{7}{8}$ yards can be bought for $\$\frac{3}{4}$.

8. Divide $\frac{4}{5}$ by $\frac{2}{3}$; $\frac{6}{7}$ by $\frac{1}{2}$; $\frac{5}{7}$ by $\frac{1}{3}$; $\frac{7}{8}$ by $\frac{3}{4}$; $\frac{7}{12}$ by $\frac{1}{4}$.

9. How do you divide one fraction by another when they have not a common denominator?

10. At $\$\frac{3}{16}$ a yard, how many yards of muslin can be bought for $\$\frac{1}{2}$? For $\$\frac{3}{4}$? For $\$\frac{7}{8}$?

11. Divide $\frac{5}{8}$ by $\frac{1}{2}$; by $\frac{1}{3}$; by $\frac{1}{4}$; by $\frac{1}{5}$; by $\frac{1}{6}$.

12. How many times is $\frac{2}{3}$ contained in $\frac{4}{5}$? In $\frac{3}{4}$? In $\frac{5}{9}$? In $\frac{6}{7}$? In $\frac{9}{10}$? In $\frac{11}{15}$?

13. When peaches are $\$\frac{3}{5}$ a bushel, how many bushels can be bought for $\$2\frac{7}{10}$?

ANALYSIS.—As many bushels as $\$\frac{3}{5}$ is contained times in $\$2\frac{7}{10}$; $\$2\frac{7}{10}$ equal $\$\frac{27}{10}$, and $\$\frac{3}{5}$ equal $\$\frac{6}{10}$; 6 tenths is contained in 27 tenths $4\frac{1}{2}$ times. Hence $4\frac{1}{2}$ bushels can be bought for $\$2\frac{7}{10}$.

14. Divide $1\frac{1}{2}$ by $\frac{1}{4}$; $2\frac{1}{3}$ by $\frac{4}{5}$; $3\frac{1}{4}$ by $\frac{3}{8}$; $2\frac{4}{5}$ by $\frac{7}{10}$.

15. How much is $2 \div \frac{3}{4}$? $3\frac{2}{3} \div \frac{3}{5}$? $4 \div \frac{7}{8}$? $5\frac{1}{2} \div \frac{5}{6}$?

16. At \$$\frac{7}{8}$ each, how many hoes can be bought for \$$2\frac{1}{4}$? For \$$3\frac{2}{3}$? For \$$1\frac{5}{6}$? For \$$5\frac{1}{8}$?

17. If $\frac{2}{7}$ of a drum of figs cost \$$\frac{5}{8}$, what will 1 drum cost?

Analysis.—Since $\frac{2}{7}$ of a drum of figs cost \$$\frac{5}{8}$, $\frac{1}{7}$ of a drum will cost $\frac{1}{2}$ of \$$\frac{5}{8}$, which is \$$\frac{5}{16}$; and 1 drum, or $\frac{7}{7}$, will cost 7 times \$$\frac{5}{16}$, or \$$\frac{35}{16}$, equal to \$$2\frac{3}{16}$. Hence 1 drum of figs will cost \$$2\frac{3}{16}$.

18. If $\frac{4}{12}$ of a dozen of penholders cost \$$\frac{3}{8}$, what will be the cost of 1 dozen?

19. If $\frac{2}{3}$ of a yard of silk cost \$$\frac{3}{4}$, what will 1 yard cost?

20. At \$$\frac{7}{9}$ a yard, how many yards of silk can be bought for \$$1\frac{2}{3}$? For \$$2\frac{1}{4}$? For \$$3\frac{1}{5}$?

WRITTEN EXERCISES.

1. Divide $\frac{7}{8}$ by $\frac{2}{3}$.

OPERATION.

$$\frac{7}{8} \div \frac{2}{3} = \frac{21}{24} \div \frac{16}{24} = 1\frac{5}{16}$$

Or, $\frac{7}{8} \div \frac{2}{3} = \frac{7}{8} \times \frac{3}{2} = \frac{21}{16} = 1\frac{5}{16}$

Analysis.—$\frac{7}{8}$ divided by $\frac{2}{3}$ is equal to 21 twenty-fourths divided by 16 twenty-fourths, which gives $1\frac{5}{16}$, the required quotient. Or,

Since $\frac{2}{3}$ is contained in 1, $\frac{3}{2}$ times, in $\frac{7}{8}$ it is contained $\frac{7}{8}$ of $\frac{3}{2}$ times, or $\frac{21}{16}$, equal to $1\frac{5}{16}$.

It will be observed that the operation is equivalent to multiplying the dividend $\frac{7}{8}$ by the divisor $\frac{2}{3}$ with its terms *inverted.*

Divide

2. $\frac{5}{8}$ by $\frac{3}{4}$.
3. $\frac{9}{10}$ by $\frac{3}{5}$.
4. $\frac{3}{5}$ by $\frac{6}{7}$.
5. $\frac{5}{9}$ by $\frac{7}{12}$.
6. $\frac{4}{14}$ by $\frac{6}{7}$.
7. $\frac{12}{25}$ by $\frac{9}{10}$.
8. $\frac{1}{2}$ of $\frac{5}{9}$ by $\frac{3}{4}$.
9. $\frac{2}{3}$ of $\frac{4}{5}$ by $\frac{12}{21}$.
10. $\frac{1}{6}$ of $\frac{5}{7}$ by $\frac{1}{4}$ of $\frac{10}{21}$.

11. Divide $3\frac{1}{2}$ by $4\frac{3}{8}$.

OPERATION.

$$3\frac{1}{2}=\frac{7}{2};\quad 4\frac{3}{8}=\frac{35}{8}$$

$$\frac{7}{2}\div\frac{35}{8}=\frac{\cancel{7}}{\cancel{2}}\times\frac{\overset{4}{\cancel{8}}}{\underset{5}{\cancel{35}}}=\frac{4}{5}$$

ANALYSIS.—Reduce both dividend and divisor to improper fractions. Then divide as in the first example.

What is the value of

12. $3\frac{1}{2}\div 2\frac{1}{3}$?
13. $4\frac{3}{4}\div 1\frac{2}{3}$?
14. $12\frac{1}{2}\div 7\frac{1}{4}$?
15. $9\frac{3}{8}\div 2\frac{4}{5}$?
16. $10\frac{4}{9}\div \frac{7}{12}$?
17. $5\frac{5}{7}\div 8\frac{1}{3}$?

RULE.—*Reduce the given fractions to equivalent fractions having a common denominator, and then divide the numerator of the dividend by the numerator of the divisor.*

Or, *Multiply the dividend by the terms of the divisor inverted.*

1. Integers and mixed numbers should first be reduced to improper fractions.
2. Use *cancellation* when convenient.

Divide

18. $\frac{7}{36}$ by $\frac{14}{27}$.
19. $\frac{20}{21}$ by $\frac{1}{2}$ of $\frac{4}{5}$.
20. $\frac{24}{63}$ by $\frac{2}{3}$ of $1\frac{5}{9}$.
21. $\frac{1}{2}$ of $\frac{3}{4}$ by $\frac{7}{8}$.
22. $\frac{4}{5}$ of $\frac{5}{8}$ by $\frac{7}{20}$.
23. $\frac{11}{15}$ by $\frac{1}{7}$ of $\frac{2}{3}$.
24. $9\frac{1}{6}$ by $\frac{1}{2}$ of 5.
25. $4\frac{5}{7}$ by $2\frac{3}{4}$.
26. $12\frac{9}{10}$ by $\frac{1}{3}$ of $6\frac{3}{4}$.
27. $\frac{3}{4}$ of $26\frac{2}{3}$ by $2\frac{6}{7}$.

28. How many pounds of tea at $\$1\frac{1}{8}$ a pound, can be bought for $\$15\frac{1}{3}$?

29. At $\$3\frac{1}{5}$ a cord, how many cords of wood can be bought for $25?

30. If a man spend $\$\frac{3}{10}$ a day for cigars, in what time will he spend $\$5\frac{3}{8}$?

31. If $1\frac{1}{3}$ bushels of oats cost $\$\frac{8}{9}$, what costs 1 bushel?

32. What is the value of $\frac{2\frac{4}{9}}{3\frac{2}{3}}$?

OPERATION.

$$\frac{2\frac{4}{9}}{3\frac{2}{3}} = \frac{\frac{22}{9}}{\frac{11}{3}} = \frac{22}{9} \div \frac{11}{3}$$

$$\text{and } \frac{22}{9} \div \frac{11}{3} = \frac{{}^{2}\cancel{22}}{{}_{3}\cancel{9}} \times \frac{\cancel{3}}{\cancel{11}} = \frac{2}{3}$$

ANALYSIS.—This example is only another *form* for expressing division of fractions. Hence reduce the mixed numbers to improper fractions, then treat the numerator $\frac{22}{9}$ as a dividend, and the denominator $\frac{11}{3}$, as a divisor, and proceed according to the rule for the division of fractions.

Expressions similar to the above are sometimes called *Complex Fractions*, and the process of performing the division is called *reducing a complex fraction to a simple one.*

33. What is the value of $\frac{2\frac{1}{4}}{3\frac{3}{5}}$, or of $2\frac{1}{4} \div 3\frac{3}{5}$?

34. What is the value of $\frac{\frac{5}{9}}{2\frac{1}{2}}$, or of $\frac{5}{9} \div 2\frac{1}{2}$?

35. What is the value of $\frac{5\frac{1}{3}}{8}$, or of $5\frac{1}{3} \div 8$?

36. What is the value of $\frac{15}{12\frac{1}{2}}$, or of $15 \div 12\frac{1}{2}$?

37. What is the value of $\frac{\frac{3}{7}}{\frac{3}{5}}$, or of $\frac{3}{7} \div \frac{3}{5}$?

38. Reduce to their simplest form $\frac{3}{\frac{2}{3}}$; $\frac{\frac{4}{7}}{\frac{6}{11}}$; $\frac{7\frac{1}{2}}{5}$; $\frac{7\frac{1}{9}}{2\frac{2}{7}}$.

39. Reduce to their simplest form $\frac{\frac{4}{9}}{\frac{7}{10}}$; $\frac{\frac{9}{25}}{2\frac{2}{5}}$; $\frac{27}{2\frac{1}{4}}$; $\frac{15}{2\frac{3}{4}}$.

40. If a horse eat $\frac{5}{7}$ of a bushel of oats a day, in how many days will he eat $12\frac{2}{3}$ bushels?

41. At \$$6\frac{1}{8}$ a bushel, how many bushels of clover seed can be bought for \$$40\frac{3}{4}$?

42. At $\frac{1}{4}$ of \$$\frac{7}{8}$ a yard, how many yards of muslin can be bought for $\frac{1}{2}$ of \$$\frac{3}{4}$?

194. To find a number when a fractional part of it is given.

1. 4 is $\frac{1}{3}$ of what number?

ANALYSIS.—4 is 1 third of 3 times 4, which is 12. Hence 4 is $\frac{1}{3}$ of 12.

2. 6 is $\frac{1}{4}$ of what number? $\frac{1}{5}$ of what number?
3. 7 is $\frac{1}{6}$ of what number? $\frac{1}{8}$ of what number?
4. $12\frac{1}{2}$ is $\frac{1}{3}$ of what number? $\frac{1}{7}$ of what number?
5. $9\frac{2}{3}$ is $\frac{1}{2}$ of what number? $\frac{1}{4}$ of what number?

6. 15 is $\frac{3}{4}$ of what number?

ANALYSIS.—15 is $\frac{3}{4}$ of 4 times $\frac{1}{3}$ of 15; $\frac{1}{3}$ of 15 is 5, and 4 times 5 are 20. Hence 15 is $\frac{3}{4}$ of 20.

7. 16 is $\frac{4}{5}$ of what number? $\frac{2}{7}$ of what number?
8. 18 is $\frac{3}{10}$ of what number? $\frac{9}{13}$ of what number?
9. $31\frac{1}{5}$ is $\frac{6}{7}$ of what number? $\frac{5}{9}$ of what number?
10. $14\frac{2}{3}$ is $\frac{4}{5}$ of what number? $\frac{2}{7}$ of what number?

11. $\frac{6}{7}$ is $\frac{3}{5}$ of what number?

ANALYSIS.—$\frac{6}{7}$ is $\frac{3}{5}$ of 5 times $\frac{1}{3}$ of $\frac{6}{7}$. $\frac{1}{3}$ of $\frac{6}{7}$ is $\frac{2}{7}$ and 5 times $\frac{2}{7}$ is $\frac{10}{7}$, or $1\frac{3}{7}$. Hence $\frac{6}{7}$ is $\frac{3}{5}$ of $1\frac{3}{7}$.

12. $\frac{9}{10}$ is $\frac{3}{8}$ of what number? $\frac{4}{9}$ of what number?
13. $\frac{5}{8}$ is $\frac{3}{10}$ of what number? $\frac{2}{7}$ of what number?
14. $\frac{3}{5}$ of $1\frac{1}{3}$ is $\frac{6}{7}$ of what number? $\frac{2}{5}$ of what number?

15. $\frac{5}{6}$ of 24 is $\frac{3}{5}$ of what number?

ANALYSIS.—$\frac{5}{6}$ of 24 is 5 times $\frac{1}{6}$ of 24, which is 20; 20 is $\frac{3}{5}$ of 5 times $\frac{1}{3}$ of 20. $\frac{1}{3}$ of 20 is $6\frac{2}{3}$, and 5 times $6\frac{2}{3}$ is $33\frac{1}{3}$. Hence, etc.

16. $\frac{3}{4}$ of 16 is $\frac{6}{7}$ of what number? $\frac{4}{5}$ of what number?
17. $\frac{2}{3}$ of 30 is $\frac{3}{8}$ of what number? $\frac{5}{7}$ of what number?
18. $\frac{7}{8}$ of 32 is $\frac{5}{6}$ of what number? $\frac{7}{10}$ of what number?

19. 28 is $\frac{4}{7}$ of how many times 8?

Analysis.—28 is $\frac{4}{7}$ of 7 times $\frac{1}{4}$ of 28, which is 49; and 8 is contained in 49, $6\frac{1}{8}$ times. Hence, etc.

20. 54 is $\frac{9}{10}$ of how many times 5? 6? 8? 9? 12?

21. 45 is $\frac{5}{9}$ of how many times 8? 9? 6? 7? 11?

22. $22\frac{1}{2}$ is $\frac{5}{8}$ of how many times 4? 8? 12? 10? 9?

23. George gave 50 cents for a book, which was $\frac{5}{6}$ of what he gave for a knife. How much did the knife cost?

24. Paid $\$13\frac{1}{2}$ for an easy chair, which was $\frac{3}{4}$ the cost of a book-case. What was the cost of the book-case?

25. Gave $\$\frac{7}{8}$ for a scarf, which was $\frac{2}{3}$ of the cost of a pair of gloves. What was the cost of the gloves?

26. A lady bought a bonnet for $\$12\frac{3}{4}$, which was $\frac{4}{5}$ of what she paid for 5 yards of silk. What was the cost of the silk a yard?

27. Henry has 45 cents, and $\frac{2}{5}$ of his money is $\frac{3}{7}$ of John's money. How much has John?

28. A man paid $\frac{5}{9}$ of all the money he had for a suit of clothes, and had $48 left. How much money had he at first?

Analysis.—Since he paid out $\frac{5}{9}$, he had $\frac{4}{9}$, or $48 left; and 48 is $\frac{4}{9}$ of 9 times $\frac{1}{4}$ of 48, which is 108. Hence he had $108 at first.

29. A farmer after selling $\frac{3}{5}$ of his farm, had 40 acres left. How many acres in his whole farm?

WRITTEN EXERCISES.

1. 48 is $\frac{3}{4}$ of what?
2. 75 is $\frac{5}{9}$ of what?
3. 136 is $\frac{8}{17}$ of what?
4. $144\frac{1}{2}$ is $\frac{9}{20}$ of what?
5. $\frac{28}{49}$ is $\frac{4}{7}$ of what?
6. $\frac{72}{108}$ is $\frac{3}{5}$ of what?
7. $121\frac{1}{3}$ is $\frac{5}{8}$ of what?
8. $\frac{7}{8}$ of 200 is $\frac{5}{11}$ of what?

9. Smith has 140 sheep, which is $\frac{5}{8}$ as many as Jones has. How many sheep has Jones?

10. What is a ship worth, if $\frac{4}{15}$ of it is worth $6380?

11. $\frac{7}{9}$ of 360 is how many times 26?

12. $\frac{13}{25}$ of 450 is $\frac{14}{15}$ of what number?

13. Paid $350 for a carriage, which was $\frac{7}{9}$ of what I paid for a horse. How much did I pay for the horse?

14. How many barrels of flour at $9 a barrel, can be bought for $\frac{7}{12}$ of $270?

REVIEW.

ORAL EXAMPLES.

195. 1. How many times 1 in $\frac{16}{5}$? In $\frac{24}{8}$? In $\frac{45}{10}$?

2. In 6 yards, how many fourths of a yard? Fifths?

3. How many sixths in 4? In $5\frac{3}{6}$? In $8\frac{1}{6}$? In $10\frac{5}{6}$?

4. Express $\frac{3}{4}$ in higher terms; $\frac{5}{6}$; $\frac{4}{5}$; $\frac{7}{8}$; $\frac{9}{10}$.

5. Express $\frac{9}{12}$ in lower terms; $\frac{16}{20}$; $\frac{24}{30}$; $\frac{20}{40}$.

6. Reduce $\frac{8}{12}$ and $\frac{12}{20}$ to their lowest terms.

7. Change $\frac{3}{5}$ and $\frac{1}{4}$ to twentieths; $\frac{1}{2}$ and $\frac{1}{3}$ to twelfths.

8. How many twenty-fourths are $\frac{1}{8}$? $\frac{3}{4}$? $\frac{5}{6}$? $\frac{7}{12}$?

9. Which is greater, $\frac{2}{7}$ or $\frac{1}{3}$? $\frac{1}{4}$ or $\frac{3}{10}$? $\frac{2}{3}$ or $\frac{5}{7}$?

10. Name 3 equivalent fractions for $\frac{1}{5}$; for $\frac{2}{3}$; for $\frac{3}{4}$.

11. Change $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{8}$ to fractions having 48 for a common denominator.

12. Find the least common denominator of the same.

13. For $\frac{16}{24}$ name 3 equivalent fractions in lower terms.

14. Name 4 fractions that can be changed to twentieths.

15. Change $\frac{2}{3}$, $1\frac{1}{2}$ and 2 to sixths. $\frac{4}{5}$ and $2\frac{1}{2}$ to twentieths.

16. How many half-barrels can be filled from 6 barrels of flour?

17. How many quarter dollars in \$$4\frac{1}{2}$? In \$$5\frac{3}{4}$?

18. How much less is $\frac{7}{15}$ than $\frac{2}{3}$? $\frac{11}{16}$ than $\frac{3}{8}$?

19. A farmer sold $\frac{1}{3}$ of his grain to one man, $\frac{2}{5}$ to another, and $\frac{1}{6}$ to another: what part of his grain did he sell, and what part had he left?

20. A lady paid $\frac{1}{3}$ of her money for a bonnet, $\frac{1}{4}$ for a shawl, and $\frac{1}{8}$ for a pair of gaiters. What had she left? Which cost the most?

21. A grocer sells $\frac{1}{3}$ of a box of tea from a box $\frac{6}{7}$ full. What part of the whole box is left?

22. If a boy earn \$$7\frac{1}{2}$ a week, and spend \$$5\frac{3}{4}$, how much would he save in 3 weeks?

23. John gave $\frac{1}{4}$ of his money for a sled, and $\frac{3}{5}$ for a cap, and had 39 cents left; how much had he at first?

24. What is the sum of $\frac{2}{3}$ of $2\frac{1}{2}+\frac{7}{8}$? Their difference? Their product? The quotient of the greater by the less?

25. When sugar is \$18 a barrel, what part of a barrel can be bought for \$6?

26. How much is the daily wages of a boy who receives \$4 a week? \$6 a week?

27. From 4 tons of coal $\frac{4}{5}$ of a ton was used one month and $1\frac{1}{3}$ tons the next: how much was left?

28. If $\frac{1}{4}$ of a box of raisins cost \$$2\frac{1}{12}$, what will $\frac{5}{6}$ of a box cost?

20. If $\frac{2}{5}$ of a yard of silk cost \$3, how many yards can be bought for \$15?

30. Henry bought $\frac{1}{2}$ a box of figs, and John $\frac{1}{2}$ as many. What part of a whole box did both buy?

31. Bought a watch and chain. The chain cost \$30, which was $\frac{3}{8}$ of what the watch cost. What did both cost?

WRITTEN EXAMPLES.

196. 1. Reduce to their lowest terms $\frac{225}{405}$; $\frac{440}{1012}$; $\frac{180}{216}$; $\frac{576}{864}$; $\frac{9504}{10368}$.

2. Change to integers or mixed numbers, $\frac{5907}{25}$; $\frac{85641}{39}$; $\frac{16380}{364}$; $\frac{39270}{102}$.

3. Reduce to improper fractions $164\frac{5}{14}$; $47\frac{42}{61}$; $104\frac{9}{112}$.

4. Change 21 to a fraction having 25 for its denominator; 34 to 50ths; 75 to 18ths.

5. Reduce $\frac{1}{2}$ of $\frac{3}{4}$, $\frac{3}{5}$, $1\frac{5}{8}$ to equivalent fractions having the least common denominator.

6. The difference between two numbers is $\frac{17}{28}$ and the smaller $9\frac{4}{7}$: what is the greater?

7. The product of two numbers is $56\frac{2}{3}$, and one of the numbers $12\frac{1}{2}$: what is the other?

8. What number divided by $\frac{9}{28}$ will give a quotient of $\frac{2}{3}$?

9. What is the difference between $\frac{3}{4}$ of $6\frac{1}{3}$ and $4\frac{1}{2} + \frac{5}{6}$?

10. A farmer had $\frac{1}{4}$ of his wheat in 1 bin, $\frac{1}{3}$ in another, and 60 bushels in the third. How many bushels had he?

11. Paid \$$1\frac{1}{3}$ a bushel for 10 bushels of wheat, and \$$\frac{5}{6}$ a bushel for 14 bushels of corn; which cost the more?

12. How many bushels of oats worth \$$\frac{3}{5}$ a bushel, will pay for 3 barrels of flour worth \$$9\frac{1}{2}$ a barrel?

13. If a man has $24\frac{7}{8}$ bushels of timothy seed, and he sells $\frac{2}{3}$ of it, how much has he left?

14. If $\frac{1}{3}$ of $5\frac{1}{3}$ cords of wood cost $\frac{1}{4}$ of \$32, what is the cost of 1 cord?

15. If $\frac{3}{8}$ of a farm is valued at \$1575, what is the value of the whole?

16. How many times can a demijohn holding $1\frac{1}{3}$ gallons be filled from a cask containing 24 gallons?

17. How many yards of cloth $\frac{4}{5}$ of a yard wide are equal to 12 yards $\frac{3}{4}$ of a yard wide?

18. How many pounds of tea at \$$\frac{7}{8}$ a pound will pay for $10\frac{1}{2}$ baskets of peaches, at \$$\frac{9}{10}$ a basket?

19. A man is 56 years old; $\frac{3}{8}$ of his age is $\frac{7}{12}$ the age of his wife: how old is his wife?

20. If $\frac{3}{10}$ of an acre of land is worth \$$79\frac{1}{6}$, what is 1 acre worth?

21. At \$$\frac{3}{8}$ a pound, how many pounds of coffee can be bought for \$$6\frac{1}{2}$?

22. If 48 is $\frac{5}{8}$ of some number, what is $\frac{3}{4}$ of the same number?

23. If $7\frac{1}{2}$ barrels of pork cost \$60, how many barrels can be bought for \$156 at the same rate?

24. A pole stands $\frac{1}{6}$ in the mud, $\frac{1}{4}$ in the water, and 35 feet above the water. What is the length of the pole?

25. A certain sum of money is divided among 4 persons. A has $\frac{2}{7}$, B, $\frac{1}{4}$, C, $\frac{3}{14}$, and D has the remainder, which is \$30. What was the whole sum divided?

26. If 3 yards of velvet cost \$$7\frac{3}{4}$, how much more than \$$10\frac{1}{2}$ will 5 yards cost?

What is the value of

27. $(75\frac{2}{3} - 7\frac{5}{6}) \div \frac{7}{8} \times 3\frac{1}{3}$?

28. $(\frac{2}{3} \text{ of } \frac{5}{6} \text{ of } 3\frac{3}{4} + 8\frac{2}{3}) \div (10\frac{1}{2} - 7\frac{5}{12})$?

29. $(\frac{3}{5} \text{ of } \frac{6}{11} \div 15) \times (15 \div \frac{3}{4} \text{ of } \frac{6}{7})$?

30. $(12\frac{3}{8} \div 1\frac{1}{12}) - (15\frac{5}{9} \times \frac{1}{3}) \times 4$?

31. $(\frac{2}{5} \text{ of } 5\frac{1}{2} - 3 \times \frac{3}{10}) \times (\frac{7}{8} \div \frac{1}{4} + 5\frac{7}{8})$?

32. $(\overline{\frac{3}{5} \times 14} \div \frac{1}{9} \text{ of } 15) \times 37\frac{1}{2}$?

33. $\frac{5}{16} \text{ of } (2\frac{4}{7} \div 6\frac{5}{8}) \times (9\frac{5}{9} \div 4\frac{6}{7})$?

34. $\dfrac{12\frac{1}{2}}{\frac{4}{7}} \div \dfrac{\frac{2}{3} \text{ of } \frac{3}{4}}{\frac{1}{2}}$?

35. $\dfrac{2\frac{2}{3} \text{ of } \frac{5}{7}}{6\frac{2}{7} \text{ of } \frac{3}{11}} \div \dfrac{1\frac{1}{9}}{9}$?

REVIEW QUESTIONS.

197. 1. Define a Fraction. A Fractional Unit. What two kinds of fractions are there? How distinguished? How is a common fraction expressed? Define the Denominator, and illustrate. The Numerator, and illustrate. The Terms of a fraction.

2. How are Common Fractions classified? Define a Proper Fraction. An Improper Fraction. A Mixed Number. What do fractions indicate? What is the Value of a Fraction? Principle. Name and illustrate the General Principles of Fractions, each in its order, I, II, III. The General Law.

3. What is Reduction of fractions? When is a fraction reduced to higher terms? Principle. Rule. When is a fraction reduced to lower terms? When to its lowest terms? Principle. Rule.

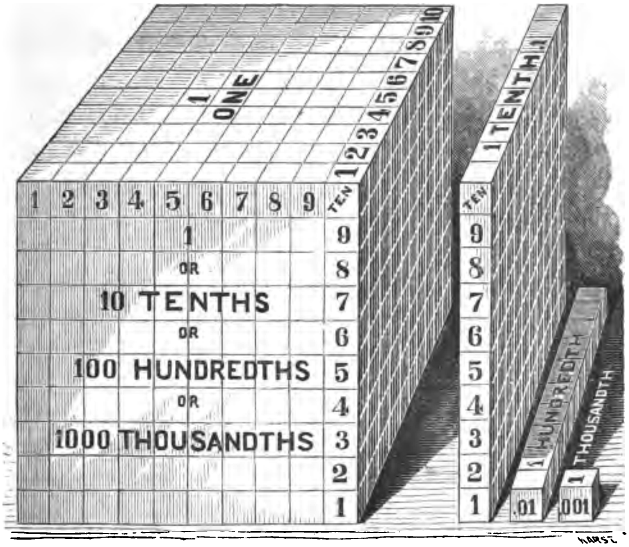
4. How is an integer, or mixed number, reduced to an improper fraction? How is an improper fraction reduced to an integer or mixed number?

5. When have two or more fractions a common denominator? When the least common denominator? Principles 1, 2. Rules.

6. What kind of fractions only can be added or subtracted? Why? Rule for each. How may integers and mixed numbers, if small, be treated? What should always be observed as necessary to render complete the solution of an example in fractions?

7. How is a fraction multiplied by an integer? Rule. An integer by a fraction? Rule. A fraction by a fraction. Rule. Why use cancellation? How is a fraction multiplied by its denominator? What are fractions connected by the word *of* sometimes called? How are they to be treated?

8. How is a fraction divided by an integer? Rule. An integer by a fraction? Rule. A fraction by a fraction? Rule. How are integers and mixed numbers to be treated? What kind of expressions are sometimes called Complex Fractions? How are they to be treated?



DECIMAL FRACTIONS.

198. 1. If a cubic block be divided into 10 equal parts, what part of the whole block is 1 part called? 2 parts? 3 parts? 4 parts? 5 parts?

2. If a unit be divided into 10 equal parts, what are the parts called?

3. What is the fractional unit?

4. If 1 *tenth* of a cubic block be divided into 10 other equal parts, what part of the whole block is 1 part? 2 parts? 3 parts? 4 parts? 6 parts?

5. What is $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{2}{10}$ of $\frac{1}{10}$? $\frac{3}{10}$ of $\frac{1}{10}$?

6. If a unit be divided into 100 equal parts, or each *tenth* into 10 equal parts, what are the parts called?

7. What is the fractional unit?

8. What part of 1 tenth is 1 hundredth? How many 1 hundredths is 1 tenth?

9. If 1 *hundredth* of a cubic block be divided into 10 equal parts, what part of the whole block is 1 part? 2 parts? 3 parts? 5 parts? 7 parts? 9 parts?

10. What is $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{1}{10}$ of $\frac{1}{100}$? $\frac{2}{10}$ of $\frac{1}{100}$? $\frac{5}{10}$ of $\frac{1}{100}$?

11. If a unit be divided into 1000 equal parts, or each 1 *hundredth* into 10 equal parts, what are the parts called?

12. What is the fractional unit?

13. What part of 1 hundredth is 1 thousandth? How many 1 thousandths is 1 hundredth?

NOTATION AND NUMERATION.

199. ***A Decimal Fraction*** is one or more of the *decimal* divisions of a unit. Thus, $\frac{1}{10}$, $\frac{5}{10}$, $\frac{7}{100}$, $\frac{25}{1000}$, etc., are decimal fractions.

200. Decimal fractions are commonly called *Decimals.*

201. Decimal Fractions are unlike *Common Fractions,* because their denominators increase and decrease by the *uniform scale* of 10, the same as integers. The fractional units are, therefore, always *tenths, hundredths, thousandths,* etc.

202. ***The Decimal Sign*** (**.**), called the *decimal point,* is used to distinguish a *decimal* from an *integer,* and *must always* be placed at the *left,* or before the figure or figures that are to be regarded as decimals.

It also indicates the *denominator*, and determines the *value* of the decimal expression.

203. Decimal fractions may be numerically written in *two* ways, viz.: either as *common fractions*, the denominator being expressed, or in *decimal notation*, the denominator being omitted, but indicated from the place the numerator occupies in the scale. Thus,

$\frac{5}{10}$,	or	.5,	read	5 *tenths.*
$\frac{25}{100}$,	"	.25,	"	25 *hundredths.*
$\frac{125}{1000}$,	"	.125,	"	125 *thousandths.*
$\frac{7}{100}$,	"	.07,	"	7 *hundredths.*
$\frac{16}{1000}$,	"	.016,	"	16 *thousandths.*
$\frac{9}{1000}$,	"	.009,	"	9 *thousandths.*

204. The denominator of a decimal fraction, when expressed, is always 10, 100, 1000, etc., or 1, with as many ciphers annexed as there are figures in the given decimal. Thus, $.3 = \frac{3}{10}$; $.04 = \frac{4}{100}$; $.005 = \frac{5}{1000}$, etc.

205. To express in *decimal form* a decimal fraction written in the form of a common fraction, omit the denominator, and if the numerator contain as many figures as there are ciphers in the denominator, place the point before it for the decimal required. Thus, $\frac{7}{10} = .7$; $\frac{36}{100} = .36$; $\frac{145}{1000} = .145$, etc.

206. But if the number of figures in the numerator are not equal to the number of ciphers in the denominator, prefix ciphers to the numerator until the number of places are equal to the ciphers in the denominator, and place the point before the whole. Thus, $\frac{5}{100} = .05$; $\frac{12}{1000} = .012$; $\frac{8}{1000} = .008$, etc.

EXERCISES.

207. 1. Express in decimal form $\frac{9}{100}$.

$\frac{9}{100} = .09$ ANALYSIS.—The numerator is 9, the denominator 100 ; the decimal is expressed by a *point, naught, nine ;* thus, .09, and is read *nine hundredths.*

In like manner analyze and express,

2. $\frac{9}{10}$.
3. $\frac{24}{100}$.
4. $\frac{4}{100}$.
5. $\frac{42}{100}$.
6. $\frac{125}{1000}$.
7. $\frac{22}{1000}$.
8. $\frac{14}{100}$.
9. $\frac{7}{1000}$.
10. $\frac{102}{1000}$.
11. $\frac{20}{100}$.
12. $\frac{8}{1000}$.
13. $\frac{30}{1000}$.

The teacher should exercise the class *orally* by rapidly dictating similar examples, thus : " The denominator is 100, the numerator 5 : what will express the decimal ?" The prompt response should be, "*Point, naught, five,* read *five-hundredths*" (.05).

Ques. " The denominator is 1000, the numerator 26 ?"

Ans. "*Point, naught, two, six,* read *twenty-six thousandths*" (.026).

208. 1. Express and read as a common fraction .09.

$.09 = \frac{9}{100}$ ANALYSIS.—The numerator is 9, the denominator 100, and the fraction is $\frac{9}{100}$.

In like manner express and read,

2. .6 ; .02 ; .28 ; .45 ; .024 ; .010.
3. .009 ; .075 ; .150 ; .204 ; .040 ; .110.

Oral exercises similar to the following should be freely used. The teacher pronounces distinctly, "*Point, naught, five.*" The pupil promptly responds, "The numerator is *five,* the denominator *one hundred,* and the fraction is *five hundredths*" ($\frac{5}{100}$).

Ques. "*Point, naught, two, six?*" *Ans.* "The numerator is *twenty-six,* the denominator *one thousand,* and the fraction is *twenty-six thousandths*" ($\frac{26}{1000}$).

209. When the decimal point stands between the integral and decimal parts of the same number it is also called the ***sepatrix,*** and is read, ***and;*** thus, 5.6 is read 5 ***and*** 6 tenths; 9.36 is read 9 ***and*** 36 hundredths.

When an integer and decimal are written together the expression is a ***Mixed Number*** **(163).**

The relation of decimals and integers to each other is clearly shown by the following

TABLE.

Hund.-millions.	Ten-millions.	*Millions.*	Hund.-thousands.	Ten-thousands.	*Thousands.*	Hundreds.	Tens.	UNITS.	Tenths.	Hundredths.	*Thousandths.*	Ten-thousandths.	Hund.-thousandths.	*Millionths.*	Ten-millionths.	Hund.-millionths.
9	8	7	6	5	4	3	2	1	.2	3	4	5	6	7	8	9

INTEGERS. DECIMALS.

210. As in integers, so in decimals, make the order of ***units*** the starting point of notation and numeration, extending the scale to the ***left*** of units place in writing ***integers,*** and to the ***right*** of units place in writing ***decimals.***

The first order to the *left* of units is *tens,* and the first order to the *right* of units is *tenths;* the second order at the left is *hundreds,* and the second order to the right is *hundredths;* the third order to the left is *thousands,* and the third order to the right is *thousandths,* and so on, the *integers* on the left, and the *decimals* on the right, corresponding in name, being equally distant from the units place.

211. Both in integers and in decimals, the value expressed by any figure is determined by the *position* of that figure, and is always *ten times* the value of the same figure in the next lower order, or 1 *tenth* the value of the same figure in the next higher order. Hence,

212. In writing decimals, as in integers, *vacant orders* must be filled with ciphers (**29,** 2).

To express 4 thousandths, write 4 in the *third* place, and ciphers in the *second* and *first* places; thus, .004.

213. Placing a cipher after the point and before the numerator of a decimal fraction, multiplies the denominator by 10, hence *divides* the decimal by 10 (**167,** II, 2).

Thus, $.3 = \frac{3}{10}$; $.03 = \frac{3}{100}$, $.003 = \frac{3}{1000}$, etc. Or, $.3 \div 10 = .03$; $.03 \div 10 = .003$, etc.

214. Rejecting a cipher from before the numerator divides the denominator by 10, hence *multiplies* the decimal by 10 (**167,** I, 2).

Thus, $.005 = \frac{5}{1000}$; $.05 = \frac{5}{100}$; $.5 = \frac{5}{10}$. Or, $.005 \times 10 = .05$; $.05 \times 10 = .5$.

215. Annexing a cipher to a decimal multiplies both numerator and denominator by 10, hence reduces to *higher terms* (**167,** III, 1).

Thus, $.3 = \frac{3}{10}$; $.30 = \frac{30}{100}$; $.300 = \frac{300}{1000}$, etc. Or, $.3 = .30 = .300$, etc.

216. Rejecting a cipher from the right of a decimal divides both numerator and denominator by 10, hence reduces to *lower terms* (**167,** III, 2).

Thus $\frac{500}{1000} = .500$; $\frac{50}{100} = .50$; $\frac{5}{10} = .5$. Or, $.500 = .50 = .5$.

From the preceding illustrations are readily deduced the following

217. PRINCIPLES.—1. *Decimals are governed by the same laws of notation as integers.* Hence

2. *The value of any decimal figure depends upon the place it occupies at the right of the decimal sign* (**211**).

3. *Every removal of a decimal figure one place to the right diminishes its value tenfold* (**213**).

4. *Every removal of a decimal figure one place to the left increases its value tenfold* (**214**).

5. *Ciphers may be annexed to, or omitted from, the right of any decimal, without changing its value* (**215, 216**).

WRITTEN EXERCISES.

1. Express in figures fifty-four thousandths.

$\frac{54}{1000} = .054$ ANALYSIS.—The numerator is 54, and the denominator 1000. Since 54 thousandths decimally expressed is equal to 5 hundredths and 4 thousandths, write 4 in thousandths place, and 5 in hundredths place, and a cipher in tenths place. Hence .054 is the decimal required.

Express decimally

2. Twelve hundredths.
3. Nine thousandths.
4. Forty-six hundredths.
5. Twenty-five thousandths.
6. Ninety-two thousandths.
7. Eight ten-thousandths.

RULE FOR DECIMAL NOTATION.—*Write the numerator of the decimal the same as if an integer, writing ciphers in the place of vacant orders to give each significant figure its proper value, and place the decimal point before tenths.*

In like manner express decimally,

8. 58 thousandths.
9. 23 ten-thousandths.
10. 126 ten-thousandths.
11. 32 hundred-thousandths.
12. 320 hundred-thousandths.
13. 1308 millionths.
14. Forty-five, and forty-five thousandths.
15. Ten, and 2037 hundred-thousandths.
16. Twelve hundred, and 120 ten-thousandths.
17. One thousand, and 500 hundred-thousandths.

18. $\frac{127}{10000}$.
19. $\frac{4056}{10000}$.
20. $\frac{132}{100000}$.
21. $\frac{208}{100000}$.
22. $48\frac{175}{10000}$.
23. $218\frac{3046}{1000000}$.

218. Read the decimal .075.

.0 7 5 = 7 5 thousandths.

Analysis.—The numerator of the fraction is 75, and the denominator 1000. The figures of the decimal express 0 tenths, 7 hundredths, and 5 thousandths, read 75 thousandths.

In like manner copy and read,

2. .007.
3. .0125.
4. .215.
5. .0342.
6. .0075.
7. .04062.
8. .3000.
9. .40500.

Rule for Decimal Numeration.—I. *Numerate from the decimal point to the right, to determine the denominator, and toward the decimal point from the right, to determine the numerator.*

II. *Read the decimal as if an integer, and give it the name of its right-hand order.*

Copy and read,

10. 4.86 ; 26.009 ; 7.4003 ; 125.00275.

11. .00104 ; 5.4300 ; 10.003146 ; .0005200.

Dictation exercises should be given by the teacher until the class can *write* and *read* decimals with rapidity and correctness.

QUESTIONS FOR REVIEW.

219. 1. What are $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$ called? Define a Decimal Fraction. What are decimal fractions commonly called? What is a decimal scale? What numbers are written in the decimal scale? What are the fractional units of a decimal fraction? How does a decimal fraction differ from a common fraction?

2. What is the Decimal Sign? For what is it used? Where placed? How is the value of a decimal expression determined? In how many ways may a decimal fraction be written? Is it a decimal in either case? Why?

3. What is the denominator of a decimal fraction? What figure of the numerator determines the name of the denominator? How is a decimal fraction in the form of a common fraction written in decimal form? When the point is placed between the integral and decimal parts of a number, what is it called? How read? What is a mixed number?

4. Where do we commence to write and read decimals? What is the scale to the right of units made up of? To the left of units? Name the first five orders of decimals. To what orders of integers do they correspond?

5. How is the value of any figure in a decimal determined? How many units of any order make one of the next higher? What is done with vacant orders in writing decimals?

6. How does placing a cipher before the numerator of a decimal fraction affect the denominator? How, the decimal? Rejecting a cipher from before the numerator has what effect upon the denominator? Upon the decimal? Illustrate. What effect is produced by annexing a cipher to a decimal? By rejecting a cipher? Illustrate.

7. Repeat principles 1, 2, 3, 4, 5. Rule for Decimal Notation. Rule for Decimal Numerations.

DECIMAL CURRENCY.

220. ***Currency*** is *money,* as coin, bank-bills, treasury notes, etc., in circulation as a medium of trade.

221. ***A Decimal Currency*** is a currency whose denominations increase and decrease by the uniform scale of 10.

222. ***The Legal Currency*** of the United States, is a *decimal* currency, and is sometimes called *Federal,* or *United States money.*

223. Its denominations and their relative values are shown in the following

TABLE.

10 mills (*m.*)	make	1 cent.	*c.* or *ct.*
10 cents	"	1 dime.	*d.*
10 dimes or 100 cents	"	1 dollar.	$.
10 dollars	"	1 eagle.	*E.*

224. The dollar is the *unit* of United States money, and all the lower denominations are *decimals* of a dollar, and separated from it by the decimal point. Thus, two dollars four dimes six cents five mills, are expressed $2.465.

225. It will thus be seen that the *dime* is 1 *tenth* part of the *unit*, or dollar; the *cent* is 1 *tenth* part of the dime, or 1 *hundredth* part of the dollar; and the *mill* is 1 *tenth* part of the cent, or 1 *hundredth* part of the dime, or 1 *thousandth* part of the dollar.

The denomination of *dimes* is not regarded in business operations, but the two places of dimes and cents, or of *tenths* and *hundredths* are appropriated to *cents*. Thus, 5 dollars 3 dimes 6 cents are written $5.36, and read *five dollars thirty-six cents*. Hence,

226. When the number expressing cents is *less* than 10, write a cipher before it in the place of *tenths*, or dimes. Thus, 7 cents is written $.07; 9 cents, $.09.

In business, *cents* are frequently written as common fractions of a dollar; thus, $3.25 is written $\$3\frac{25}{100}$, and read 3 and $\frac{25}{100}$ dollars. The *half-cent* may be written either as a common fraction, $\frac{1}{2}$, or as 5 mills. Thus, 37 and one-half cents is written $\$.37\frac{1}{2}$, or $.375.

In business calculations, if the mills in the final *result* is 5, or more than 5, they are considered a *cent;* if *less* than 5, they are not regarded. Thus, $3.198 would be called $3.20, and $1.254, would be called $1.25.

227. PRINCIPLES.—1. *Decimal Currency is expressed according to the decimal system of notation.*

2. *All the operations upon Decimal Currency are the same as the corresponding operations on Decimals.*

United States money was established by act of Congress, 1786. Previous to that, pounds, shillings and pence were in use.

WRITTEN EXERCISES.

Express in figures,

1. Five dollars twenty cents. $5.20.
2. Three dollars nine cents.
3. Forty dollars ten cents.
4. Sixty-six dollars nine mills.
5. 74 cents 5 mills. $.745.
6. 9 dollars 20 cents 6 mills.
7. 20 dollars twelve and a half cents.
8. 96 cents seven mills.

Read

$ 7.84.
$10.05.
$25.12½.
$ 45.094.
$ 50.50.
$100.005.
$100.658.
$ 40.404.
$ .872.

REDUCTION.

Case I.

228. To reduce decimals to lower or higher orders.

1. In 1 melon how many *tenths* of a melon? How many *hundredths?* How many *thousandths?*

2. In 3 tenths of a mile how many hundredths? In 30 hundredths how many thousandths?

3. How many tenths in 2 units? In 3? In 4? In 5?

4. How many tenths in 30 hundredths? In .40?

5. How many hundredths in 5 tenths? In .6? .7? .8?

6. How many thousandths in 2 units? In 4? In 6?

7. How many thousandths in .06? In .08? .12?

8. How many hundredths in 300 thousandths? How many tenths?

9. How many hundredths in .200? In .320? In .1500? In .2500?

10. How many tenths in 4 units? How many hundredths? How many thousandths?

11. In 3000 thousandths how many hundredths? How many tenths? How many units?

12. How many cents in $1? $2? $3? $5? $7? $8?

13. How many mills in 6 cents? In 7 cents? 8 cents? 9 cents? 10 cents?

14. How many cents in 2 dollars 20 cents? In 5 dollars 18 cents?

15. How many mills in $2? In $3? In 45 cents?

16. How many dollars in 400 cents? In 525 cents?

17. How many hundredths of a dollar are 5 cents? 25 cents? 56 cents?

18. How many thousandths of a dollar are 5 mills? 5 cents 5 mills? 23 cents 8 mills? 37½ cents?

19. How many cents in 2000 mills? How many dollars?

20. Express decimally 5 cents.

Let the pupil answer, "*Point, naught, five*, read *five hundredths*" (.05).

21. Express decimally 7 cents; 8 cents; 9 cents.

22. Express decimally 5 mills; 6 mills; 6 cents 6 mills; 14 cents 5 mills; 22 cents 7 mills.

23. What part of a dollar is 9 cents? 12 cents? 24 cents? 48 cents? 50 cents? 75 cents?

24. Change .5 to hundredths. To thousandths. To ten-thousandths.

25. Change .5000 to thousandths. To hundredths. To tenths.

26. What is a common denominator of .3 and .05?

27. Change .2, .04 and .005 to equivalent decimals having a common denominator. .15, .6, and .125.

Decimals are reduced to a common denominator by annexing ciphers until the number of places in each are equal.

Dictation exercises similar to the preceding should be continued until the class are thoroughly familiar with the subject.

WRITTEN EXERCISES.

1. Reduce .4 and .23 to ten-thousandths.

OPERATION.

$.4 = .4000$

$.23 = .2300$

ANALYSIS.—Since annexing ciphers to a decimal does not change its value (**215**), annex ciphers to the given decimals until the right-hand cipher occupies the place of the required order.

2. Reduce .06 and .034 to ten-thousandths.
3. Reduce .9 and .240 to hundred-thousandths.
4. Reduce .0041 and .07 to millionths.
5. Reduce .326 to ten-millionths.
6. Reduce .32016 to hundred-millionths.
7. Reduce .006 to hundred-thousandths.
8. Reduce .8, .104 and .0031 to equivalent decimals having a common denominator.
9. Reduce 24, 2.73, .0062 and .9 to equivalent decimals having a common denominator.

An integer is reduced to a decimal by placing the decimal point after units, and annexing ciphers; one cipher reducing it to *tenths*, two ciphers, to *hundredths*, and so on. Thus, 2 = 2.0 = 2.00.

10. Change $25 to cents.

OPERATION.

$25 × 100 = 2500 cents.

Analysis.—Since $1 is 100 cents, $25 are 25 times 100 cents, or 2500 cents (**89**).

Hence,

229. To change dollars to cents omit the sign ($) and annex *two* ciphers; dollars to mills annex *three* ciphers; cents to mills annex *one cipher*. Thus, $9 = 900 *cents* = 9000 *mills;* $.45 = 450 *mills.*

Reduce

11. $47 to cents.
12. $163 to cents.
13. $35 to mills.
14. $104 to mills.
15. 67 cents to mills.
16. $.87 to mills.
17. How many cents are $56? $200? $107? $321?
18. How many mills are $27? $.96? $.72? $.87?

230. To change dollars and cents, or dollars cents and mills, to mills, remove the decimal point and sign ($). Thus, $4.28 equals 428 *cents;* $2.375 equals 2375 *mills.*

19. How many cents in $18.75? $84.09? $30.06?
20. How many mills in $5.625? $.76? $.085?
21. Reduce.03400 to thousandths.

OPERATION.

.034|00 = .034

Analysis.—Since omitting ciphers from the right of a decimal does not change its value (**216**), cut off as many ciphers or figures from the right of the given decimal as will reduce it to the required order.

Reduce

22. .23000 to hundredths.
23. .10200 to thousandths.
24. .9000 to tenths.
25. .0700 to hundredths.

26. Reduce 4.04000 to thousandths.
27. Reduce 3.120400 to ten-thousandths.
28. Reduce .75030000 to hundred-thousandths.
29. Reduce 2.00800000 to millionths.

30. Change 1875 cents to dollars.

OPERATION.

```
1|00)18|75
     $18.75
```

ANALYSIS.—Since 100 cents make $1, 1875 cents are as many dollars as 100 cents are contained times in 1875, which is 18.75 times (**113**). Hence,

231. To change cents to dollars, prefix the sign ($) and point off *two* figures to the right; mills to dollars, point off *three* figures from the right.

Reduce

31. 3467 cents to dollars.
32. 10408 cents to dollars.
33. 46725 mills to dollars.
34. 30200 cents to dollars.
35. 762 mills to cents.
36. 5607 mills to dollars.
37. 3009 cents to dollars.
38. 850 mills to cents.

CASE II.

232. To reduce a decimal to a common fraction.

1. How many halves in $\frac{5}{10}$? In $\frac{50}{100}$?
2. How many fifths in $\frac{4}{10}$? In $\frac{6}{10}$? In $\frac{8}{10}$?
3. How many fourths in $\frac{25}{100}$? $\frac{50}{100}$? $\frac{75}{100}$?
4. How many twentieths in $\frac{5}{100}$? $\frac{15}{100}$? $\frac{35}{100}$?
5. In .50 how many halves? Fourths? Tenths? Twentieths?
6. How many fourths in .25? In .75?
7. How many eighths in .40? In .80? In 1.20?

WRITTEN EXERCISES.

1. Reduce .125 to a common fraction expressed in lowest terms.

OPERATION.

$.125 = \frac{125}{1000} = \frac{1}{8}$

ANALYSIS.—The numerator is 125, the denominator 1000, and the fraction $\frac{125}{1000}$, which reduced to its lowest terms is $\frac{1}{8}$. Hence $.125 = \frac{1}{8}$.

Reduce to equivalent common fractions,

2. $.4. *Ans.* $$\frac{2}{5}$.
3. .16.
4. $.75.
5. .008.
6. .08.
7. .225.
8. .024.
9. .0075.
10. .1875.
11. .0625.
12. .00125.
13. .0068.

RULE.—*Omit the decimal point, then write the numerator over the proper denominator, and reduce to its lowest terms.*

Reduce to common fractions in lowest terms,

14. $.375.
15. .475.
16. $.875.
17. .0024.
18. .068.
19. .3125.
20. .5625.
21. .0016.
22. .00060.
23. $.125.
24. $.625.
25. $.1875.

26. Express by an integer and common fraction 8.25.

27. Express by an integer and common fraction $12.75. 25.005. $36.125.

28. Express by an integer and common fraction 37.0375. $9.625. 42.1875.

CASE III.

233. To reduce a common fraction to a decimal.

1. How many tenths in $\frac{1}{2}$? How many hundredths?
2. How many tenths in $\frac{1}{5}$? In $\frac{3}{5}$? In $\frac{4}{5}$?
3. How many hundredths in $\frac{1}{5}$? In $\frac{1}{4}$? In $\frac{1}{20}$?

WRITTEN EXERCISES.

1. Reduce $\frac{3}{4}$ to an equivalent decimal.

1ST OPERATION.

$$\frac{3}{4}=\frac{300}{400}=\frac{75}{100}=.75$$

Or, 2D OPERATION.

$$\begin{array}{r} 4\underline{)3.00} \\ .75 \end{array}$$

ANALYSIS.—Annex the same number of ciphers to both terms of the fraction, and divide the resulting terms by 4, the significant figure of the denominator, to obtain the *decimal* denominator 100. Then change to decimal *form* (**205**). Or, the same result may be obtained more directly as in the second operation.

2. Reduce $\frac{1}{16}$ to its equivalent decimal.

1ST OPERATION.

$$\begin{array}{r} 16\underline{)1.0000} \\ .0625 \end{array}$$

2D OPERATION.

Or, $\frac{1}{16}=\frac{10000}{160000}=\frac{625}{1000}=.0625$

The reason for prefixing a cipher to the quotient is apparent in the second operation.

Reduce to equivalent decimals,

3. $\frac{1}{4}$.
4. $\frac{5}{8}$.
5. $\frac{2}{25}$.
6. $\frac{4}{5}$.
7. $\frac{17}{20}$.
8. $\frac{3}{16}$.
9. $\frac{14}{20}$.
10. $\frac{3}{80}$.
11. $\frac{4}{125}$.
12. \$$\frac{3}{8}$.
13. \$$\frac{3}{4}$.
14. \$$\frac{7}{8}$.

RULE.—I. *Annex decimal ciphers to the numerator and divide by the denominator.*

II. *Point off as many decimal places in the result as are equal to the number of ciphers annexed.*

The sign + is sometimes placed after the result to show that there is still a remainder. Thus, $\frac{1}{3}=333\frac{1}{3}$, or 333+.

Reduce to equivalent decimals,

15. $\frac{2}{3}$.
16. $\frac{3}{25}$.
17. \$$\frac{9}{20}$.
18. \$$\frac{9}{16}$.
19. $\frac{19}{40}$.
20. $\frac{49}{125}$.
21. \$$5\frac{3}{5}$.
22. $3\frac{3}{4}$.
23. \$$12\frac{14}{70}$.

ADDITION.

234. 1. What is the sum of $\frac{3}{10}$ and $\frac{4}{10}$? .5 and .3?

2. What is the sum of $\frac{4}{100}$ and $\frac{15}{100}$? .09 and .12?

3. What is the sum of $\frac{3}{10}$ and $\frac{75}{100}$? .4 and .08?

4. Find the sum of .15, .2 and .07. Of .6, .20 and .06.

5. Find the sum of $\frac{5}{100}$ and $\frac{5}{1000}$. Of .06 and .006.

6. Find the sum of $2.5, $3 and $.6.

7. Find the sum of $.16, $.08 and $1.3.

8. How many decimal figures in the sum of *tenths* and *tenths?* Of *tenths* and *hundredths?* Of *hundredths* and *thousandths?* Of *tenths* and *thousandths?*

235. Since decimals and integers increase and decrease uniformly by the scale of 10, decimals expressing like parts of a unit may be *added, subtracted, multiplied* and *divided* in the same manner as integers.

The pupil should be required to obtain and express all results in *decimal* form.

WRITTEN EXERCISES.

1. What is the sum of 2.125, 13.07, 7.8 and .3142?

OPERATION.

$$\begin{array}{r@{.}l} 2&1250\\ 13&0700\\ 7&8000\\ &3142\\ \hline 23&3092 \end{array} \quad \text{Or,} \quad \begin{array}{r@{.}l} 2&125\\ 13&07\\ 7&8\\ &3142\\ \hline 23&3092 \end{array}$$

ANALYSIS. — Since figures of the same unit value only can be added (**56**), write the numbers so that figures of the same order shall stand in the same column, and the decimal points directly under one another. Reducing the decimals to a common denominator by annexing ciphers (**228**), or supposing them to be annexed, add as in integers, placing the decimal point before tenths in the sum.

In like manner add,

(2.)	(3.)	(4.)	(5.)
42.3	12.326	4031.06	.608242
13.06	204.09	108.304	.0315044
8.049	8.3024	9.001345	.8034
1.6	52.007	76.739	.086
.037	324.1	250.0007	.9106
65.046			2.4397464

6. Find the sum of \$28.50, \$125, \$3.875 and \$.945.

7. Find the sum of \$5.08, \$.69, \$250.46 and \$17.375.

RULE.—I. *Write the numbers so that figures of the same order shall stand in the same column.*

II. *Add as in addition of integers, and place the decimal point before the order of tenths in the sum.*

8. What is the sum of 36 hundredths, 207 thousandths, 321 ten-thousandths, and 75 hundred-thousandths?

9. A grocer bought a barrel of sugar for \$21.84, a box of tea for \$42.375, a cheese for \$6.08, and a tub of butter for \$10.125: what was the cost of all?

10. How many acres of land in 3 farms, containing respectively $87\frac{1}{2}$ acres, $126\frac{7}{10}$ acres, and $140\frac{1}{4}$ acres?

11. Add 275 dollars 12 cents 5 mills, 50 dollars 50 cents, 32 dollars 5 cents 5 mills, and 75 cents.

12. A lady paid \$75.50 for some furs, \$$20.18\frac{3}{4}$ for a dress, \$$3\frac{4}{5}$ for a parasol, and \$$15\frac{3}{4}$ for a shawl. How much did she pay for all?

13. What is the sum of 800 dollars 800 cents 800 mills?

14. What is the sum of 15 hundred, 15 tens, 15 tenths, and 15 hundredths?

15. A man divides his farm into 6 fields, containing respectively 15 acres, $22\frac{5}{8}$ acres, 30.06 acres, $41\frac{1}{8}$ acres, 28.325 acres, and $17\frac{1}{2}$ acres. How many acres in his farm?

16. Bought a ton of coal for \$7.46, a barrel of sugar for \$$28.37\frac{1}{2}$, a bag of coffee for \$$12\frac{3}{4}$, and a barrel of flour for \$$9.06\frac{1}{4}$. What was the cost of all?

Find the value

17. Of $16.5 + .0348 + 7\frac{1}{5} + .07\frac{1}{2} + 1.008 + \frac{4}{5}$.

18. Of $\$.87\frac{1}{2} + \$5\frac{3}{4} + \$86 + \$.338 + \$\frac{7}{8} + \$102.95$.

19. Of 26 tenths + 416 hundredths + 9 thousandths + 5 millionths.

SUBTRACTION.

236. 1. From $\frac{5}{10}$ take $\frac{3}{10}$. From .7 take .4.

2. From $\frac{16}{100}$ take $\frac{8}{100}$. From .24 take .08.

3. From $\frac{25}{1000}$ take $\frac{15}{1000}$. From .018 take .007.

4. From $\frac{4}{10}$ take $\frac{32}{100}$. From .8 take .75.

5. What is the difference between $\frac{3}{5}$ and $\frac{6}{10}$? $\frac{1}{2}$ and .5?

6. What is the difference between 2 and .2? 1 and .75?

7. Find the value of $\frac{7}{10} - .3$; $.25 - \frac{1}{4}$; $.60 - .5$.

8. Find the value of $\$.75 - \$.5$; $\$\frac{1}{2} - \$.25$; $\$\frac{9}{10} - \$.4$.

9. Which is greater $\$\frac{1}{2}$ or \$.5? \$.35 or $\$\frac{1}{4}$? \$.7 or $\$\frac{3}{4}$? $\$\frac{4}{5}$ or \$.75? 30 cents or \$.3? $\$\frac{5}{20}$ or 40 cents?

10. If there be *three* decimal figures in the minuend and *one* in the subtrahend, how many, in the remainder? If *two* in the minuend and *four* in the subtrahend? If *none* in the minuend and *three* in the subtrahend? If *five* in the minuend and *none* in the subtrahend?

WRITTEN EXERCISES.

1. From 7.6325 subtract 5.4.

OPERATION.

7.6325	Or,	7.6325
5.4000		5.4
2.2325		2.2325

Analysis.—Write the given numbers as in addition, the subtrahend under the minuend, reducing the decimals to a common denominator by annexing ciphers (**228**), or supposing them to be annexed until each are equal in number of places, then subtract as in integers.

	(2.)	(3.)	(4.)	(5.)
From	18.5	2.8706	.50376	.36
Subtract	2.3476	.49	.065	.127084
	16.1524	2.3806	.43876	.232916

6. From $125.75 subtract $41.095.

7. From $1.375 subtract $.88.

8. From .84032 subtract .0047.

Rule.—I. *Write the subtrahend under the minuend, so that figures of the same order shall stand in the same column.*

II. *Subtract as in subtraction of integers and place the decimal point before the order of tenths in the remainder.*

From

9. 297.5 take 4.273.
10. 9.00372 take .009.
11. 5001.1 take .10002.

From

12. Two take 2 ten-thousandths.
13. Ten take 10 thousandths.
14. Four take 24 millionths.

15. From .00038 take 33 ten-millionths.
16. From 30000 take 3 millionths.

17. From 578 tenths take 736 hundredths.

18. From 1 take 9999 thousandths.

19. From 30.034 take .11034.

20. From 24763 take 87 hundredths.

21. From 25 hundred take 25 hundredths.

22. From 600 dollars take 600 cents.

23. From four hundred twenty-seven thousandths take four hundred twenty-seven millionths.

24. A man having $1000 in bank drew out $450.62 : how much remained in bank ?

25. Sold a horse for $300, which was $65.25 more than he cost : how much did he cost ?

26. Bought a farm for $2560.50 ; paid at one time $1046, and at another time $807.87 : how much remains unpaid ?

27. From a piece of cloth containing 56.25 yards, 24⅝ yards were cut : how many yards remained ?

28. Bought groceries to the amount of $7.18 : how much change must I receive for two five-dollar bills ?

29. A coal dealer bought 570.5 tons of coal, and sold at different times 80.54 tons, 120⅘ tons, and 114⅜ tons : how much had he left ?

30. A man having $14725, gave $3560 for a store, and $7015.87½ for goods ; how much money had he left ?

Find the value of

31. (4.5 + .036) — (1.9 — .0027).

32. $50 — $15.36 + $6¾ — $10½.

33. (.375 + 34) — (56.2 — 45.002).

34. ($.87½ + $14⅜) — ($5.10 + $.75).

35. (155.006 — .32) — (80.0032 + 55.1).

36. $70 — ($25.4 — $10.12½) + $7⅗.

MULTIPLICATION.

237. 1. How much is 4 times $\frac{1}{10}$? 3 times .2?

2. How much is 6 times $\frac{8}{100}$? 4 times .04?

3. What is the product of $\frac{2}{100} \times 3$? $3 \times .02$?

4. What is the product of $\frac{2}{10} \times \frac{3}{10}$? $.3 \times .2$?

5. What is the product of $\frac{2}{100} \times \frac{3}{10}$? $.3 \times .02$?

6. What is the product of $\frac{2}{100} \times \frac{3}{100}$? $.03 \times .02$?

7. How much is 6 times .6 of a dollar? 7 times $\frac{9}{100}$ of a dollar? 8 times .005 of a dollar?

8. How much is $2 \times .5$? $2 \times .05$? $2 \times .005$? $2 \times .0005$?

9. How many decimal places in the product of *tenths* multiplied by *units?* *Tenths* by *tenths?* *Tenths* by *hundredths?* *Hundredths* by *hundredths?*

10. If there are *two* decimal figures in the multiplicand, and *two* in the multiplier, how many will there be in the product? If *three* in the multiplicand and *one* in the multiplier? If *none* in the multiplicand and *four* in the multiplier?

238. PRINCIPLE.—*The number of decimal places in the product of two decimals, is equal to the number of decimal places in both factors.*

WRITTEN EXERCISES.

1. Multiply .56 by .4.

OPERATION.

$$\begin{array}{r} .56 \\ .4 \\ \hline .224 \end{array}$$

ANALYSIS.—Multiply as in common fractions (**189**); thus, $.56 \times .4 = \frac{56}{100} \times \frac{4}{10} = \frac{224}{1000} = .224$. Or,

Multiply as in integers, and since *hundredths* multiplied by *tenths* produces *thousandths*, the product must contain *three* decimal places, or as many as both factors contain. (PRIN.)

	(2.)	(3.)	(4.)	(5.)
Multiply	4.64	53.062	.1346	675.1
By	3.35	4.53	.203	.008

What is the product of

6. 6 times $2.45.
7. 9 times .326.
8. 26 times 4.007.
9. 15 times .0038.
10. .5 of $6.25.
11. .22 of 3.84.
12. .07 of $12.031.
13. .042 of .506.
14. $3.6 × .045.
15. .723 × 6.04.
16. 28.7 × .029.
17. .186 × $4.02.

RULE.—*Multiply as in multiplication of integers, and from the right of the product point off as many figures for decimals as there are decimal places in both factors.*

1. If there are not as many figures in the product as there are decimals in both factors, supply the deficiency by prefixing ciphers.

2. To multiply by 10, 100, 1000, etc., remove the decimal point in the multiplicand as many places toward the *right* as *there are ciphers* in the multiplier (**89**).

Multiply and express the product in decimals.

18. 25 by 25 hundredths.
19. 76 by 2¼ hundredths.
20. 42 units by 42 tenths.
21. .56 by .0007.
22. 2.3042 by .0234.
23. $48½ by 6¼.
24. .0012 by 1000.
25. $2.032 by 100.
26. .87½ by .08⅓.
27. .046 by .046.
28. $242.5 by ¾.
29. ⅞ by 4.5.

Find the value

30. Of 28 × .25 × 6.
31. Of .014 × 6.2 × .007.
32. Of $37½ × .08 × ½.
33. $200 × 3⅔ × .006.
34. .304 × 100 × 10¼.
35. $59.36 × 2½ × .9.

36. Multiply sixty-five thousandths by two hundred twenty ten-thousandths.

37. If an acre of land produce 35.7 bushels of wheat, how many bushels will 5.125 acres produce?

38. If a boy earn $.87½ in 1 day, how much will he earn in 9¼ days?

What is the value

39. Of 46 mules, at $75.375 each?

40. Of 100 barrrels of flour at $9.62½ each?

41. Of 14⅛ yards of cloth at $4½ a yard?

42. Of 125 bushels of oats at $.625 a bushel?

43. Of 85 pounds of sugar at 10 cents a pound?

44. Of 25¾ bushels of timothy seed, at $3.25?

45. Of 3000 pounds of wool, at 37½ cents?

46. In 1 rod is 16.5 feet: how many feet in 30.005 rods?

47. If 1 dollar in gold be worth 109½ cents in currency, what is $500 in gold worth?

48. A farmer took 3 tons of hay to market, for which he received $12.62½ a ton. He bought 2 barrels of flour at $9¼ a barrel, 25 pounds of coffee at $.37½ cents a pound, and 3 hand-rakes at $.75 each. How much money had he left?

Find the value of

49. $46.37 \times .032 + 46.5 - .0084.$

50. $320.48 - \overline{24.5 \times 4.2} + .9805.$

51. $(72.06 - 9.1425) \times (16 - 4.25).$

52. $\overline{1.45 \times 26} + \overline{10 \times 3.74}.$

53. $\overline{25.1 \times .008} + \overline{8 - .0326} \times 5.$

54. $\overline{(100 - 4.75} \times \overline{.09 + 6.31}) + 400.$

DIVISION.

239. 1. What is $\frac{1}{3}$ of $\frac{6}{10}$? $\frac{1}{4}$ of $\frac{16}{100}$? $\frac{1}{5}$ of $\frac{25}{1000}$?

2. What is $\frac{1}{2}$ of .8? $\frac{1}{5}$ of .35? $\frac{1}{6}$ of .048?

3. Divide $\frac{6}{10}$ by $\frac{2}{10}$; $\frac{36}{100}$ by $\frac{9}{100}$; $\frac{48}{1000}$ by $\frac{8}{1000}$.

4. Divide .8 by 4; .8 by .4; .08 by .04; .008 by .004.

5. Divide $\frac{6}{10}$ by $\frac{12}{100}$; $\frac{25}{100}$ by $\frac{25}{1000}$; .25 by .025.

6. Divide 3.6 by 6.

Analysis.—3.6 equals 36 tenths, and $\frac{1}{6}$ of 36 tenths is 6 tenths. Hence $3.6 \div 6 = .6$.

7. Divide .36 by 6; .36 by .06; .036 by .006.

8. Divide $\frac{3}{10}$ by $\frac{6}{100}$; .3 by .06; 1.5 by .05.

Reduce the fractions to a common denominator.

9. Multiply $\frac{7}{10}$ by $\frac{5}{10}$; .7 by .5. Divide .35 by .7.

10. Multiply $\frac{7}{10}$ by $\frac{5}{100}$; .7 by .05. Divide .035 by .7.

11. The product of two factors is .35, one of which is .5: what is the other?

12. The product of two factors is .035, one of which is .07: what is the other?

13. The product of two factors is .0035, one of which is .05: what is the other?

14. Divide .14 by 7; .48 by .04; .0060 by .012.

15. How many decimal places in the quotient when *tenths* are divided by *units?* *Tenths* by *tenths?* *Hundredths* by *tenths?* *Thousandths* by *hundredths?* *Ten-thousandths* by *hundredths?*

16. How many decimal places in the quotient when the product of two factors is divided by one of the factors?

17. If there be *two* decimal figures in the divisor and *three* in the dividend, how many will there be in the quotient? If *three* in the divisor and *three* in the dividend? If *two* in the divisor and *none* in the dividend? If *none* in the divisor and *three* in the dividend?

240. Principle.—*The quotient must contain as many decimal places as there are decimal places in the dividend, less the number of decimal places in the divisor.*

WRITTEN EXERCISES.

1. Divide 4.265 by 5.

OPERATION.

$$5\overline{)4.265}$$
$$.853$$

Analysis.—4.265 is 4265 thousandths; $\frac{1}{5}$ of 4265 thousandths is 853 thousandths. Hence 4.265 divided by 5 is .853.

2. What is the quotient of .624 divided by .6?

OPERATION.

$$.6\overline{).624}$$
$$1.04$$

Analysis.—Divide as in common fractions (**193**). Thus, $.624 \div .6 = \frac{624}{1000} \div \frac{6}{10} = \frac{104}{100} =$ 1.04, the required quotient. Or,

Divide as in integers, and since the dividend contains *three* decimal places and the divisor *one*, the quotient must have *two* decimal places. (Prin.)

Divide
3. 8.176 by 7.
4. .675 by .15.
5. $26.64 by 8.

Divide
6. $41.42 by $5.45.
7. 3.76 by .8.
8. 17.6 by 44.

Rule.—*Divide as in division of integers, and from the right of the quotient point off as many figures for decimals as the number of decimal places in the dividend exceed those in the divisor.*

1. If the number of figures in the quotient be *less* than the excess of the decimal places in the dividend over those in the divisor, the deficiency must be supplied by *prefixing* ciphers.

2. If there be a remainder after dividing the dividend, annex ciphers, and continue the division: the ciphers annexed are decimals of the dividend.

3. The dividend must always contain at least as many decimal places as the divisor, before commencing the division.

4. In most business transactions, the division is considered sufficiently exact when the quotient is carried to 4 decimal places, unless great accuracy is required.

5. To divide by 10, 100, 1000, etc., remove the decimal point in the dividend as many places to the *left* as *there are ciphers* in the divisor (**113**).

Find the value of

9. 75 ÷ .75.
10. .75 ÷ 75.
11. 7.5 ÷ .75.
12. 2.5 ÷ 12.5.
13. .56 ÷ 1.12.
14. 17.6 ÷ 10.
15. .0992 ÷ .32.
16. $2.25 ÷ 9.
17. .084 ÷ 8.
18. $68 ÷ 32.
19. 44 ÷ .4.
20. $1248 ÷ 100.

21. Divide 3.475 by 25; by 2.5; by .25; by .025.

22. Divide 7.2 by 6; by 12; by 8; by 9.

23. What is $\frac{1}{8}$ part of 7.2? Of 8.32? Of 1.095? Of .0368? Of 2.00728?

24. How many times is 7 contained in 8.4? In .84? In .084? In .0084?

25. How many building lots can be made of 2.16 acres of land, allowing .18 of an acre to a lot?

Find the *price* of 1 bushel, 1 pound, etc., of each of the following:

26. If 125 bushels of oats cost $62.50?

27. If 35 pounds of sugar cost $2.80?

28. If 144 bushels of wheat cost $168.48?

29. If 100 acres of land cost $\$3156\frac{1}{2}$?

30. If 9 turkeys cost $\$7.87\frac{1}{2}$?

31. If 396 pounds of sugar cost $44.748?

32. If 2500 pounds of butter cost $625?

33. If 2450 pounds of pork cost $\$153\frac{1}{8}$?

34. If 894 pine-apples cost $80.46?

35. If 7.5 acres of land produce 192.225 bushels of wheat, how much does 1 acre produce?

36. If a man build 812.5 rods of fence in 100 days, how many rods does he build each day?

37. At $12\frac{1}{2}$ cents each, how many cocoa-nuts can be bought for $1.75?

38. At $11 a ton, how much hay can be bought for $13.75?

39. A farmer sold 120 bushels of rye, at $\$1.12\frac{1}{2}$ a bushel, for which he received 27 barrels of flour: what was the cost of the flour a barrel?

REVIEW.

241. Reduce .0125 to a common fraction?

2. Express $\frac{23}{10000}$ as a decimal.

3. Reduce $\frac{9}{14}$ to four decimal places.

4. Change $\$142\frac{7}{8}$ to a decimal form.

5. Express $7.625 by an integer and common fraction.

6. Express decimally the difference between $2\frac{1}{2}$ times $\$\frac{5}{8}$ and $\$9.37\frac{1}{2}$.

7. What is the difference between 20 thousand and twenty-thousandths?

8. What is the product of 6000 by 6 thousandths?

9. What is the quotient of .125 divided by 8 thousand?

10. From the sum of 256.07 and 5.0125 take their difference.

11. Reduce $\frac{3}{8}$, $\frac{4}{5}$, $7\frac{9}{16}$, $.9\frac{1}{2}$ and $.15\frac{1}{4}$ to decimals and find their sum.

12. If the product of two factors is .00207 and one of the factors is .009, what is the other?

13. If 45.3 is the dividend and .015 the quotient, what is the divisor?

14. If the divisor is .004 and the quotient .04, what is the dividend?

15. A man bought 26 boxes of lemons at $\$4\frac{1}{8}$ a box, and sold them at \$3.90 a box. How much did he lose?

16. At $\$.31\frac{1}{4}$ a bushel, how many bushels of potatoes can be bought for \$9?

17. If 18.24 yards of cloth cost \$27.36, what will 7.25 yards cost? $9\frac{3}{4}$ yards? $15\frac{3}{8}$ yards?

18. A laborer receives $\$2\frac{1}{2}$ a day wages, and spends \$1.125 a day, including Sunday, for his support. How much does he save in 1 week? In 4 weeks? In 10 weeks?

19. If $4\frac{3}{4}$ tons of coal cost \$32.3, what will $9\frac{3}{5}$ tons cost?

What is the value of

20. $1.12 \times 5 \div 14$.
21. $\$15.5 \div 8 \times 100$.
22. $\$.75 \times 25 \div .625$.
23. $7.1 \times 8.2 - \overline{34\frac{3}{4} \div 2.5}$.
24. $.25 \times .5 \times 12 + 20 \div 100$.
25. $.07 \times 2.4 \times .015 \times 1000$.

26. $\overline{\$38\frac{1}{2} \times 15\frac{1}{4}} \div \overline{\$13.02 - \$8.323}$.
27. $6\frac{1}{2} \times \frac{8}{10} \times 3.16 \div 100 \times 10$.

28. A farmer sold 40 bushels of barley at \$.75 a bushel, 50 bushels of oats at $\$\frac{5}{8}$ a bushel, 3 tons of hay at $\$15\frac{3}{4}$ a ton, for all of which he received \$62.25 in money, and 5 barrels of flour. What was the cost of the flour a barrel?

SHORT METHODS.

Case I.

242. To find the cost of a number of articles, or quantity, when the price of a unit is an aliquot part of one dollar.

1. How many cents in ½ of $1? In ¼ of $1?
2. What part of $1 is 50 cents? Is 25 cents?
3. How many cents in ⅕ of $1? In ⅒ of $1?
4. What part of $1 is 20 cents? Is 10 cents?
5. How do we obtain ½ of $1? ⅓? ¼? ⅕? ⅒?
6. How many cents in ⅓ of $1? In ⅙ of $1?
7. What part of $1 is 8⅓ cents? Is 16⅔ cents?
8. What will be the cost of 18 pounds of butter at 25 cents a pound?

Analysis.—Since 1 pound cost 25 cents, or $¼, 18 pounds will cost 18 times $¼, or $$\frac{18}{4}$, equal to $4½, or $4.50.

9. What will 24 table books cost, at 12½ cents each? At 16⅔ cents each? At 20 cents each?

10. What will be the cost of 15 bushels of oats at 33⅓ cents a bushel? At 50 cents a bushel?

11. What cost 24 pounds of sugar at 16⅔ cents a pound? At 20 cents a pound? At 25 cents a pound?

12. At 16⅔ cents each, what will be the cost of 12 slates? Of 20 slates? Of 24 slates?

13. At 25 cents a yard, what will be the cost of 9 yards of muslin? Of 15 yards? Of 28 yards? Of 40 yards?

14. What will be the cost of 16 pounds of rice at 6¼ cents a pound? At 8⅓ cents? At 10 cents?

243. *An Aliquot Part* of a number is such a part as will exactly divide that number. Thus, $1\frac{1}{2}$, 2, 3, 4 and 6 are aliquot parts of 12 ; 3, 5, 6, $7\frac{1}{2}$ and 10 of 30.

ALIQUOT PARTS OF ONE DOLLAR.

5 cents = $\frac{1}{20}$ of $1.	$6\frac{1}{4}$ cents = $\frac{1}{16}$ of $1.
10 cents = $\frac{1}{10}$ of $1.	$8\frac{1}{3}$ cents = $\frac{1}{12}$ of $1.
20 cents = $\frac{1}{5}$ of $1.	$12\frac{1}{2}$ cents = $\frac{1}{8}$ of $1.
25 cents = $\frac{1}{4}$ of $1.	$16\frac{2}{3}$ cents = $\frac{1}{6}$ of $1.
50 cents = $\frac{1}{2}$ of $1.	$33\frac{1}{3}$ cents = $\frac{1}{3}$ of $1.

WRITTEN EXERCISES.

1. What must be paid for 216 bushels of potatoes at $33\frac{1}{3}$ cents a bushel?

OPERATION.

$$\$\tfrac{1}{3} \times 216 = \frac{\$216}{3} = \$72$$

ANALYSIS.—Since the price is $\$\frac{1}{3}$ a bushel, 216 bushels will cost 216 times $\$\frac{1}{3}$, or $72. Or, the cost will be $\frac{1}{3}$ as many dollars as there are bushels, or $216 ÷ 3 = $72.

2. What cost 428 dozens of eggs at 25 cents a dozen ?

3. What cost 148 yards of sheeting at $16\frac{2}{3}$ cents a yard?

RULE.—*Take such fractional part of the given number, or quantity, as the price is of one dollar.*

4. At $.50 each, what will 1263 apple barrels cost ?

5. At $8\frac{1}{3}$ cents a pound, what will be the cost of 2 barrels of sugar, each containing 174 pounds ?

6. What cost 2344 pounds of cheese at $8\frac{1}{3}$ cents a pound ? At $12\frac{1}{2}$ cents ? At $16\frac{2}{3}$ cents ?

7. What will be the cost of 5 sacks of coffee, each containing 46 pounds, at $33\frac{1}{3}$ cents a pound ?

CASE II.

244. To find the quantity, when the cost is given, and the price is an aliquot part of one dollar.

1. How many pounds of raisins at $16\frac{2}{3}$ cents a pound, can be bought for $3?

ANALYSIS.—Since $16\frac{2}{3}$ cents, or $\$\frac{1}{6}$, will buy 1 pound, $3 will buy as many pounds as $\$\frac{1}{6}$ is contained times in $3, which is 18 times. Hence 18 pounds can be bought for $3.

2. How many cocoa-nuts at 25 cents each, can be bought for $4? For $5? For $6?

3. At 20 cents each, how many primers can be bought for $2? For $5? For $7?

4. For $9, how many pounds of sugar can be bought at $8\frac{1}{3}$ cents a pound? At 10 cents? At $12\frac{1}{2}$ cents?

WRITTEN EXERCISES.

1. At $33\frac{1}{3}$ cents a yard, how many yards of cambric can be bought for $16?

OPERATION.

$$\$16 \div \$\tfrac{1}{3} = 48$$

Or,

$$16 \times 3 = 48$$

ANALYSIS.—Since the price is $\$\frac{1}{3}$ a yard, $16 will buy as many yards as $\$\frac{1}{3}$ is contained times in $16, or 48 times. Or, $1 will buy 3 yards, and $16 will buy 16 times 3 yards, or 48 yards.

2. How many pounds of castile soap can be bought for $34, at $16\frac{2}{3}$ cents a pound? At 20 cents? At 25 cents?

RULE.—*Divide the cost by such fraction as will express the price as an aliquot part of one dollar.*

3. How many baskets of peaches can be bought for $242, at $33\frac{1}{3}$ cents a basket? At 50 cents?

4. How many pounds of cheese can be purchased for \$192, at $6\frac{1}{4}$ cents a pound? At $8\frac{1}{3}$ cents? At $12\frac{1}{2}$ cents?

CASE III.

245. To find the cost, when the quantity and the price of 100, or 1000, are given.

1. What cost 325 feet of boards, at \$4.36 per 100 feet?

1ST OPERATION.

$$\begin{array}{r} 4.36 \\ 325 \\ \hline 100)1417.00 \\ \hline \$14.17 \end{array}$$

ANALYSIS.—At \$4.36 a *foot*, the cost would be \$4.36 × 325 = \$1417. But since \$4.36 is the cost of a *hundred* feet, \$1417 is 100 times too much; hence divide \$1417 by 100 (**113**); the result is \$14.17.

When the price is by the *thousand*, divide the product by 1000.

2D OPERATION.

$$\begin{array}{r} 325 = 3.25 \\ \$4.36 \\ 3.25 \\ \hline \$14.17 \end{array}$$

ANALYSIS.—325 feet is equal to 3.25 times 100 feet. Since 100 cost \$4.36, 325 feet will cost 3.25 times \$4.36, or \$14.17.

In like manner, when the price is by the *thousand*, reduce the quantity to thousands and decimals before multiplying.

2. What will 3240 feet of timber cost at \$6.20 per 100?
3. What cost 12345 bricks, at \$8.50 per thousand?
4. What cost 2465 pounds of lead, at \$.75 per 100?

RULE.—I. *Reduce the given quantity to hundreds and decimals of a hundred, or to thousands and decimals of a thousand.*

II. *Multiply the price by the quantity and point off in the product as in multiplication of decimals.*

In business transactions the letter C is sometimes used for *hundreds*, and M for *thousands*.

5. What cost 9320 envelopes at $3.75 per thousand?

6. What cost 5800 shingles, at $5.12½ per M.?

7. What cost 6700 laths, at $.62½ per C.?

8. What will be the freight on 784 pounds of castings, at $1.12½ per 100 pounds?

9. What is the cost of 3200 feet of boards at $90 per M.; 952 feet of scantling at $11½ per M.; and 1624 pickets at $1.87½ per C.?

CASE IV.

246. To find the cost, when the quantity and the price of a ton of 2000 pounds are given.

1. What cost 2460 pounds of hay, at $16.50 per ton?

OPERATION.

$$\begin{array}{r} 2\underline{)\$16.50} \\ 8.25 \\ \underline{2.460} \\ \$20.29500 \end{array}$$

ANALYSIS.—Since $16.50 is the cost of 2000 pounds, one-half of $16.50, or $8.25, is the cost of 1000 pounds. 2460 pounds will cost 2.460 times $8.25, or $20.295.

2. At $12¾ a ton, what will 450 pounds of hay cost?

3. What cost 1400 pounds of coal, at $6.80 a ton?

RULE.—*Multiply one-half the price of a ton by the number of thousands and decimals of a thousand in the given quantity, as in Case III.*

4. What is the freight on 3244 pounds of iron, at $1.80 per ton?

5. At $18.50 a ton, what will 1580 pounds of hay cost?

6. At $6.84 a ton, what cost 3142 pounds of plaster?

7. What cost 48 sacks of guano, each sack containing 156½ pounds, at $20¼ a ton?

ACCOUNTS AND BILLS.

247. ***An Account,*** in commercial transactions, is a record of *debts* and *credits*.

248. ***A Debtor*** is one who owes another money, goods or services.

249. ***A Creditor*** is one to whom money, goods or services are due from another.

250. ***A Bill*** is a written statement of money paid, of goods sold or of services rendered, with the price or value annexed to each item, and the entire cost.

251. The following abbreviations are in general use in business transactions :

@	At.	Dr.	Debtor.
℀ or Acc't	Account.	Gal.	Gallon.
Am't	Amount.	Inst.	This month.
Bal.	Balance.	Int.	Interest.
Bbl.	Barrel.	Mdse.	Merchandise.
Bo't	Bought.	Net	Without discount.
¢ or ct.	Cents.	No.	Number.
%	Per cent.	Pay't	Payment.
Co.	Company.	Per	By.
Cr.	Creditor.	Rec'd	Received.
Do.	The same.	Ult.	The last month.
Doz.	Dozen.	Yd.	Yards.

The character @ is always followed by the *price of a unit ;* thus, 3 yd. of cloth @ $2, signifies, 3 yards of cloth at $2 a yard ; ¼ lb. of tea @ $.75, signifies ¼ pound of tea at $.75 a pound.

252. Copy the following bills and find the amount due:

(1.)

New York, Sept. 14, 1874.

Mr. James E. Rice,

Bought of Robert Walker & Co.

15 yd.	Broadcloth	@	$4.20	$63	
24 "	Satinet	"	1.37½	33	
10 "	Vesting	"	.90	9	
42 "	Flannel	"	75	31	50
60 "	Drilling	"	.16	9	60
12 "	Silk	"	2.62½	31	50
38 "	Ticking	"	.52	19	76
				$197	36

(2.)

Baltimore, Jan. 18, 1875.

Mr. Amos Kent,

Bought of Smith & Perkins.

	10 chests Green Tea .	@	$36.25		
	5 " Black " .	"	28.12½		
	12 sacks Java Coffee .	"	19.80		
	5 bbl. Coffee Sugar (A)	"	26.87½		
	9 boxes Raisins . .	"	5.42		
	116 lb. Cheese . . .	"	.12½		
	132 lb. Rice	"	.08⅓		
	25 lb. Pearl Starch .	"	.18		
	6 boxes Oranges . .	"	3.25		
				$647	13

Received Payment,

Smith & Perkins.

(3.)

CHICAGO, Dec. 7, 1875.

MESSRS. JONES & MILLS,

Bo't of MASON & SON.

21000 ft. Pine Boards	@	$16	per M.		
9420 " Plank . .	"	12.50	" "		
7075 " Scantling .	"	1.70	" C.		
762 " Timber . .	"	2.62½	" "		
6840 " Lath . . .	"	.87½	" "		
4790 " Pickets . .	"	15.60	" M.		
				$728	60

Received Payment,

MASON & SON,
Per JOHN WISE.

(4.)

NEW ORLEANS, Jan. 2, 1875.

MR. GEORGE SOULE,

To STEVENS & SEYMORE, *Dr.*

1874						
Sept.	3	To 5 Blank Books . . .	@	$2.30		
"	"	" 7 gross Spencerian Pens	"	1.12½		
"	26	" 15 B. & S. Bookkeeping	"	1.75		
Oct.	10	" 4 reams Cap Paper .	"	3.40		
"	19	" 20 Townsend's Com. Law	"	2.87½		
Nov.	22	" 12 packs Plain Cards .	"	.37½		
Dec.	4	" Note Paper and Ink			2	78
					$	

Received Payment by note at 30 days,

STEVENS & SEYMORE.

(5.)

Springfield, April 10, 1877.

Mr. John H. Wilson,

To William E. Turner, *Dr.*

1876				
Nov.	15	For Repairing House as per contract	$60	
"	"	" 600 ft. Pine Boards @ $4.50 per C.		
"	26	" 6 days work of self " 3.25		
"	"	" 4 days work of son " 1.75		
"	"	" Nails, Hinges, etc.	3	75
			$117	25

Received Payment,

Wm. E. Turner.

(6.)

Statement of Account.

San Francisco, Nov. 10, 1875.

Mr. William P. Banks,

In acc t with Carter Brothers, *Dr.*

1875			
March 6	To 5 doz. Scythes . . @ $9.		
" "	" 16 Buckeye Ploughs " 10.37½		
Apr. 25	" 4½ doz. Hoes . . . " 5.48		
June 16	" 564 lb. Chain . . . " .16⅔		
" "	" 6¼ doz. Steel Shovels " 11.72		
Sept. 7	" 120 sets Blind Hinges " .33⅓		
" "	" 14 gross Screws . . " 1.62½		
	Cr.		
July 12	By 10 bbl. Flour . . @ $7.50		
Aug. 20	" 1470 ft. Pine Lumber " $2 per C.		
Sept. 1	" Draft on New York	200	
Oct. 26	" 146 bu. Oats . . @ $.40		
	Bal. due Carter Brothers,	$	

REVIEW QUESTIONS.

253. 1. What is Currency? Decimal Currency? Currency of the United States? What is it sometimes called? Table? What is the unit? What are all lower denominations? How distinguished from dollars?

2. To what order of decimals does the dime correspond? The cent? The mill? How are dimes regarded in business? When the cents are less than 10, how are they written? How are cents sometimes expressed? The half-cent? Principles of Decimal Currency, 1, 2.

3. How are decimals reduced to a common denominator? How is an integer reduced to a decimal? Illustrate. How are dollars changed to cents? To mills? Cents to mills? Dollars and cents to mills? How are cents reduced to dollars? Mills to dollars?

4. How is a decimal reduced to a common fraction? A common to a decimal fraction? What does the sign + placed after a quotient show? How are decimals added, subtracted, etc.? Rule for addition of decimals? For subtraction. Principle for multiplication of decimals. Rule. When the number of decimal places in both factors is greater than the number of decimals in the product, what is done? How multiply a decimal by 10, 100, etc.?

5. Principle of division of decimals. Rule. If the excess of decimal figures in the dividend over those in the divisor is greater than the number of figures in the quotient, what is done? If there be a remainder after dividing the dividend? How divide a decimal by 10, 100, etc.?

6. Define an aliquot part of a number. Name some of the aliquot parts of $1? How obtain the cost, when the price is an aliquot part of $1? How, the quantity, when the cost is given and the price is an aliquot of $1? How obtain the cost, when the price is by the 100 or 1000? When the price is by the ton of 2000 pounds?

7. Define an Account. A Debtor. A Creditor. A Bill. Define the abbreviations most commonly used in business.

DENOMINATE NUMBERS.

254. ***A Denominate Number*** is a number consisting of concrete units of one denomination; as 4 weeks; 6 feet; 9 pounds.

255. ***A Simple Number*** is either an abstract number, or a denominate number of but *one* denomination; as 12; 6 quarts; 3 pounds.

256. ***A Compound Denominate Number*** is a number consisting of *two* or *more* denominations of the same nature or kind; as 8 pounds 6 ounces; 5 gallons 2 quarts 1 pint.

257. In simple numbers and in decimals the law of increase and decrease is by the *uniform scale* of 10; but in compound numbers the scale varies according to the *standard unit.*

MEASURES.

258. ***A Measure*** is a *standard unit* established by law or custom, and by which *extent, dimension, capacity amount,* or *value* is estimated.

259. Thus, the *standard unit* of measures of Extension is the *yard;* of Liquid Measure, the *wine gallon* ; of Dry Measure, the *Winchester bushel;* of Weight, the *Troy pound,* etc. Hence the length of a piece of cloth is ascertained by applying a *yard* measure; the capacity of a cask, by the use of the *gallon* measure; of a bin, by the use of the *bushel* measure; the weight of a body, by the *pound* weight, etc.

260. Measures may be classified into six kinds:

Money, or Value,
Extension,
Capacity,
Weight,
Time,
Angles, or Arcs.

MEASURES OF VALUE.

261. ***Money*** is the measure of the *value* of things or of services, and the medium of exchange in trade.

262. ***Coin,*** or ***Specie,*** is metal stamped or pressed with a die to give it a legal, fixed value, for the purpose of circulating as money.

263. ***Paper Money*** consists of notes and bills duly authorized by the government to circulate as money.

264. ***Currency*** is a term applied to all kinds of money in circulation, both *coin* and *paper*.

Treasury notes of a less value than $1, are called *Fractional Currency.*

UNITED STATES MONEY.

265. ***United States Money*** is the legal currency of the United States, and was authorized by Congress in 1786.

The *unit* of United States Money is the *Gold Dollar.*

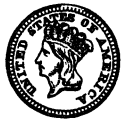


TABLE.

10 Mills (*m.*)	=	1 Cent . .	*ct.*
10 Cents	=	1 Dime . .	*d.*
10 Dimes	=	1 Dollar .	$.
10 Dollars	=	1 Eagle .	*E.*

$$1\ E. = \begin{cases} 10000\ m. \\ 1000\ ct. \\ 100\ d. \\ 10\ \$. \end{cases}$$

266. *The Coin* of the United States consists of *gold, silver, nickel* and *bronze,* and as fixed by the "New Coinage Act" of 1873, is as follows:

267. *Gold.* The double-eagle, eagle, half-eagle, quarter-eagle, three-dollar and one-dollar pieces.

268. *Silver.* The *Trade* dollar, half-dollar, quarter-dollar and the ten-cent piece.

269. *Nickel.* The five-cent and three-cent pieces.

270. *Bronze.* The one-cent piece.

1. The *old* silver dollar, half-dime and three-cent pieces, the bronze two-cent, and the nickel one-cent pieces are no longer coined.

2. The *Trade-dollar* is designed solely for purposes of commerce and not for currency. The weight is 420 grains.

CANADA MONEY.

271. *Canada Money* is the legal currency of the Dominion of Canada. The denominations are *dollars, cents* and *mills,* and have the same *nominal* value as the corresponding denominations of U. S. Money.

The Currency of the Dominion of Canada was made *uniform* July 1st, 1871. Before the adoption of the decimal system in 1858, pounds, shillings and pence were used.

272. *The Coin* of the Dominion of Canada is *silver* and *bronze.*

273. The ***Silver Coins*** are the fifty-cent, twenty-five-cent, ten-cent and five-cent pieces.

274. *The Bronze Coin* is the one-cent piece.

The *gold coins* in use are the British sovereign and half-sovereign.

ENGLISH MONEY.

275. ***English,*** or ***Sterling Money,*** is the legal currency of Great Britain.

The *unit* of English Money is the *Sovereign,* or *Pound Sterling*

TABLE.

			U. S. VALUE.
4 Farthings (*far.*)	=	1 Penny . . . . *d.*	$0.02027+.
12 Pence	=	1 Shilling . . . *s.*	2433+.
20 Shillings	=	1 Sovereign . . *sov.* or 1 Pound . . . £.	$4.8665.

OTHER DENOMINATIONS.

			U. S. VALUE.
2 Shillings (*s.*)	=	1 Florin. . . . *fl.*	$0.48665.
5 Shillings	=	1 Crown. . . . *cr.*	1.2166+.

276. ***The Coin*** of Great Britain in general use consists of *gold, silver* and *copper,* as follows :

277. ***Gold.*** The sovereign and half-sovereign.

278. ***Silver.*** The crown, half-crown, florin, shilling, six-penny and three-penny pieces.

279. ***Copper.*** The penny, half-penny and farthing.

1. How many farthings in 2 pence ? In 6 pence ?
2. How many pence in 3 shillings ? In 4 shillings ?
3. How many pence in 8 farthings ? In 16 farthings ?

4. How many shillings in 24 pence? In 48 pence?

5. How many pounds in 40 shillings? In 80 shillings?

6. How many shillings in 2 pounds? In 3 pounds?

7. At 5 shillings a yard, how many pounds will 8 yards of cloth cost? 10 yards?

8. At 8 shillings a pair, how many pairs of shoes can be bought for 2 sovereigns? For 4 pounds?

9. At 3 shillings a bushel, how many crowns will pay for 10 bushels of potatoes? How many florins?

10. At 6 pence a yard, how many yards of ribbon can be bought for a crown?

11. In 2 pounds how many shillings? Crowns?

12. In 2 pounds 6 shillings, how many shillings?

13. In 9 shillings 10 pence, how many pence?

14. How many shillings in 112 pence?

FRENCH MONEY.

280. ***French Money*** is the legal currency of France, and is *decimal*.

The Franc of the REPUBLIC.

The legal *unit* of French Money is the *Silver Franc*.

The Franc of the EMPIRE.

TABLE.

				U. S. VALUE.
10 Millimes (*m.*)	=	1 Centime . . .	*ct.*	$0.00186.
10 Centimes	=	1 Decime . . .	*dc.*	.0186.
10 Decimes	=	1 Franc	*fr.*	.186.
20 Francs	=	1 Napoleon . .	*Nap.*	$3.86.

Decimes are not used in business calculations, but are expressed in *centimes.* Thus, 5 decimes are expressed by 50 centimes.

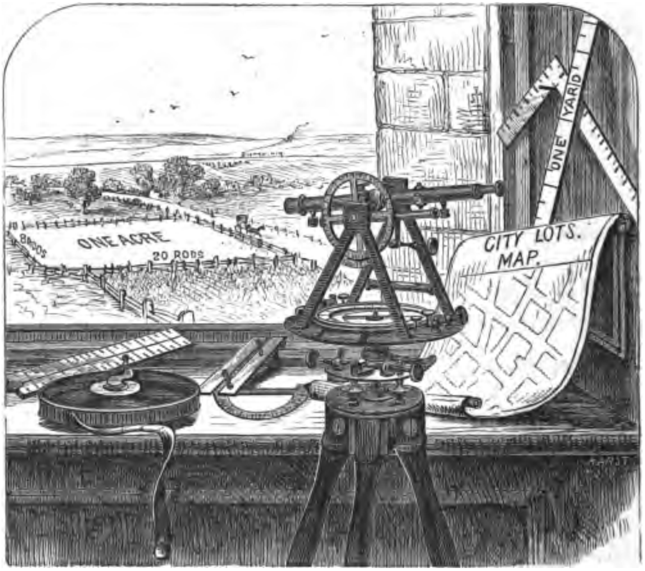
281. ***The Coin*** of France consists of *gold, silver* and *bronze,* as follows:

282. ***Gold.*** The 100, 40, 20, 10 and 5 franc pieces.

283. ***Silver.*** The 5, 2 and 1 franc, the 50 and the 25 centime pieces.

284. ***Bronze.*** The 10, 5, 2 and 1 centime pieces.

1. How many decimes in 1 franc? How many centimes? Millimes?
2. How many francs in 2 Napoleons? In 4? In 5?
3. How many Napoleons in 60 francs? In 75 francs?
4. How many francs in 100 centimes? In 400 ct.?
5. At 4 francs a pair, how many pairs of kid gloves can be bought for 2 Napoleons? For 3 Nap.?
6. At 8 francs a meter, how many meters of silk can be bought for 4 Napoleons? For 5 Nap.?
7. What part of a Napoleon are 5 francs? 10 francs?
8. What part of a franc are 25 ct.? 50 ct.? 75 ct.?
9. What is the value of 2 francs in U. S. money? Of 3 fr.?
10. At 50 ct. each, how many neck-ties can be bought for 4 fr.? For 6 fr.?



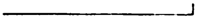
MEASURES OF EXTENSION.

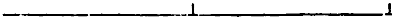
285. *Extension* is that which has one or more of the dimensions, *length, breadth* and *thickness.* It may be a *line,* a *surface,* or a *solid.*

286. *A Line* has only one dimension—*length.*

LINEAR MEASURE.

287. *Linear,* or ***Long Measure,*** is used in measuring lines and distances.


1 Inch.


2 Inches.

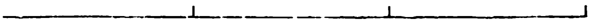

3 Inches.

TABLE.

12 Inches (*in.*)	= 1 Foot . . . *ft.*	1 *Mi.* =	63360 *in.*
3 Feet	= 1 Yard . . *yd.*		5280 *ft.*
5½ Yards, or 16½ Feet	= 1 Rod . . . *rd.*		1760 *yd.*
320 Rods	= 1 mile . . . *mi.*		320 *rd.*

OTHER DENOMINATIONS.

3 Barley-corns, or sizes	= 1 Inch.	Used by shoemakers.
4 Inches	= 1 Hand.	" to measure height of horses.
6 Feet	= 1 Fathom.	" " depth at sea.
1.15 Statue Miles	= 1 Geog. Mi.	" " distances "
3 Geographic Miles	= 1 League.	
60 Geographic, or 69.16 Statute Miles	= 1 Degree	of Latitude on a Meridian, or of Longitude on the Equator.
360 Degrees	= the Circumference of the Earth.	

A *barley-corn* is 1 third of an inch. A *knot* is 1 geographic, or nautical mile.

288. ***Cloth Measure*** is practically out of use. In measuring goods sold by the yard, the yard is divided into *halves, fourths, eighths* and *sixteenths.*

289. ***Gunters Chain*** is the *linear unit* used by land surveyors in measuring distances and boundaries of land, and is 4 rods, or 66 feet long, and consists of 100 links.

TABLE.

7.92 Inches	= 1 Link . . . *l.*	1 *Mi.* =	63360 *in.*
25 Links	= 1 Rod . . . *rd.*		8000 *l.*
4 Rods	= 1 Chain . . *ch.*		320 *rd.*
80 Chains	= 1 Mile . . . *mi.*		80 *ch.*

Engineers commonly use a chain or *measuring tape* 100 feet long, each foot divided into *tenths.*

Measurements are recorded in *chains* and *hundredths.*

1. How many inches in 3 feet? In 5 feet?

2. How many feet in 36 inches? In 72 inches?

3. How many feet in 4 yd.? In 7 yd.? 10 yd.?

4. How many feet in 4 rd.? In 2 rd.? In 3 rd.?

5. How many yards in 2 rd.? In 4 rd.? In 5 rd.?

6. In 4 ft. 6 in., how many inches? In 1 yd. 2 ft.?

7. In 6 yd. 2 ft., how many feet? In 9 yd. 4 ft.?

8. How many leagues in 12 mi.? In 20 mi.?

9. How many yards in 72 in.? In 108 in.?

10. If a man walk 4 miles an hour, how long will it take him to walk 10 leagues?

11. If a vessel sail 3 leagues an hour, how long will she be in sailing 90 miles?

12. How many feet high is a horse that measures 15 hands?

13. How many feet in 5 fathoms? In 8 fathoms?

14. How many rods in $\frac{1}{4}$ of a mile? In $\frac{1}{5}$? In $\frac{1}{8}$?

15. What part of a foot is 6 in.? 4 in.? 3 in.?

16. What part of a yard is 1 ft.? Is 9 in.? Is 1 ft. 6 in.?

17. What part of a mile are 80 rd.? 160 rd.? 40 chains? 60 ch.?

18. In 96 in., how many yards and feet?

19. In 108 in., how many yards?

20. At 4 cents a foot, what will 5 yards of wire cost?

21. At 16 cents a yard, what will $2\frac{1}{4}$ yards of ribbon cost? $3\frac{1}{8}$ yd.?

22. What will be the cost of a cable 10 fathoms long, at $1 a foot?

In connection with the Oral Questions of each table, the teacher may require the pupil to solve the *Written Examples* arranged in corresponding order, on pages 220. and 223.

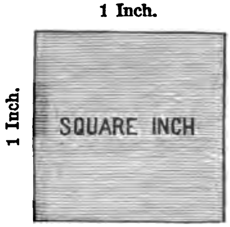
SURFACE, OR SQUARE MEASURE.

290. ***Surface,*** or ***Square Measure,*** is used in computing areas, or surfaces; as of land, boards, painting, plastering, etc.

291. ***A Surface*** has two dimensions—*length* and *breadth.*

292. ***A Square*** is a plane figure having four equal sides and four right angles (**329**).

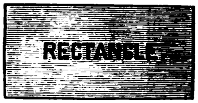
A *Square Inch* is a square, each side of which is 1 *inch* in length.



A *Square Foot* is a square, each side of which is 1 *foot* in length.

293. ***A Rectangle*** is a plane figure having four right angles, and its *opposite* sides equal.

When *all* its sides are equal it is called a *square.*

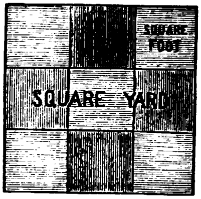


294. ***The Area*** of a plane figure is the surface included within the lines which bound it, and is expressed by the number of times it contains a given *unit of measure.*

295. ***The Unit of Measure*** for surfaces is a *square,* each side of which is a unit of some known length.

Thus, the *unit* of square inches is 1 *inch square;* of square feet, 1 *foot square,* etc.

296. ***A Square Yard*** is a square, each side of which is 1 yard, or 3 feet in length.



3 sq. ft. × 3 = 9 sq. ft.

If each square in the diagram represents 1 square foot, or a surface 1 foot square, the *area* is, 3 sq. ft × 3 = 9 sq. ft.

A rectangular surface may have the two dimensions alike, or both different. A blackboard 15 ft. long and 4 ft. wide contains 15 × 4 = 60 square feet of surface. Hence,

297. *The area of a square or rectangle is found by multiplying the length by the breadth.*

298. The *area* and *one side* being given, the *other* side is found by *dividing* the *area* by the given side.

TABLE.

144 Square Inches (*sq. in.*) = 1 Square Foot . . *sq. ft.*
9 Square Feet = 1 Square Yard . . *sq. yd.*
30¼ Square Yards = 1 Sq. Rod or Perch *sq. rd.*; *P.*
160 Square Rods = 1 Acre . . . . . . *A.*
640 Acres = 1 Square Mile . . *sq. mi.*

sq. mi.		*A.*		*sq. rd.*		*sq. yd.*		*sq. ft.*		*sq. in.*
1	=	640	=	102400	=	3097600	=	27878400	=	4014489600
		1	=	160	=	4840	=	43560	=	6272640
				1	=	30¼	=	272¼	=	39204
						1	=	9	=	1296
								1	=	144

Artificers **estimate their work as follows:**

By the *square foot;* as in glazing, stone-cutting, etc.

By the *square yard,* or by the *square of* 100 *square feet;* as in plastering, flooring, roofing, paving, etc.

Brickwork is usually estimated by the 1000 bricks; sometimes in cubic feet.

299. ***The Acre*** is the *unit of land measure* used by surveyors in computing the area of land, and is subdivided as follows:

TABLE.

625 Square Links (*sq. l.*) = 1 Pole . . . . . *P.*
16 Poles = 1 Square Chain . *sq. ch.*
10 Square Chains = 1 Acre . . . . . *A.*
640 Acres = 1 Square Mile . *sq. mi.*
36 Square Miles (6 miles square) = 1 Township . . *Tp.*

Tp.		*sq. mi.*		*A.*		*sq. ch.*		*P.*		*sq. l.*
1	=	36	=	23040	=	230400	=	3686400	=	2304000000

A square mile of land is also called a *Section*, and is divided into *half-sections*, *quarter-sections*, etc.

Measurements of land are commonly recorded in *square miles*, *acres* and *hundredths* of an acre.

1. How many square yards in 36 sq. ft.? In 72 sq. ft.?
2. How many square rods in $\frac{1}{2}$ of an acre? In $\frac{1}{4}$?
3. What part of an acre is 40 sq. rd.? 20 sq. rd.?
4. What part of an acre is 5 sq. ch.? 6 sq. ch.?
5. How many acres are 20 sq. ch.? 40 sq. ch.?
6. How many acres in $\frac{1}{2}$ of a section? In $\frac{1}{4}$ of a section?
7. How many square feet in a surface 6 ft. square?
8. How many square inches in the surface of a piece of copper 12 in. long and 9 in. wide?
9. How many square rods in a lot 8 rd. long and 5 rd. wide? What part of an acre does it contain?
10. How many square yards in a floor 10 ft. long and 9 ft. wide? In a ceiling 12 ft. square? 11 ft. by 12 ft.?
11. What is the difference between 4 ft. square and 4 square feet?
12. How many half-sections in a township?

13. What will it cost to paint a ceiling 12 ft. long and 9 ft. wide, at $½ a square yard?

14. At 3 cents a square foot, what will be the cost of a board 15 ft. long and 2 ft. wide?

15. At 6 cents a square foot, what will it cost to dress a stone that is 6 ft. long and 5 ft. wide?

16. How many yards of carpeting a yard wide will cover a floor 6 yd. long and 12 ft. wide?

17. How many square ft. are there on one side of a roof that is 20 ft. long and 15 ft. wide? On both sides?

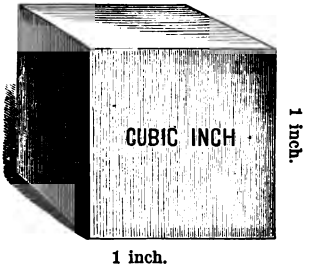
18. How many perches in a field 20 rd. long and 16 rd. wide? How many acres?

19. How many sections of land in a township 4 miles long and 3 miles wide? How many half-sections? Quarter-sections?

CUBIC, OR SOLID MEASURE.

300. *Cubic,* or ***Solid Measure,*** is used in computing the *volume* or contents of solids; as of timber, stone, boxes of goods, capacity of rooms, bins, ships, etc.

301. *A Solid* has *three* dimensions—*length, breadth* and *thickness.*



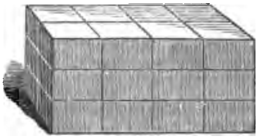
302. *A Cube* is a body bounded by six equal squares called *faces.* The sides of the squares are called the *edges* of the cube.

A *Cubic Inch* is a cube, each edge of which is 1 *inch* in length.

A *Cubic Foot* is a cube, each edge of which is 1 *foot* in length.

303. *A Rectangular Body* is one bounded by six rectangular faces, the opposite ones being equal and parallel.

When *all* the faces are equal, it is called a *cube*.

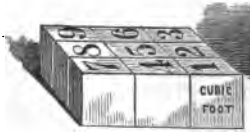


304. *The Volume,* or contents of a body, is the space included within the surfaces which bound it, and is expressed by the number of times it contains a given *unit of measure.*

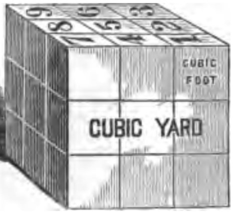
305. *The Unit of Measure* for a *Solid,* is a *cube,* the edge of which is a unit of some known length.

Thus, the *unit* of cubic inches, is a cube the edge of which is 1 *inch* long; of cubic feet, is a cube, the edge of which is 1 *foot* long, etc.

306. *A Cubic Yard* is a cube, each edge of which is 1 yard, or 3 feet in length.



3 cu. ft. × 3 = 9 cu. ft.



9 cu. ft. × 3 = 27 cu. ft.

If each small cube in the diagram represents one cubic foot, the solidity or contents of each layer is 3 cu. ft. × 3 = 9 cubic feet; and the solidity of the 3 layers, or large cube, is 3 times 9 cu. ft., or 3 cu. ft. × 3 × 3 = 27 cubic feet.

A solid or body may have three dimensions all alike, or all different. Thus, a cubic foot is 12 in. long, 12 in. wide, and 12 in. thick, and contains 12 × 12 × 12=1728 cubic inches.

A block of marble 6 ft. long, 4 ft. wide, and 3 ft. thick contains 6 × 4 × 3=72 cubic, or solid feet. Hence,

307. *The contents of a cubic or rectangular solid are found by multiplying the length by the breadth, and their product by the thickness.*

TABLE.

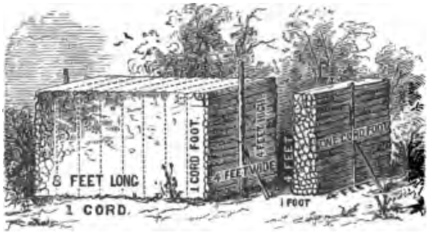
1728 Cubic Inches (*cu. in.*) = 1 Cubic Foot . . . *cu. ft.*
27 Cubic Feet = 1 Cubic Yard . . . *cu. yd.*

1 cu. yd. = 27 cu. ft. = 46656 cu. in.

308. *Wood Measure* is used to measure wood and rough stone.

TABLE.

16 Cubic Feet = 1 Cord Foot . . . . *cd. ft.*
8 Cord Feet, or 128 Cubic Feet = 1 Cord . . . . *Cd.*
24¾ Cubic Feet = 1 Perch of Stone, or of Masonry *Pch.*



A *Cord* of wood is a pile 8 ft. long, 4 ft. wide and 4 ft. high.

A *cord-foot* is 1 foot in length of such a pile; that is, 1 ft. long, 4 ft. wide and 4 ft. high.

A *Perch* of stone or of masonry is 16½ ft. long, 1½ ft. wide and 1 ft. high.

For fuller tables and notes, see "Arithmetical Examples," or "Complete Arithmetic."

1. How many cubic feet in 2 cubic yards?

2. How many cubic inches in 1 cu. ft. 9 cu. in.?

3. What part of a cubic yard is 9 cu. ft.? Is 18 cu. ft.?

4. How many cubic feet in 2 cord feet? In 3 cd. ft.?

5. In $\frac{1}{4}$ of a cord, how many cubic feet? How many cord feet?

6. How many cubic feet in $\frac{1}{2}$ of a cord? How many cord feet?

7. How many cord feet in 32 cu. ft.? In 64 cu. ft.?

8. In 2 perch of stone, how many cu. ft.? In 3 perch?

9. A block of granite 5 ft. long, 4 ft. wide and 3 ft. thick contains how many cubic feet?

10. How many blocks, each containing 1 cubic foot, are equal to a block 6 ft. long, 5 ft. wide and 2 ft. thick?

11. How many cubic inches in a 4 inch cube? In a 5 inch cube?

12. What is the difference between 3 cubic inches and a 3 inch cube?

13. How many cubic feet in a load of wood 8 ft. long, 4 ft. wide and 2 ft. high? How many cord feet? What part of a cord?

14. What are the solid contents of a brick 8 in. long, 4 in. wide and 2 in. thick?

15. How many cubic feet in a box whose inner edge is 4 ft.?

16. How many cubic feet in a bin 8 ft. long, $4\frac{1}{2}$ ft. wide and 3 ft. deep in the inside?

17. What is the cost of a block of marble 4 ft. long, 4 ft. wide and 3 ft. thick, at $2 a cubic foot.

18. What part of a cord is 6 cd. ft.? 32 cu. ft.?

19. What part of a cubic yard is 18 cu. ft.?

MEASURES OF CAPACITY.

309. ***Capacity*** signifies extent of room or space.

310. Measures of capacity are divided into two classes: Measures of Liquids and Measures of Dry Substances.

LIQUID MEASURE.

311. ***Liquid Measure*** is used in measuring liquids; as spirituous liquors, oil, molasses, milk, water, etc.

TABLE.

4 Gills (*gi.*)	= 1 Pint . . . *pt.*		1 *Gal.* =	32 *gi.*
2 Pints	= 1 Quart . . *qt.*			8 *pt.*
4 Quarts	= 1 Gallon . . *gal.*			4 *qt.*

In estimating the capacity of cisterns, reservoirs, etc.:

31½ Gallons	make	1 Barrel	*bbl.*
63 Gallons	"	1 Hogshead . .	*hhd.*

The barrel and hogshead are not fixed measures, but vary. In Massachusetts the barrel is estimated at 32 gallons.

The *Standard Liquid Gallon* of the United States contains 231 cubic inches.

The *Imperial Gallon* of Great Britain contains 277.274 cubic inches.

The *Old Ale or Beer Measure* is out of use. The gallon contained 282 cubic inches.

312. ***Apothecaries Fluid Measure*** is used by physicians and apothecaries in prescribing and compounding liquid medicines.

TABLE.

60 Minims, or drops (♏)	make	1 Fluidrachm . .	*f* ʒ.
8 Fluidrachms	"	1 Fluidounce . .	*f* ℥.
16 Fluidounces	"	1 Pint	*O.*
8 Pints	"	1 Gallon	*Cong.*

1 *Cong.* = 8 *O.* = 128 *f* ℥ = 1024 *f* ʒ = 61440 ♏.

1. *O* stands for the Latin *Octarius*, a Pint; *C*, for *Congius*, a Gallon.

2. For ordinary purposes, 4 teaspoons = 1 tablespoon; 2 tablespoons = 1 ounce; 2 ounces = 1 wineglass; 2 wineglasses = 1 teacup; and 4 teacups = 1 pint.

These values necessarily vary, since the vessels named are not of uniform capacity.

1. How many gills in 3 pints? In 6 pints?
2. How many pints in 4 quarts? How many gills?
3. In 3 gal., how many quarts? How many pints?
4. In 32 pt., how many quarts? How many gallons?
5. What part of a quart is 4 gills? Is 1 pint?
6. What part of a gallon is 2 pt.? Is 4 pt.? Is 6 pt.?

7. How many gills in ¼ of a quart? In ⅛?
8. How many pints in 1 gal. 2 qt.? In 2 gal. 1 qt.?
9. How many fluidrachms in 5 fluidounces?
10. In 2 pints how many fluidounces?
11. If a pint of molasses cost 8 cents, what will a gallon cost?
12. At 4 cents a pint, what will 6 qt. of milk cost?
13. If a gallon of wine cost $4, what will 1 pint cost?

DRY MEASURE.

313. ***Dry Measure*** is used in measuring articles not liquid; as grain, fruit, salt, roots, etc.

TABLE.

2 Pints (*pt.*)	=	1 Quart . . .	*qt.*	1 *Bu.* =	64 *pt.*
8 Quarts	=	1 Peck . . .	*pk.*		32 *qt.*
4 Pecks	=	1 Bushel . .	*bu.*		4 *pk.*

The *Standard Bushel* of the United States contains 2150.42 cubic inches, and is a cylindrical measure 18½ inches in diameter and 8 inches deep.

The half-peck, or *dry gallon*, contains 268.8 cubic inches. Six quarts dry measure are equal to nearly 7 quarts liquid measure.

1. How many pints in 3 qt.? In 1 pk. 2 qt.?
2. How many quarts in 3 pk.? In 1 bu. 1 pk.?
3. In 64 qt., how many pecks? How many bushels?
4. How many pecks in 5½ bu.? In 6¾ bu.?
5. What part of a bushel is 3 pk.? 24 qt.? 8 qt.?
6. At 5 cents a pint, what will 1 pk. of chestnuts cost?
7. How many quart boxes will 3 pk. 6 qt. of cherries fill?
8. Bought a peck of peanuts for $.80, and sold them for 6 cents a pint; how much was the gain?

MEASURES OF WEIGHT.

314. ***Weight*** is the measure of the quantity of matter a body contains, determined by the force with which it tends toward the earth.

TROY WEIGHT.

315. ***Troy Weight*** is used in weighing gold, silver, jewels, and in philosophical experiments.

TABLE.

24 Grains (*gr.*) = 1 Pennyweight . *pwt.*
20 Pennyweights = 1 Ounce . . . . *oz.*
12 Ounces = 1 Pound . . . . *lb.*

$$1\ lb. = \begin{cases} 5760\ gr. \\ 240\ pwt. \\ 12\ oz. \end{cases}$$

The quality or fineness of gold is expressed in *carats;* a carat meaning $\frac{1}{24}$ part. Gold that is 18 carats fine contains 18 parts of pure gold and 6 parts alloy, or baser metal.

APOTHECARIES WEIGHT.

316. ***Apothecaries Weight*** is used by apothecaries and physicians in compounding dry medicines.

TABLE.

20 Grains (*gr.* xx)	=	1 Scruple	*sc.*, or ℈.
3 Scruples (℈iij)	=	1 Dram	*dr.*, or ʒ.
8 Drams (ʒ viij)	=	1 Ounce	*oz.*, or ℥.
2 Ounces (℥ xij)	=	1 Pound	*lb.*, or ℔.

1 ℔ = 12 ℥ = 96 ʒ = 288 ℈ = 5760 *gr.*

1. *Medicines* are bought and sold in quantities by avoirdupois weight.

2. The pound, ounce and grain are the same as those of Troy weight, the ounce being differently divided.

1. How many grains in 2 pwt.? In 3 pwt.?
2. How many grains in 3 scruples? In 5 ℈?
3. In 60 pwt., how many ounces? In 80 pwt.?
4. How many scruples in 5 dr.? In 8 dr.?
5. How many drams in 30 ℈? In 45 ℈?
6. How many pounds in 72 oz.? In 96 oz.? In 144 oz.?
7. How many ounces in 3½ lb.? In 2¾ lb.? In ⅔ lb.?
8. What part of a pound is 4 oz.? 6 oz.? 8 oz.? 9 oz.?
9. How many powders of 6 grains each, can be made of 1 dram of medicine?
10. How many spoons, each weighing 2 oz., can be made of 2 lb. 6 oz. of silver?
11. Which is heavier, an ounce of calomel or an ounce of silver?
12. If 5 grains of medicine cost 10 cents, what will 2 drams cost?

AVOIRDUPOIS WEIGHT.

317. ***Avoirdupois Weight*** is used for all the ordinary purposes of weighing.

TABLE.

16 Ounces (*oz.*)	= 1 Pound	*lb.*	1 *T.* =	32000 *oz.*
100 Pounds	= 1 Hundredweight	*cwt.*		2000 *lb.*
20 cwt., or 2000 lb.	= 1 Ton	*T.*		20 *cwt.*

The Long Ton of 2240 lb. is seldom used, except in U. S. Custom Houses.

318. COMPARATIVE TABLE OF WEIGHTS.

	TROY.	APOTHECARIES.	AVOIRDUPOIS.
1 Pound =	5760 Grains =	5760 Grains =	7000 Grains.
1 Ounce =	480 " =	480 " =	437.5 "
	175 Pounds =	175 Pounds =	144 Pounds.

319. The following denominations are also used:

100 Pounds of Grain or Flour	make	1 Cental.
100 Pounds of Dry Fish	"	1 Quintal.
100 Pounds of Nails	"	1 Cask or Keg.
196 Pounds of Flour	"	1 Barrel.
200 Pounds of Pork or Beef	"	1 Barrel.

320. The weight of the *bushel* of certain grains, seeds and vegetables has been fixed by statute in many of the States, and though the laws of different States are *not uniform* in this respect, the following are the prevailing standards:

Wheat . .	60 lb.	Beans	60 lb.	Wheat Bran . .	20 lb.
Rye . . .	56 "	Buckwheat . .	42 "	Rye Meal . . .	56 "
Corn . . .	56 "	Flax Seed . .	56 "	Corn Meal . . .	50 "
Barley . .	48 "	Hemp Seed . .	44 "	Corn in Ear . .	68 "
Oats . . .	32 "	Potatoes . . .	60 "	Clover Seed . .	60 "
Peas . . .	60 "	Onions	57 "	Timothy Seed .	45 "

1. How many ounces in 3 lb.? In 1 lb. 10 oz.?

2. How many pounds in 4 hundredweight? In 6 cwt.?

3. How many hundredweight in 300 lb.? In 500 lb.?

4. In 5 cwt. 20 lb., how many pounds?

5. In 1 T. 5 cwt., how many hundredweight?

6. What part of a cwt. is 25 lb.? 50 lb.? 75 lb.?

7. How many cwt. in ¼ of a ton? In ½? In ¾? In ⅖?

8. How many tons are 50 cwt.? 60 cwt.? 75 cwt.?

9. What will 1 ton of hay cost, at 1 cent a pound?

10. At 5 cents an ounce, what will 1½ lb. of licorice cost?

11. At 8 cents a pound, what will 3 cwt. of sugar cost?

12. At $2 a bushel, how much must be paid for a bag of wheat weighing 120 pounds?

13. What will 20 pounds of clover seed cost, at $10 a bushel?

14. How many bushels of corn meal can be bought for $3, at 2 cents a pound?

15. If 8 oz. of tea cost $½, what will 4 pounds cost?

16. How many barrels are 1200 pounds of beef?

17. How many pounds in ½ bbl. of pork? In ½ bbl. of flour?

18. How much is ½ cental of wheat? 2 centals of flour?

19. At $3 a bushel, what will be the cost of 30 lb. of beans?

20. When pork is $24 a barrel, what is 25 lb. worth?

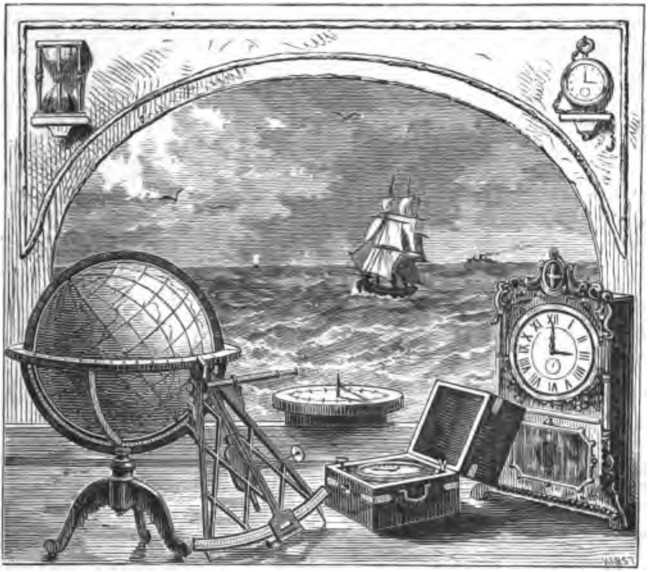
21. What will 3 lb. 8 oz. of beef cost, at 12 cents a lb.?

22. At $.50 a cwt., what will 1 T. 6 cwt. of hay cost?

23. What will ½ quintal of fish cost, at 6 cents a pound?

24. At $12 a barrel, what will a sack of flour containing 49 lb. cost?

25. If 25 lb. of cheese cost $2, what will 3 cwt. cost?



MEASURES OF TIME.

321. ***Time*** is the measure of duration.

322. The ***Unit*** is the *mean solar day*.

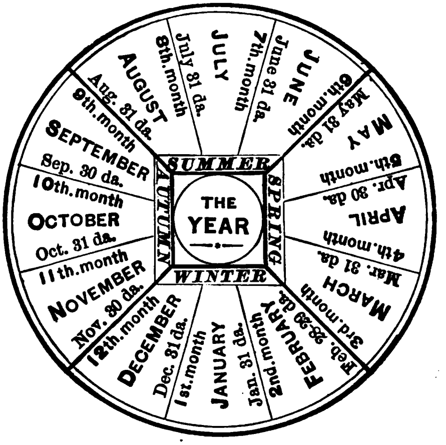
TABLE.

60	Seconds (*sec.*)	= 1 Minute . .	*min.*
60	Minutes	= 1 Hour	*hr.*
24	Hours	= 1 Day	*da.*
7	Days	= 1 Week . . .	*wk.*
365	Days, or	= 1 Common Year	*yr.*
12	Calendar Mo.		
366	Days	= 1 Leap Year .	*yr.*
100	Years	= 1 Century . .	*Cen.*

COMMON YEAR.

$$1\ \textit{Yr.} = \begin{cases} 525600\ \textit{min.} \\ 8760\ \textit{hr.} \\ 12\ \textit{mo.} \\ 365\ \textit{da.} \end{cases}$$

The calendar year is divided as shown in the diagram:



365 or 366 days.

The exact length of a solar year is 365 da. 5 h. 48 min. 46 sec.; but for convenience it is reckoned 11 min. 14 sec. more than this, or 365 da. 6 h. = $365\frac{1}{4}$ da. This $\frac{1}{4}$ day in 4 years makes one day, which, every fourth, bissextile, or leap year, is added to the shortest month, giving it 29 days.

Every year divisible by 4 is a leap year, the centennial years excepted; as 1876, 1880.

In most business transactions 30 days are considered a month, and 12 months a year.

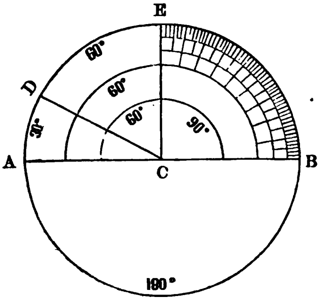
1. How many seconds in $\frac{1}{2}$ min.? In $\frac{1}{3}$ min.?
2. How many minutes in 120 sec.? In 150 sec.?
3. How many minutes in $\frac{1}{2}$ hour? In $\frac{1}{4}$? In $\frac{1}{3}$?
4. How many hours in 90 min.? In 120 min.?
5. What part of an hour is 20 min.? Is 40 min.?
6. How many weeks in 44 da.? In 63 da.? In 84 da.?
7. In 3 yr., how many months? In 5 yr.? In 6 yr.?
8. Name the months that have 30 da. each. 31 da. each?
9. How many days from Jan. 1st to March 10th, inclusive?
10. How many days from April 15th to July 10th?

CIRCULAR MEASURE.

323. ***Circular,*** or ***Angular Measure,*** is used in measuring angles and arcs of circles, in determining latitude and longitude, the location of places, the motion of the heavenly bodies, etc.

324. The ***Unit*** is the *degree,* which is $\frac{1}{360}$ part of the circumference of any circle.

FIG. 1.



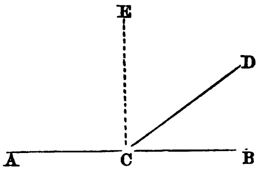
325. ***A Circle*** is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the *center.*

326. ***A Circumference*** is the line that bounds the circle.

327. ***An Arc*** is any part of the circumference; as A D, D E.

328. ***An Angle*** is the difference in the direction of two lines proceeding from a common point, called the *vertex.* Thus, in Fig. 2, A C D and D C B are *angles,* and C their *vertex.*

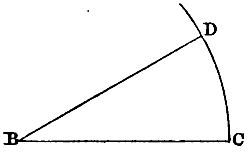
FIG. 2.



329. ***A Right Angle*** is formed by drawing one line perpendicular to another. Thus, A C E and E C B are right angles.

330. ***A Degree*** is one of the 360 equal parts into which the circumference of a circle is supposed to be divided. Thus, E and B (Fig. 1) are at the distance of 90°, or a *right angle* from each other, the vertex being at the center of the circle.

FIG. 3.



331. ***The Measure of an Angle*** is the *arc* of the circle included between its sides. Thus, the arc D B, in Fig. 3, is the measure of the angle D C B.

TABLE.

60 Seconds (″)	=	1 Minute	′
60 Minutes	=	1 Degree	°
30 Degrees	=	1 Sign	*S.*
12 Signs, or 360°	=	1 Circle	*C.*

$$1\ C. = \begin{cases} 1296000'' \\ 21600' \\ 360^\circ \\ 12\ S. \end{cases}$$

A Semi-Circumference is *one-half* of a circumference, or 180°.
A Quadrant is *one-fourth* of a circumference, or 90°.
A Sextant is *one-sixth* of a circumference, or 60°.
A Sign is *one-twelfth* of a circumference, or 30°.
A Degree (1°) is *one-nintieth* of a *right angle.*

The length of a degree varies with the size of the circle; thus, a degree of longitude at the Equator is 69.16 statute miles, at 30° of latitude it is 59.81 miles, at 60° of latitude it is 34.53 miles, and at 90°, or the poles, it is nothing.

A *minute* of the earth's circumference is called a geographic, or nautical mile, and is about $1\frac{1}{6}$ common miles.

1. How many degrees in a sign? In a sextant? In a quadrant?
2. How many degrees in $\frac{1}{2}$ of a circle? In $\frac{1}{3}$? In $\frac{1}{4}$?
3. How many geographic miles in 2'? In 3°?

4. What is the length of a degree at the Equator? At the poles?

5. How many degrees in 240 nautical miles?

6. How many degrees in ½ of a quadrant? In ⅓ of a sextant?

7. How many signs in 2 quadrants? How many degrees?

8. How many sextants in a circle? How many quadrants?

9. What part of a quadrant is 15°? 30°? 45°?

MISCELLANEOUS MEASURES.

COUNTING.

332. This measure is used in counting certain classes of articles for market purposes.

TABLE.

12 Units	=	1 Dozen . . .	*doz.*
12 Dozen	=	1 Gross	*gro.*
12 Gross	=	1 Great Gross .	*G. gro.*
20 Units	=	1 Score	*sc.*

1 *G. gro.* = 1728 *units.*
144 *doz.*
12 *gro.*

Two things of a kind are often called a *pair*, and six things a *set*; as a pair of horses, a set of chairs, etc.

PAPER.

333. The denominations of this table are used in the paper trade:

TABLE.

24 Sheets	=	1 Quire . .	*qr.*
20 Quires	=	1 Ream . .	*rm.*
2 Reams	=	1 Bundle .	*bun.*
5 Bundles	=	1 Bale . .	*B.*

1 *B.* = 4800 Sheets.
200 Quires.
10 Reams.
5 Bundles.

1. How many things in 3 doz.? In 6 doz.? In 1 gro.?

2. How many dozens in 3½ gro.? In 5 gro.?

3. How many dozens in ½ of a great gross?

4. In 1 ream of paper, how many quires? In 2½ reams?

5. In 1 bale, how many reams? In 3 bales? 5 bales?

6. How many score in 40? In 60? In 100?

7. How many sets in 24? In 36? In 72?

8. How many pairs in 12? In 20? In 28?

9. What will be the cost of a gross of pens at 10 cents a dozen?

10. At 20 cents a quire, what will ½ a ream of paper cost?

REVIEW QUESTIONS.

334. 1. Define a Denominate Number. A Simple Number. A Compound Denominate Number. The difference between simple and compound numbers. Define a Measure. Illustrate. Name the different classes of measures.

2. Define Money. Coin. Paper Money. Currency. Fractional Currency. United States Money. What is the unit? Repeat table. Name the gold coins. The silver coins. Other coins. Design of the Trade Dollar.

3. Define Canada Money? The denominations? When made uniform? What was used previous to the decimal system? Name the coins in use. Define English Money. What is the unit? Repeat the table. Name the principal gold coins. Silver coins. Other coins. Value of the Sovereign in U. S. Money.

4. Define French Money? The unit? Repeat the table. Name the gold coins in use. The silver coins. Other coins. Value of the Franc in U. S. Money?

5. Define Extension. A Line. Linear Measure. Repeat table. Other denominations. How is cloth measured? What is Gunters Chain? Table? How are measurements recorded?

6. For what is Square Measure used? Define a Surface. A Square. A square inch. A square foot. A Rectangle. When is a rectangle a square? What is the area of a plane figure? The unit of square measure? Define a square yard. Rule to find area. To find one side, when the area and other side are given. How do artificers estimate their work? Define the Acre. For what used? What is a Township? A Section?

7. For what is Cubic Measure used? Define a Solid. A Cube. A cubic inch. A cubic foot. A rectangular solid. Solidity, or Volume. The unit of cubic measure. Rule to find contents of a cube. Repeat table. Wood Measure? Repeat table. What is a cord of wood? A cord foot? A perch of stone?

8. Define Capacity. How divided? For what is Liquid Measure used? Table? What kind of measures are the barrel and hogshead? How many cubic inches in the standard liquid gallon? In the Imperial gallon? In the old beer gallon? Define Apothecaries Flüid Measure. For what used? For what is Dry Measure used? Repeat table. How many cubic inches in the standard bushel? In the dry gallon?

9. Define Weight. For what is Troy Weight used? Repeat table. How is the quality of gold expressed. Illustrate. For what is Apothecaries Weight used? Repeat table. How are medicines bought and sold in quantities? How do Troy and Apothecaries Weight differ? For what is Avoirdupois Weight used? Repeat table. Comparative table. Other denominations.

10. Define Time. The unit. Repeat table. How is the year divided? Name the Seasons. Months of each season. Days of each month. How are leap years distinguished from the common year?

11. For what is Circular Measure used? Define a Circle. A Circumference. An Arc. An Angle. A Right Angle. A Degree. The measure of an angle. Repeat the table. Define a Semi-circumference. A Quadrant. A Sextant. A Sign. How does a degree vary in length? Repeat the table for counting. Table for paper.

REDUCTION.

335. ***Reduction of Denominate Numbers*** is the process of changing numbers from one denomination to another without altering their value.

Reduction is of two kinds—*Descending* and *Ascending.*

336. ***Reduction Descending*** is the process of changing numbers of a higher denomination to equivalent ones of a lower denomination; as feet to inches.

337. ***Reduction Ascending*** is the process of changing numbers of a lower denomination to equivalent ones of a higher denomination; as inches to feet.

REDUCTION DESCENDING.

338. To reduce integers of higher denominations to lower.

1. How many inches in 4 ft.? In 5 ft. 7 in.?
2. How many feet in 2 yd. 1 ft.? How many inches?
3. Change 5 qt. 1 pt. to pints? To gills?
4. Reduce 1 gal. 2 qt. 1 pt. to pints?
5. How many cord feet in 3 cd.? In 5½ cd.?
6. Reduce 2 lb. 6 oz. to ounces. 3 lb. 10 oz. 4½ lb.
7. How many pounds in 6 cwt.? In 5 cwt.? 15 cwt.?
8. In 3 bu. 2 pk., how many pecks? How many quarts?
9. Reduce 9 oz. 12 pwt. to pennyweights.
10. In 4 gross, how many dozens? In 5 gross 6 doz.?

339. PRINCIPLE.—*Denominate numbers are changed to lower denominations by* MULTIPLICATION.

WRITTEN EXERCISES.

1. How many inches in 26 rd. 3 yd. 2 ft. 9 in.?

OPERATION.

26 rd. 3 yd. 2 ft. 9 in.
$5\frac{1}{2}$
146 yd.
3
440 ft.
12
5289 inches.

ANALYSIS. — Since 1 rd. is equal to $5\frac{1}{2}$ yd., there are $5\frac{1}{2}$ times as many yards as rods, plus the given number of yards; 26 yd. × $5\frac{1}{2}$ + 3 yd. = 146 yd.

Since 1 yd. is equal to 3 ft., there are 3 times as many feet as yards, plus the given number of feet; 146 ft. × 3 + 2 ft. = 440 ft.

Since 1 ft. is equal to 12 in., there are 12 times as many inches as feet; 440 in. × 12 + 9 in. = 5289 in., the number of *inches* in 26 rd. 3 yd. 2 ft. 9 in.

2. Reduce £7 12s. 8d. to pence.
3. Reduce 6 bu. 3 pk. 6 qt. to pints.
4. Reduce 4 lb. 9 oz. 12 pwt. to pennyweights.

RULE.—I. *Multiply the highest denomination of the given number by that number of the scale that will reduce it to the next lower denomination, and to the product add the given number of that lower denomination, if any.*

II. *Proceed in like manner with this and each successive denomination obtained, until the required denomination is reached.*

Reduce

5. £21 9s. 4d. to pence.
6. £12 7s. $9\frac{3}{4}$d. to farthings.
7. 26 cr. 2s. 6d. to pence.
8. 14s. 3 far. to farthings.
9. £18 to U. S. money.
10. 23 crowns to U. S. money.

Change

11. 10 florins to farthings.
12. 16 sov. to shillings.
13. 45 francs to centimes.
14. 14 Napoleons to francs.
15. 21 Nap. to U. S. money.
16. 18 fr. to U. S. money.

17. Reduce 5 mi. 120 rd. 4 yd. to yards.
18. Reduce 15 yd. 1 ft. 10 inches to inches.
19. In 4 fathoms 4 ft., how many inches?
20. In 3 miles, how many feet?
21. In 2 mi. 20 ch., how many links?
22. Reduce 14 yd. of cloth to fourths.
23. Reduce 10½ yd. of flannel to eighths.
24. In 9¼ yd. of silk, how many sixteenths?

25. Reduce 75 sq. rd. 12 sq. yd. 4 sq. ft. to square feet.
26. In 5 A., how many square yards?
27. Change 14 sq. yd. 5 sq. ft. 84 sq. in. to square inches.
28. In 2 A., how many sq. links?
29. How many square rods in 12 A. 4 sq. ch.?
30. In a township of land, how many quarter sections?

31. Reduce 16 cd. 5 cd. ft. to cubic feet.
32. Reduce 5 cu. yd. to cubic inches.
33. How many cubic feet in 28 perch of stone?
34. How many cubic inches in a perch of stone?

35. Reduce 2 hhd. 24 gal. 3 qt. to quarts.
36. How many pints in 44 gal.? In 2 bbl. 10 gal.?
37. Reduce 3 Cong. 5 O. 10 f ʒ to fluid ounces.

38. Reduce 16 bu. 6 qt. to pints.
39. In 3 bu. 2 pk., how many pints?

40. Reduce 5 lb. 10 oz. 12 pwt. to pennyweights.
41. Change 9 oz. 16 pwt. to grains.
42. Reduce 3 ℥ 5 ʒ to grains.

43. Reduce 4 T. 12 cwt. 60 lb. to pounds?
44. In 6 cwt. 28 lb. 10 oz., how many ounces?

How many pounds in

45. 14 centals of grain.
46. 38 bbl. of flour.
47. 19 bbl. of pork.
48. 16 bbl. of beef.
49. 12.6 quintals of fish.
50. 27.25 kegs of nails.
51. 26½ bu. of wheat.
52. 48 bu. of oats.
53. 29 bu. of corn meal.
54. 12½ bu. of onions.
55. 7¾ bu. of clover seed.
56. 65 bu. of rye.

57. Reduce 4 wk. 3 da. 5 hr. to minutes.

58. How many hours in a leap year?

59. If a clock tick seconds, how many times will it tick during February, 1876?

60. How many hours in the three Autumn months?

61. Reduce 14° 20′ 25″ to seconds.

REDUCTION ASCENDING.

340. To reduce integers of lower denominations to higher.

1. How many feet are 72 in.? How many yards?

2. How many pecks are 64 qt.? How many bushels?

3. How many pounds are 64 oz. of butter? 48 oz. of silver? 96 oz. of magnesia?

4. Reduce 144 in. to feet. To yards.

5. How many cords in 48 cord feet?

6. Reduce 50 pints to quarts. To gallons.

7. How many tons are 42 cwt.? 65 cwt.?

8. How many hundredweight are 600 lb.? 750 lb.?

9. Reduce 91 days to weeks. To months.

10. How many dozens are 144 eggs? How many gross?

341. PRINCIPLE.—*Denominate numbers are changed to higher denominations by* DIVISION.

WRITTEN EXERCISES.

1. Reduce 427 pints to bushels.

OPERATION.

2) 427 pt.
8) 213 qt. + 1 pt.
4) 26 pk. + 5 qt.
6 bu. + 2 pk.

427 pt. = 6 bu. 2 pk. 5 qt. 1 pt.

ANALYSIS.—Since 2 pt. make 1 qt., in 427 pt. there are as many quarts as 2 pt. are contained times in 427 pt., or 213 qt. and 1 pt. remaining.

Since 8 qt. make 1 pk., in 213 qt. there are as many pecks as 8 qt. are contained times in 213 qt., or 26 pk. and 5 qt. remaining.

And since 4 pk. make 1 bu., in 26 pk. there are as many bushels as 4 pk. are contained times in 26 pk., or 6 bu. and 2 pk. remaining. Hence 427 pt. = 6 bu. 2 pk. 5 qt. 1 pt.

2. How many rods are 440 feet ?

OPERATION.

3) 440 ft.
5½) 146 yd. + 2 ft.
2
11) 292 half-yd.
26 rd. + 3 yd.

440 ft. = 26 rd. 3 yd. 2 ft.

ANALYSIS.—In order to divide the second dividend 146 yd. by 5½, both dividend and divisor must be reduced to *halves;* thus 5½ = 11 *halves;* and 146 = 292 *halves;* 11 is contained in 292, 26 times and 6 remainder. Since the dividend is *halves* the remainder 6 is *halves,* and 6 *half-yards* = 3 yd. Hence 440 ft. are equal to 26 rd. 3 yd. 2 ft.

3. Reduce 6292 pence to pounds.
4. Reduce 5630 pennyweights to pounds.
5. Reduce 840 gills to gallons.

RULE.—I. *Divide the given denomination by that number of the scale which is equal to a unit of the next higher denomination, and write the remainder as a part of the answer.*

II. *In like manner divide this and each successive quotient until the denomination required is reached. The last quotient with the remainders annexed will be the result required.*

Reduction descending and ascending *prove each other*.

How many
6. Cents are 650 mills ?
7. Dollars are 1240 cents ?
8. Francs are 500 centimes?
9. Napoleons are 100 fr. ?
10. Pounds are 3600 pence ?
11. Shillings are 556 far. ?
12. Sovereigns are 21368 far.?
13. Crowns are 1260 pence ?
14. Florins are 599 pence ?

15. Yards are 1242 inches ?
16. Miles are 21120 feet ?
17. Fathoms are 1435 feet ?
18. Rods are 1968 inches ?
19. Yards are 325 eighths ?
20. Miles are 8470 links ?
21. Acres are 25600 sq. rd. ?
22. Sq.rd. are 346720 sq.in.?
23. Sections are 1920 A. ?
24. Acres are 650 sq. ch. ?
25. Cu. yd. are 786 cu. ft. ?
26. Cords are 43860 cu. ft.?
27. Cd. ft. are 165888 cu.in.?
28. Quarter-sections are 6400 sq. chains.

Reduce
29. 2347 pints to gallons.
30. 6048 gills to barrels.
31. 3696 cu. in. to gallons.
32. 6748 f ʒ to Cong.
33. 1075210 cu. in. to bu.
34. 1236 pints to pecks.

35. 23597 gr. to lb. Troy.
36. 1246 ℈ to ounces.
37. 7342 pwt. to pounds.
38. 8425 oz. to cwt.
39. 4536 lb. to bu. of wheat.
40. 6912 lb. to bu. of oats.
41. 900 lb. to bbl. of beef.
42. 14000 lb. to bu. of corn.

43. 86400 min. to weeks.
44. 28635 sec. to hours.
45. 10800 hr. to mo. of 30 da.
46. 10000″ to degrees.

47. 2736 pens to gross.
48. 1728 eggs to dozens.
49. 4060 balls to score.
50. 17647 sheets to reams.
51. 960 quires to bundles.

REVIEW.

WRITTEN EXAMPLES.

342. What will be the cost of

1. 1 T. 15 cwt. 36 lb. of sugar, at 9 cents a pound?
2. 2 lb. 8 oz. 12 pwt. of gold dust, at $.72 a pwt.?
3. 3 mi. of telegraph wire, at 12 cents a foot?
4. 1 hhd. of wine, at 40 cents a pint?
5. 2 bu. 1 pk. 6 qt. grass seed, at 14 cents a quart?
6. 2760 lb. of wheat, at $1.50 a bushel?
7. 12 weeks board, at $2.62½ a day?
8. 1 half acre of land, at 25 cents a square foot?
9. 128 pints of chestnuts, at $3.50 a bushel?
10. 240 cord feet of wood, at $4½ a cord?
11. 4000 lb. of hay, at $.75 a hundredweight?
12. 360 quarts of milk, at 32 cents a gallon?
13. 16 reams of paper, at 20 cents a quire?
14. 5 bbl. of pork, at 9 cents a pound?
15. 8 gross of lead pencils, at 5 cents a piece?

16. At 12 cents a pint, how much molasses can be bought for $8.52?

17. Paid $52.20 for a gold chain, at the rate of $.90 a pennyweight. How much did it weigh?

18. A township 4 miles long and 3½ miles wide, is equal to how many farms of 80 acres each?

19. Paid $17 for a barrel of beef: how much is that a pound?

20. How many minutes more are there in the summer, than in the winter months of a common year?

21. How many acres in a field 90 rd. long and 75 rd. wide?

22. What will be the cost of a board 14 ft. long and 2½ ft. wide, at 6 cents a square foot?

23. What is the value of a pile of wood 18 ft. long, 8 ft. high and 8 ft. wide, at $4.50 a cord?

24. How many spoons, each weighing 1 oz. 6 pwt., can be made from 1 lb. 3 oz. 12 pwt. of silver?

25. If 1 bushel of wheat make 49 lb. of flour, how many barrels will 300 bushels make?

26. How many cubic yards of earth must be removed to make a cellar 44 ft. long, 27 ft. wide and 5 ft. deep?

27. What will it cost to excavate a reservoir 120 ft. long, 54 ft. wide and 5 ft. deep, at $2.25 a cubic yard?

28. How many square yards of oil cloth will cover an office floor 15 ft. long and 12 ft. wide?

29. A grocer bought 2 bbl. of pork at $14 a barrel, and retailed it at 10 cents a pound: how much was his gain?

30. How many rods of fence will enclose a farm 1 mile square?

31. What will be the cost of slating a roof 40 ft. long, and each of the two sides 20 ft. wide, at $14 per square of 100 feet?

32. How many bricks 8 in. long and 4 in. wide, will pave a walk 16 ft. long and 4 ft. wide?

33. A man bought a quarter section of land for $4160: how much did he pay an acre?

34. Find the cost of carpeting a room 18 ft. long and 15 ft. wide, with carpeting 1 yd. wide, at $3.37½ a yard?

35. What will it cost to build a wall 48 ft. long 6 ft. high and 1½ ft. thick, at 12½ cents a cubic foot?

36. What will it cost to paint the ceiling of a room 18 ft. square, at $.62½ a square yard?

REDUCTION OF DENOMINATE FRACTIONS.

343. *A Denominate Fraction* is a fraction whose integral *unit* is a denominate number. Thus, $\frac{5}{8}$ of a pound, .5 of a mile, are denominate fractions.

1. *Denominate Fractions* are written either as *Common* Fractions, or as *Decimals*.

2. The principles, operations and analyses of *Denominate Fractions* are essentially the same as those of *Denominate Integers*.

CASE I.

344. To reduce a denominate fraction to integers of lower denominations.

1. How many inches in $\frac{3}{4}$ of a foot?

ANALYSIS.—Since 1 foot is 12 in., $\frac{3}{4}$ of 1 foot is $\frac{3}{4}$ of 12 in., or 9 in. Hence $\frac{3}{4}$ ft. = 9 in.

2. How many ounces in $\frac{1}{2}$ lb.? In $\frac{1}{4}$ lb.? In $\frac{5}{8}$ lb.?
3. How many quarts in $\frac{3}{8}$ pk.? In $\frac{3}{4}$ pk.? In $\frac{3}{8}$ bu.?
4. How many hours in $\frac{1}{6}$ da.? In $\frac{2}{3}$ da.? In $\frac{5}{8}$ da.?
5. Change to months $\frac{1}{3}$ yr. $\frac{5}{6}$ yr. $\frac{3}{4}$ yr. $\frac{7}{12}$ yr.
6. Change to pounds $\frac{1}{2}$ bu. oats. $\frac{3}{4}$ bu. corn. $\frac{5}{6}$ bu. beans.
7. How many pounds in $\frac{1}{4}$ bbl. flour? In $\frac{3}{5}$ bbl. beef. In $\frac{1}{8}$ bbl. pork?
8. How much is .5 of a pound Troy?

ANALYSIS.—Since 1 pound is 12 oz., .5 of 12 oz., or $12 \times .5 = 6$ oz. Hence .5 lb. = 6 oz.

9. How much is .6 of a £? .5 of a foot? .25 of a cwt.?
10. Change to minutes .3 hr. .25 hr. .7 hr. .8 hr.
11. How much is .25 of a bu.? .5 of a gal.?
12. Change to pounds .36 of a cwt. .2 of a ton.

WRITTEN EXERCISES.

1. Reduce $\frac{4}{5}$ bu. and .8 bu. each to integers of lower denominations.

1ST OPERATION.	1ST OPERATION.
$\frac{4}{5}$ bu. $\times 4 = \frac{16}{5} = 3\frac{1}{5}$ pk.	.8 bu. $\times 4 = 3.2$ pk.
$\frac{1}{5}$ pk. $\times 8 = \frac{8}{5} = 1\frac{3}{5}$ qt.	.2 pk. $\times 8 = 1.6$ qt.
$\frac{3}{5}$ qt. $\times 2 = \frac{6}{5} = 1\frac{1}{5}$ pt.	.6 qt. $\times 2 = 1.2$ pt.
Hence	Hence
$\frac{4}{5}$ bu. = 3 pk. 1 qt. $1\frac{1}{5}$ pt.	.8 bu. = 3 pk. 1 qt. 1.2 pt.

Analysis.—Since there are 4 pk. in 1 bu., there are 4 times as many pecks as bushels; and since there are 8 qt. in 1 pk., there are 8 times as many quarts as pecks; and last, since there are 2 pt. in 1 qt., there are 2 times as many quarts as pints. Hence $\frac{4}{5}$, or .8 bu. = 3 pk. 1 qt. $1\frac{1}{5}$, or 1.2 pt.

The following methods may be regarded as most convenient in practice, since the operations are performed without *rewriting the fractional part of each product.*

2D OPERATION.

```
          4
          4
      5)1 6(3 pk. 1 qt. 1⅕ pt.
  3     1 5
  2       1
5)6       8
  5     5)8
  1       5
```

2D OPERATION.

```
 .8 bu.
   4
 3.2 pk.
   8
 1.6 qt.
   2
 1.2 pt.
```

.8 bu. = 3 pk. 1 qt. 1.2 pt.

Reduce to integers of lower denominations:

2. $\frac{3}{5}$ of a day.
3. $\frac{3}{5}$ of a pound Troy.
4. $\frac{9}{10}$ of a yard.
5. .125 of a barrel.
6. .92 of a day.
7. .4156 of a cwt.

RULE.—I. *Multiply the given fraction or decimal by that number in the scale that will reduce it to the next lower denomination.*

II. *Proceed in like manner with the fractional part of each successive product until reduced to the denomination required.*

III. *The integral parts of the several products arranged in their proper order will be the required result.*

Change to integers of lower denominations:

8. $\frac{4}{5}$ of a month.
9. $\frac{5}{6}$ of an acre.
10. $\frac{1}{5}$ of a week.
11. $\frac{7}{8}$ of a rod.
12. $\frac{9}{10}$ of a yard.
13. $\frac{5}{8}$ of a hogshead.
14. $\frac{4}{7}$ of a great gross.
15. .625 of a bushel.
16. .3746 of a £.
17. .715 of a degree.
18. .008 of a mile.
19. .659 of a week.
20. .9 of a pound avoir.
21. .75 of a bale of paper.

CASE II.

345. To reduce a compound denominate number to a fraction of a higher denomination.

1. What part of a pound avoir. are 4 oz.? 8 oz.? 12 oz.?
2. What part of an hour are 15 min.? 20 min.? 30 min.?
3. What part of a £ are 5s.? 10s.? 12s.? 15s.?
4. What part of 3 gal. are 6 qt.?

ANALYSIS.—Since there are 12 qt. in 3 gal., 6 qt. are $\frac{6}{12}$ or $\frac{1}{2} =$ .5 of 3 gal. Hence 6 qt. $= \frac{1}{2}$ or .5 of 3 gal.

5. What part of 2 lb. Troy are 6 oz.? 8 oz.? 1 lb. 6 oz.?
6. What part of 4 yd. are 4 ft.? 2 yd. 2 ft.? 1 yd. 2 ft.?

WRITTEN EXERCISES.

1. Reduce 1 da. 9 hr. 36 min. to the decimal of a week.

1ST OPERATION.

$$60\,\underline{)\,36.0\text{ min.}}$$
$$24\,\underline{)\,9.6\text{ hr.}}$$
$$7\,\underline{)\,1.4\text{ da.}}$$
$$.2\text{ wk., or }\tfrac{1}{5}\text{ wk.}$$

ANALYSIS.—Since 60 min. are 1 hr., there are $\frac{1}{60}$ as many hours as minutes; $\frac{1}{60}$ of 36 min. is .6 hr., which added to 9 hr. make 9.6 hour.

Since 24 hr. are 1 da., there are $\frac{1}{24}$ as many days as hours; $\frac{1}{24}$ of 9.6 hr. is .4 da., which added to 1 da. make 1.4 days.

And since 7 da. are 1 wk., there are $\frac{1}{7}$ as many weeks as days; $\frac{1}{7}$ of 1.4 da. is .2 wk. This may be changed to the form of a *common* fraction, $\frac{1}{5}$ wk. (**232**).

2D OPERATION.

$$1\text{ da. }9\text{ hr. }36\text{ min.} = 2016\text{ min.}$$
$$1\text{ wk.} = 10080\text{ min.}$$
$$\tfrac{2016}{10080} = \tfrac{1}{5}\text{ wk., or .2 wk.}$$

ANALYSIS.—To find what part one denominate number is of another, both must be reduced to the same denomination.

1 da. 9 hr. 36 min. are equal to 2016 min., and 1 wk. is equal to 10080 min. Since 1 min. is $\frac{1}{10080}$ of a week, 2016 min. is $\frac{2016}{10080} = \frac{1}{5}$ wk. This may be changed to the form of a *decimal*, .2 wk. (**233**).

2. Reduce 1 oz. 6 pwt. 16 gr. to the fraction of a pound.

3. Reduce 3 cd. ft. 8 cu. ft. to the decimal of a cord.

RULE.—I. *Divide the lowest denomination given by that number in the scale which is equal to a unit of the next higher, and add the quotient as a decimal to that higher denomination.*

II. *Proceed in like manner until the whole is reduced to the denomination required.* Or, *Reduce the given number to its lowest denomination for the numerator, and a unit of the required denomination to the same denomination for the denominator of the required fraction, which reduce to its lowest terms, or to a decimal.*

1. In finding *what part* one compound number is of another, they must be *like numbers,* and *both* reduced to the lowest denomination in *either.*

2. If the given number contain a fraction, the *denominator* of this fraction must be regarded as the *lowest* denomination.

What part of

4. 1 wk. is 3 da. 2 hr. 40 min.?
5. 2 lb. Troy is 7 oz. 4 pwt.?
6. 4 bu. is 1¾ pk.?
7. 1 bbl. is 15 gal. 3 qt.?
8. 1 yd. is 1 ft. 9.6 in.?
9. 2 pk. is 2 pk. 4 qt.?
10. 1 da. is 7 hr. 12 min.?
11. 2 lb. avoir. is 10⅔ oz.?
12. £5 is 7s. 6d.?
13. 2 cd. is 4 cd. ft. 8 cu. ft.?
14. 3 ft. square is 3 sq. ft.?
15. 2 rd. 8 ft. is 4 yd. 1½ ft.?

What decimal of

16. £4 is 48s. 6d.?
17. 3 bu. is 3 pk. 1.12 qt.?
18. 1 fathom is 3¾ ft.?
19. 1 T. 5 cwt. is 25 lb.?
20. 6 gal. is 3 qt. 1 pt. 2 gi.?
21. 1 wk. 3 da. is 4 da. 9 hr.?
22. 5 yd. 1 ft. is 2 yd. 2 ft.?
23. 2 lb. Troy is 9 oz. 7 pwt. 12 gr.
24. 1 lb. is 10 ℥ 2 ℈?
25. 1° is 13′ 1.2″?
26. 1 ream is 150 sheets?

27. What part of 1 barrel of flour is 24½ lb.?
28. What part of 15 cwt. 21 lb. is 2 cwt. 11 lb.?
29. What part of 4 gal. 3 qt. is 3 qt. 2 gi.?
30. What decimal of a pound is 3¼ ℥?
31. What part of 4 oz. is 2 oz. 16 pwt. 19.2 gr.?
32. What decimal of 1 f℥ is 5 fʒ 36 ♏?
33. What decimal of 1¼ bu. is .45 of a peck?

ADDITION.

346. Denominate Numbers are *added, subtracted, multiplied* and *divided* by the same general methods as are employed for like operations in Simple Numbers.

The corresponding processes are based upon the *same principles;* the only modification of the rules is that required in reducing by a *varying* instead of a *uniform scale of* 10.

WRITTEN EXERCISES.

1. What is the sum of 32 bu. 2 pk. 6 qt.; 24 bu. 1 pk. 4 qt.; 16 bu. 3 pk. 7 qt.?

OPERATION.

bu.	pk.	qt.
32	2	6
24	1	4
16	3	7
74	0	1

ANALYSIS.—Write the numbers so that units of the same denomination stand in the same column, and begin at the right to add.

The sum of the quarts is 17 qt., equal to 2 pk. 1 qt. Write the 1 qt. under the column of quarts and add the 2 pk. to the column of pecks.

Add, in like manner, the columns of pecks and bushels.

(2.)

A.	P.	sq. yd.	sq. ft.	sq. in.
26	128	25	8	125
19	38	30	7	150
456	70	16	6	98
502	78	12(½)	5	85
		(½) =	4	72
503	78	13	1	13

(3.)

lb.	oz.	pwt.
16	11	18
26	9	0
11	0	6
	5	10
9	0	4
64	2	18

RULE.—I. *Write the numbers so that units of the same denomination will stand in the same column, and begin at the right to add.*

II. *Add each denomination separately as in simple numbers, and if the sum be less than a unit of the next higher denomination, write it under the column added.*

III. *When the sum of any denomination is equal to, or greater than a unit of the next higher denomination, reduce to that denomination, and write the excess, if any, under the column added, and add the quotient to the next column.*

4. What is the sum of 4 ℔ 5 ℥ 3 ʒ 1 ℈ 16 gr.? 10 ℥ 6 ʒ 4 gr.; 1 ℔ 2 ℥ 2 ℈ 10 gr.; and 4 ℥ 7 ʒ 1 ℈?

5. Add 5 cd. 6 cd. ft. 10 cu. ft.; 12 cd. 5 cd. ft.; 3 cd 12 cu. ft.; and 7 cd. ft. 9 cu. ft.

6. What is the sum of 12° 40′ 25″; 3 S. 15° 50′; 25° 36′ 42″; and 1 S. 18° 29′ 30″?

7. A man divided his farm into 3 fields. The first contained 22 A. 60 P.; the second 36 A. 44 P.; the third 42 A. 120 P. How many acres in his farm?

8. Add the sum of 1 T. 14 cwt. 26 lb. 9 oz.; 10 cwt. 20 lb. 14 oz.; 2 T. 44 lb.; 16 cwt. 34 lb., and 56 lb. 12 oz.?

9. If a printer one day uses 2 bundles 1 ream 9 quires of paper; the next day 2 bundles 1 ream 15 quires 18 sheets; and the next 4 bundles 6 quires, how much does he use in the three days?

10. A miller bought 4 loads of wheat; the first contained 46 bu. 36 lb.; the second 43 bu. 50 lb.; the third 49 bu. 23 lb.; and the fourth 50 bu. 56 lb. What was the amount of the 4 loads?

SUBTRACTION.

WRITTEN EXERCISES.

347. 1. From 16 lb. 8 oz. 6 pwt. 10 gr., take 7 lb. 4 oz. 12 pwt. 6 gr.

OPERATION.

lb.	oz.	pwt.	gr.
16	8	6	10
7	4	12	6
9	3	14	4

ANALYSIS.—Write the subtrahend under the minuend so that units of the same denomination shall stand in the same column, and begin at the right to subtract.

Subtract 6 gr. from 10 gr., and write the difference, 4 gr., under the grains.

Since 12 pwt. cannot be subtracted from 6 pwt., take 1 oz. equal to 20 pwt. from the 8 oz., leaving 7 oz., and add it to the 6 pwt., making 26 pwt. Subtract 12 pwt. from 26 pwt., and write the difference, 14 pwt., under the pennyweights.

Since 1 oz. was taken from 8 oz., subtract 4 oz. from 7 oz., and write the difference, 3 oz., under the ounces.

Subtract 7 lb. from 16 lb., and write the difference, 9 lb., under the pounds. Hence 9 lb. 3 oz. 14 pwt. 4 gr. is the required difference.

(2.)

hhd.	gal.	qt.	pt.
7	28	2	1
3	14	3	0
4	13	3	1

(3.)

wk.	da.	hr.	min.	sec.
10	4	16	40	34
3	5	10	45	20
6	6	5	55	14

RULE.—I. *Write the subtrahend under the minuend, so that units of the same denomination shall stand in the same column, and begin at the right to subtract.*

II. *Subtract each denomination separately, as in simple numbers, and write the difference underneath.*

III. *When the number of any denomination in the subtrahend exceeds that in the minuend, add to the number in the minuend as many units as make one of the next higher denomination, and then subtract.*

IV. *Diminish the next higher denomination of the minuend by* 1, *or, which will give the same result, add* 1 *to the next higher denomination of the subtrahend before subtracting. Proceed in the same manner with each denomination.*

Find the remainder in the following:

4. 74 yd. 2 ft. 6 in. — 15 yd. 2 ft. 9 in.

5. 125 A. 84 P. — 40 A. 110 P.

6. 7 T. 5 cwt. 18 lb. 6 oz. — 2 T. 9 cwt. 12 lb. 10 oz.

7. 5 ℔ 7 ℥ 3 ʒ 1 ℈ 8 gr. — 1 ℔ 9 ℥ 7 ʒ 2 ℈ 5 gr.

8. 120 cd. 4 cd. ft. 7 cu. ft. — 84 cd. 10 cu. ft.

9. 29 sq. yd. 6 sq. ft. 84 sq. in. — 16 sq. yd. 2 sq. ft. 96 sq. in.

10. 8 reams 16 quires 5 sheets — 20 quires 10 sheets.

11. A grocer bought 1 hhd. of molasses and sold 48 gal. 2 qt. 1 pt.: how much remained?

12. From a tub of butter containing 1 cwt. 28 lb., was taken 56 lb. 8 oz.: how much remained?

13. From a *section* of land, there was sold 320 A. 85 P.: how much remained?

14. A merchant tailor bought English cloths to the amount of £425 14s. 8½d., and paid £275 18s. 6¼d.: how much did he still owe?

15. From a section of land there was sold at one time 140 A. 96 P., at another time 200 A. 125 P., and at another time 75 A. 28 P.: how much remained?

348. To find the interval of time between two dates.

1. How many years, months and days from May 10, 1869, to July 4, 1875?

OPERATION.

yr.	mo.	da.
1875	7	4
1869	5	10
6	1	24

ANALYSIS.—Since the later date expresses the greater period of time, write it as the minuend, and the earlier date as the subtrahend, writing first on the left the year, next the number of the month, and next the number of the day; then subtract as in subtraction of compound numbers.

When *hours* are to be obtained, reckon from 12 at night, and if *minutes* and *seconds*, write them still at the right of hours.

In finding the time between two dates, 12 mo. are usually considered a year, and 30 days, a month.

2. How long from Apr. 22, 1870, to Oct. 9, 1876?

3. A note given June 10, 1871, was paid Sept. 4, 1874. How long did it run?

4. Washington was born Feb. 22, 1732, and died Apr. 17, 1790. What was his age?

5. How much time has elapsed since the declaration of independence of the United States?

6. How many days from the 20th of May to the 14th of December?

7. What length of time will elapse from 40 min. past 3 o'clock, P.M., Sept. 4th, 1872, to 20 min. before 10 o'clock, A.M., April 24th, 1874?

8. How many years, months and days from your birthday to this date, or what is your age?

MULTIPLICATION.

WRITTEN EXERCISES.

349. 1. Multiply 8 cd. 5 cd. ft. 9 cu. ft. by 4.

OPERATION.

Cd.	cd. ft.	cu. ft.
8	5	9
		4
34	6	4

ANALYSIS.—Write the multiplier under the lowest denomination of the multiplicand, and begin at the right to multiply.

4 times 9 cu. ft. are 36 cu. ft., equal to 2 cd. ft. 4 cu. ft. Write the 4 cu. ft. under the cubic feet in the product, and reserve the 2 cd. ft. to add to the product of cord feet.

4 times 5 cd. ft. are 20 cd. ft., and 2 cd. ft. added make 22 cd. ft., equal to 2 cd. 6 cd. ft. Write the 6 cd. ft. under the cord feet in the product and reserve the 2 cd. to add to the product of cords.

4 times 8 cd. are 32 cd. and 2 cd. added make 34 cd., which write under the cords.

(2.)

bu.	pk.	qt.	pt.
9	2	6	1
			7

(3.)

lb.	oz.	pwt.	gr.
23	9	16	12
			9

RULE.—I. *Write the multiplier under the lowest denomination of the multiplicand.*

II. *Multiply each denomination separately, as in simple numbers, and if the product is less than a unit of the next higher denomination, write it under the term multiplied.*

III. *When the product of any denomination is equal to, or greater than a unit of the next higher denomination, reduce it to that denomination and write the excess, if any, under the term multiplied, and add the quotient to the product of the next higher denomination.*

When the multiplier is a *composite* number and more than 12, we may multiply first by one of the factors, and that partial product by another, and so on (**88**).

What is the result

4. Of £9 10s. 8½d. × 6? by 7? by 9?
5. Of 6 hhd. 20 gal. 3 qt. × 5? by 8? by 15?
6. Of 2 rd. 4 yd. 2 ft. 6 in. × 6? by 18?
7. Of 9 cu. yd. 15 cu. ft. 520 cu. in. × 7? by 14?
8. Of 7 ℔ 8 ℥ 5 ʒ 1 ℈ 18 gr. × 9? by 250?
9. Of 5 hr. 42 min. 50 sec. × 12? by 36?
10. Of 26 cd. 3 cd. ft. 12 cu. ft. × 18? 24?
11. 14 A. 26 P. × 24? 36? 42?

12. How much paper will be required to print 10 editions of a book, each requiring 40 reams 7 quires 12 sheets?

13. What amount of cranberries in 16 boxes, each box containing 1 bu. 2 pk. 6 qt.?

14. How many bushels of wheat in 42 sacks, each containing 145 lb.?

15. What is the weight of 1 doz. spoons, each weighing 1 oz. 12 pwt. 16 gr.?

16. How much hay in 7 stacks, each containing 6 T. 250 lb.?

17. If 5 lb. 8 oz. of grapes be required to fill a box, what will be the weight of enough to fill a half-gross of boxes of the same capacity?

18. How much land in 3 farms, each farm being divided into 10 fields, and each field containing 14 A. 20 P.?

19. What will be the cost of 4 pieces of cloth, each piece containing 30 yards, at 16s. 9d. a yard?

20. If a pipe discharge 4 hdd. 26 gal. 3 qt. of water in 1 hour, how much will it discharge in 48 hours?

DIVISION.

WRITTEN EXERCISES.

350. 1. Divide 88 bu. 3 pk. 7 qt. by 6.

OPERATION.

	bu.	pk.	qt.	pt.
6)	88	3	7	0
	14	3	2	1

ANALYSIS.—Write the divisor at the left of the dividend, as in simple numbers. The object in this example is to find $\frac{1}{6}$ of a compound number.

1 sixth of 88 bu. is 14 bu. and a remainder of 4 bu. Write the 14 bu. in the quotient, and reduce the 4 bu. to pecks. 4 bu. equal 16 pk., plus 3 pk. make 19 pk.

1 sixth of 19 pk. is 3 pk. and a remainder of 1 pk. Write the 3 pk. in the quotient and reduce the 1 pk. to quarts. 1 pk. equals 8 qt., plus 7 qt. make 15 qt.

1 sixth of 15 qt. is 2 qt. and a remainder of 3 qt. Write the 2 qt. in the quotient, and reduce the 3 qt. to pints. 3 qt. equal 6 pt.

1 sixth of 6 pt. is 1 pt., which write in the quotient.

(2.)

	lb.	oz.	pwt.	gr.
5)	52	4	16	16
	10	5	15	8

(3.)

	wk.	da.	hr.	min.	sec.
7)	33	5	23	45	10
	4	5	20	32	10

4. Divide 88 bu. 3 pk. 7 qt. by 14 bu. 3 pk. 2 qt. 1 pt.

OPERATION.

88 bu. 3 pk. 7 qt. = 5694 pt.
11 bu. 3 pk. 2 qt. 1 pt. = 949 pt.
5694 pt. ÷ 949 pt. = 6

ANALYSIS.—The object in this example is to divide one compound number by another.

88 bu. 3 pk. 7 qt. are equal to 5694 pints; and 14 bu. 3 pk. 2 qt. 1 pt. are equal to 949 pt.; and 949 pt. is contained in 5694 pt., 6 times.

5. Divide 31 lb. 8 oz. by 2 lb. 10 oz. Troy.

6. Divide £98 16s. by £4 2s. 4d.

RULE.—When the divisor is a simple number.

I. *Write the divisor at the left of the dividend, and begin with the highest denomination and divide as in simple numbers.*

II. *When there is a remainder after dividing any denomination, reduce it to the next lower denomination, and to the result add the given number of that denomination, if any, and divide as before.*

When the divisor is a compound number.

Reduce both dividend and divisor to the lowest denomination contained in either, and divide as in simple numbers.

What is the quotient of

7. 376 gal. 3 qt. 1 pt. ÷ 9?
8. 328 yd. 1 ft. 3 in. ÷ 6?
9. 281 A. 85 P. ÷ 14?
10. 9 hhd. 28 gal. 2 qt. ÷ 12?
11. 153° 16′ 12″ ÷ 9?
12. £98 16s. ÷ 24?
13. 3 da. 11 hr. 10 min. ÷ 7 hr. 45 min. 50 sec.?

14. A teamster drew 19 cd. 2 cd. ft. 11 cu. ft. of wood at 15 loads: how much did he average per load?

15. Bought 6 spoons, which weighed 11 oz. 3 pwt.: what was the weight of each spoon?

16. If a man feed his horse 1 pk. 4 qt. of oats a day, how long will 5 bu. 2 pk. 4 qt. last him?

17. How many iron rails, each 18 ft. long, will be required to lay 3 miles of railroad track?

18. The total weight of 18 hhd. of sugar is 7 T. 15 cwt. 66 lb. 4 oz.: what is their average weight?

19. If a township of land 3 miles square be divided equally into farms of 120 A. each, what number of farms will it make?

MEASUREMENTS.

351. RECTANGULAR SURFACES.*

WRITTEN EXERCISES.

1. What is the area of a floor 27 ft. long and 18 ft. wide? **(297.)**

2. How many feet wide is a hall that is 36 ft. long, and contains 252 square feet? **(298.)**

3. What is the area of a rectangle whose length is 13 ch. and breadth 9 ch.?

4. A hall floor 8 ft. 4 in. wide contains 450 sq. ft. What is its length?

5. What will it cost to cover the floor with carpeting $\frac{3}{4}$ yd. wide, at $2.25 a yard?

6. How many acres in a piece of land 60 rd. square?

7. What is the difference between the area of a floor 25 ft. square and that of two others 12 ft. 6 in. square?

8. What will be the cost of a piece of land 80 rd. long and 75 rd. wide, at 68\frac{1}{2}$ an acre?

9. A rectangular field 120 rd. long contains 24 A. What is its width?

10. What will be the cost of covering the floor of a hall 45 ft. long and 6 ft. 6 in. wide, with oilcloth, at $1.35 a square yard?

11. What will be the cost of glazing 5 windows, each 8 ft. 2 in. by 4 ft., at $.75 a square foot?

12. How many small squares, each containing 6 sq. in., are equal to one large one, 3 feet square.

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* For Definitions, Illustrations, etc., see pages 197, 198, and 199.

13. What is the area of a square field, the distance around which is 140 chains?

14. Find the cost of painting a roof 52 ft. long and 30 ft. wide, at $.75 a square.

15. How many planks 16 ft. long and 9 in. wide, will be required to floor a room 36 ft. long and 24 ft. wide?

16. What will be the cost of flagging a side-walk 156 ft. long and 6½ ft. wide, at $2.40 a square yard?

17. What is the difference in area between a rectangle 60 ft. by 40 ft., and a square the sum of the sides of which is equal to that of the rectangle?

18. What will be the cost of covering a floor 35 ft. by 24 ft., with matting 4 ft. wide, worth $1.25 a yard?

19. What is the difference between the area of a field 60 rd. square and that of two others each 30 rd. square?

20. How many yards of Brussels carpet, 2 ft. 4 in. wide, will cover the floor of a room 52 ft. long by 35 ft. wide; and what will be its cost at $1.87½ a yard.

21. How many yards of cambric ⅝ yd. wide, will line 24 yd. of silk ¾ yd. wide?

22. What will be the cost of wainscoting a room 16 ft. 8 in. by 15 ft. 10 in., to the height of 6 ft., at $1.20 a square yard?

23. How many yards of silk ¾ yd. wide, will line 2 sets of curtains, each set containing 12 yd. of serge 1¼ yd. wide?

24. What will be the cost of plastering a room 27 ft. 6 in. long by 18 ft. wide, and 12 ft. high, at $.37½ a square yard, allowing 219 sq. ft. for doors, windows, etc.?

25. A piece of land 16 ch. long and 15 ch. wide, that cost $200 an acre, was divided into lots, each 10 rd. by 3 rd., and sold for $150 each. What was the gain?

352. RECTANGULAR SOLIDS.*

1. What is the volume of a rectangular solid 22 ft. long, 13 ft. wide, and 5 ft. thick? **(307.)**

2. What is the volume of a cube whose edge is 18 in.?

3. A tank 14½ ft. long and 12 ft. wide contains 1566 cu. ft. What is its depth?

OPERATION.—1566 ÷ $\overline{14\frac{1}{2} \times 12}$ = 9 ft., the depth. Hence,

To find a *required dimension* of a rectangular solid,

RULE.—*Divide the volume by the product of the numbers denoting the other two dimensions.*

4. How many cords in a pile of wood 38 ft. long, 8 ft. wide, and 6½ ft. high? **(308.)**

5. How many cubic yards of earth must be removed in digging a cellar 40 ft. long, 31 ft. wide, and 6 ft. 9 in. deep?

6. What will be the cost of a block of granite, 7 ft. 6 in. long, 4 ft. wide, and 3 ft. 2 in. thick, at $.25 a cu. ft.?

7. What is the length of a wall 2 ft. thick and 6½ ft. high, which cost $520, at the rate of $1 per cubic foot?

8. How many cubic blocks whose edges are 2 in., are equal in volume to one whose edge is 2 ft.?

9. What is the volume of a bin, the inside dimensions of which are 10 ft. 6 in. by 8 ft. by 4 ft. 3 in.?

10. What will be the cost of digging a cellar 36 ft. long, 25 ft. wide, and 7½ ft. deep, at $.45 a cubic yard?

11. How many cans, 6 in. by 4 in. by 3 in. can be packed in a box 24 in. by 18 in. square, in the clear?

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* **For Definitions, Illustrations, etc., see pages 200, 201, and 202.**

12. How many cubic feet of air in a room that is 21 ft. 9 in. long, 16 ft. 4 in. wide, and 11 ft. 6 in. high?

13. How much wood is there in a pile 8 ft. long, 6 ft. wide, and 5 ft. 4 in. high?

14. What will it cost to remove an embankment 120 ft. long, 64 ft. wide, and 6.5 ft. high, at $.54 a cubic yard?

15. At $3.80 a cord, what will be the value of the wood that can be piled under a shed 40 ft. long, 20 ft. wide, and 15 ft. high?

353. CAPACITY OF BINS, ETC.

RULES.—1. To find the *exact* capacity of a bin or any rectangular space, in bushels.

Divide the contents in cubic inches by 2150.42; *the quotient will represent the number of bushels.* **(313.)**

2. To find the *cubic* contents in a given number of bushels.

Multiply the number of bushels by 2150.42; *the product will be the number of cubic inches, which may be reduced to higher denominations if required.*

Since a standard bushel contains 2150.42 cu. in., and a cubic foot, 1728 cu. in., a bushel is to a cubic foot nearly as 4 to 5; or a bushel is equal to $1\frac{1}{4}$ cu. ft., nearly. Hence for all practical purposes,

3. *Any number of cubic feet diminished by* $\frac{1}{5}$, *will represent an equivalent number of bushels.*

Thus, 200 cu. ft. $-$ $\frac{1}{5}$ of 200 cu. ft., or 40 cu. ft. $=$ 160, the number of bushels in 250 cu. ft.

4. *Any number of bushels increased by* $\frac{1}{4}$, *will represent an equivalent number of cubic feet.*

Thus, 160 bu. $+$ $\frac{1}{4}$ of 160 bu., or 40 bu. $=$ 200, the number of cu. ft. in 160 bu.

1. How many bushels of grain will a bin 10 ft. square and 4 ft. high, contain?

2. How many bushels of oats can be put in a bin 8 ft. long, 6 ft. wide, and 5 ft. deep?

3. What is the volume in cubic feet, of a bin that will hold 320 bushels of grain?

4. A bin 8 ft. 6 in. long, 6 ft. wide, and 4 ft. 4 in. deep, is full of wheat. What is its value at $2.25 a bushel?

5. What must be the depth of a bin, whose length is 9 ft., and width 5 ft., to contain 180 bushels?

OPERATION.—180 bu. + 45 bu. = 225; $225 \div \overline{9 \times 5} = 5$ ft., the depth.

6. What must be the width of a bin that is 12 ft. long and 6 ft. deep, to contain 684 bu. of barley?

7. How long must a bin be made, that is 8 ft. wide and 3 ft. 6 in. deep, to contain 448 bu. of grain?

354. CAPACITY OF VESSELS, ETC.

RULES.—1. To find the *exact* capacity of a vessel or space in gallons:

Divide the contents in cubic inches by 231 *for liquid gallons, or by* 268.8 *for dry gallons.* **(311)**.

2. To reduce gallons to cubic inches:

Multiply the given number of liquid gallons by 231; *then reduce to higher denominations if required.*

A *cubic foot* of pure water weighs 1000 oz., 62½ lb. Avoir.

1. How many gallons of water will a cistern hold, that is 4 ft square and 6 ft. 9 in. deep?

OPERATION.—$(4 \times 4 \times 6\frac{3}{4} \times 1728) \div 231 = 807\frac{69}{77}$ gal.

2. How many gallons will a tank hold, that is 6 ft. long, 2 ft. 6 in. wide, and 3 ft. 4 in. deep?

3. Find the depth of a cistern that is 6 ft. long and 4½ ft. wide, the capacity of which is 26 hhd.

4. Find the weight of water in a bathing tub 6 ft. 8 in. long, 3 ft. wide, and 1 ft. 9 in. deep, when ¾ full.

5. A cellar 35 ft. long and 24 ft. wide, contains 3 ft. of water. What will it cost to pump it out, at 5 cents a hogshead?

6. A tank in the attic of a house is 4 ft. long, 3 ft. wide, and 1 ft. 8 in. deep. How many gallons of water will it hold, and what will be its weight?

7. How many hogsheads will a cistern hold, that is 9 ft. long, 6 ft. wide, and 6 ft. deep?

8. How many cubic feet in a space that holds 52 barrels?

9. How many more cubic inches in 89.5 gal., dry measure than in the same number of gallons liquid measure?

10. If a man buy 6 bu. of chestnuts at $4 a bushel, dry measure, and sell the same at 20 cents a quart, liquid measure, how much does he gain?

REVIEW QUESTIONS.

355. 1. Define Reduction of Denominate Numbers. Reduction Descending. State the principle. Rule. Define Reduction Ascending. State the principle. Rule. What are Denominate Fractions? How are they written? Case I? Rule? Case II? Rule?

2. How are Denominate Numbers Added, Subtracted, etc.? How do they differ from Simple Numbers? Give the rule for addition. For subtraction. How is time between two dates found? Give the rule for multiplication. Rule for division.

3. How do you find any required dimension of a rectangular solid? The capacity of any rectangular space in bushels? The cubic contents of any number of bushels? The capacity of a vessel in gallons? The cubical contents of any number of gallons? Weight of a cubic foot of water?

PERCENTAGE.

356. 1. How much is $\frac{1}{100}$ of 100 ft.? $\frac{3}{100}$ of 100 ft.? $\frac{4}{100}$ of 100 ft.? $\frac{5}{100}$ of 400 ft.?

2. How much is $\frac{2}{100}$ of $300? $\frac{4}{100}$ of $500?

3. How many *hundredths* of a number or quantity is $\frac{1}{2}$ of it? $\frac{1}{4}$ of it? $\frac{1}{5}$ of it? $\frac{1}{10}$ of it?

4. What *part* of 100 hundredths is 10 hundredths? Is 20 hundredths? is 25 hundredths? Is 50 hundredths?

357. ***Percentage*** is a term applied to all computations in which 100 is used as a fixed measure or standard.

358. ***Per Cent.*** is an abbreviation of the Latin phrase *per centum,* which signifies *by the hundred,* and is used in business transactions to signify *hundredths* of any thing or quantity.

Thus 5 *per cent* means $\frac{5}{100}$, the 5 standing for the numerator, and the words "*per cent*" always for the denominator 100.

The phrase "per cent." wherever it occurs, should invariably suggest to the mind a horizontal line with 100 below it, or a *decimal* to the *hundredths* place. Thus, *per cent* $= \overline{100}$; 25 *per cent* $= \frac{25}{100}$, or .25.

359. ***The Sign %*** is generally used by business men in place of the words *per cent.* Thus, 6% signifies 6 per cent.

5 per cent, 5%, $\frac{5}{100}$ and .05 are equivalent expressions; the first two are used in the statement of questions, the other two in performing the operations.

5. How many *hundredths* of a number is 4 per cent of it? 6 per cent? 7%? 9%? 15%? 34%?

6. What *per cent* of a number is $\frac{3}{100}$ of it? $\frac{7}{100}$ of it? $\frac{8}{100}$? $\frac{11}{100}$? $\frac{26}{100}$? $\frac{42}{100}$? $\frac{75}{100}$?

7. How many *hundredths* of a number is $2\frac{1}{2}\%$ of it? $5\frac{1}{4}\%$? $6\frac{3}{4}\%$? $7\frac{2}{5}\%$? $9\frac{7}{10}\%$? $12\frac{1}{2}\%$? $21\frac{4}{5}\%$?

$2\frac{1}{2}\%$ is written $.02\frac{1}{2}$, or .025; $5\frac{1}{4}\%$ is written $.05\frac{1}{4}$, or .0525, etc.

8. What *per cent* of a number is .06? $.07\frac{1}{2}$? .045? $.08\frac{4}{5}$? .0225? .015? $.00\frac{1}{2}$? $.00\frac{5}{8}$? .0075?

360. 9. What per cent of a number is $\frac{1}{4}$ of it?

ANALYSIS.—Since $\frac{100}{100}$ is the *whole* of any thing or number, $\frac{1}{4}$ of the same is $\frac{1}{4}$ of $\frac{100}{100}$, or $\frac{25}{100}$, equal to 25%. Hence $\frac{1}{4}$ of a number is 25% of it.

10. What per cent of a number is $\frac{1}{2}$ of it? $\frac{1}{3}$ of it? $\frac{1}{5}$? $\frac{1}{8}$? $\frac{1}{12}$?

11. What % of a number is $\frac{3}{4}$ of it? $\frac{2}{3}$ of it? $\frac{4}{5}$? $\frac{5}{8}$? $\frac{5}{6}$? $\frac{7}{10}$? $\frac{9}{20}$? $\frac{12}{25}$? $\frac{11}{50}$?

361. 12. What *part* of a number is $8\frac{1}{3}\%$ of it?

ANALYSIS.—$8\frac{1}{3}\%$ is $\frac{8\frac{1}{3}}{100}$, or $\frac{25}{300}$, equal to $\frac{1}{12}$. Hence $8\frac{1}{3}\%$ is $\frac{1}{12}$ of a number.

13. What part of a number is 5% of it? 8%? $12\frac{1}{2}\%$?

14. What part of a number is $16\frac{2}{3}\%$? $6\frac{1}{4}\%$? $37\frac{1}{2}\%$? $33\frac{1}{3}\%$? $66\frac{2}{3}\%$?

362. 15. What part of a number is $\frac{1}{3}\%$ of it?

ANALYSIS.—$\frac{1}{3}\%$ is $\frac{\frac{1}{3}}{100}$, equal to $\frac{1}{300}$. Hence $\frac{1}{3}\%$ of a number is $\frac{1}{300}$ part of it.

16. What is $\frac{1}{2}\%$ of a number? $\frac{1}{5}\%$? $\frac{1}{6}\%$? $\frac{3}{8}\%$? $\frac{5}{6}\%$?

The pupil should be drilled on questions similar to the preceding until he is prompt and correct in answering them.

363. Since any *per cent* is equivalent to the same number of *hundredths,* it may be expressed either as a *decimal* or as a *common* fraction, as shown in the

TABLE.

Per cent.	Dec.	Com. Fractions.	Per cent.	Dec.	Com. Fractions.
2%	.02	$\frac{1}{50}$	75%	.75	$\frac{3}{4}$
4%	.04	$\frac{1}{25}$	125%	1.25	$1\frac{1}{4}$
6%	.06	$\frac{3}{50}$	$\frac{1}{2}$%	.005	$\frac{1}{200}$
10%	.10	$\frac{1}{10}$	$\frac{3}{4}$%	.0075	$\frac{3}{400}$
20%	.20	$\frac{1}{5}$	$8\frac{1}{3}$%	$.08\frac{1}{3}$	$\frac{1}{12}$
25%	.25	$\frac{1}{4}$	$12\frac{1}{2}$%	.125	$\frac{1}{8}$
50%	.50	$\frac{1}{2}$	$16\frac{2}{3}$%	$.16\frac{2}{3}$	$\frac{1}{6}$

WRITTEN EXERCISES.

364. Change to equivalent expressions having the sign of *per cent.*

1. .15. *Ans.* 15%.
2. .0825. " $8\frac{1}{4}$%.
3. $.00\frac{7}{8}$. " $\frac{7}{8}$%.
4. $.08\frac{1}{2}$.
5. $.33\frac{1}{3}$.
6. .755.
7. 2.50.
8. 1.125.
9. .0075.

365. Change to the form of decimals.

10. 4%. *Ans.* .04.
11. $10\frac{2}{5}$%. " .104.
12. $5\frac{2}{3}$%. " $.05\frac{2}{3}$.
13. $\frac{5}{8}$%.
14. $\frac{3}{5}$%.
15. $16\frac{3}{5}$%.
16. $26\frac{1}{2}$%.
17. $33\frac{1}{3}$%.
18. $206\frac{1}{2}$%.

366. Change to equivalent expressions having the sign of *per cent.*

19. $\frac{3}{4}$. *Ans.* 75%.
20. $\frac{5}{8}$. " $62\frac{1}{2}$%.
21. $2\frac{1}{4}$. " 225%.
22. $\frac{1}{8}$.
23. $2\frac{1}{8}$.
24. $\frac{5}{16}$.
25. $\frac{4}{15}$.
26. $\frac{16}{40}$.
27. $\frac{7}{12}$.

367. Change to the form of *common fractions.*

28. 12%. *Ans.* $\frac{3}{25}$.
29. $\frac{1}{4}$%. " $\frac{1}{400}$.
30. $6\frac{1}{4}$%. " $\frac{1}{16}$.
31. 120%.
32. $\frac{3}{8}$%.
33. $2\frac{1}{4}$%.
34. $66\frac{2}{3}$%.
35. 75%.
36. 28%.

DEFINITIONS.

368. In the applications of percentage, three elements or parts at least are considered, viz.: the Rate, the Base, and the Percentage. Any two of these being given, the other can be found.

369. ***Rate,*** or ***Rate Per Cent,*** is the decimal which denotes how many *hundredths* of a number are to be taken. Thus, $5\% = \frac{5}{100} = .05$: .05 is the *rate per cent.*

If the decimal be reduced to a common fraction in its *lowest terms,* this fraction will still be the equivalent *rate,* though not the rate *per cent.*

370. ***The Base*** is the number to which the rate is applied. Thus, in the expression 6% of $25, the *base* is $25.

371. ***Percentage*** is the result obtained by applying the rate to the base. Thus, in the statement, 5% of $40 is $2, .05 is the *rate per cent,* $40, the *base,* and $2, the *percentage.*

372. ***The Amount*** is the sum of the base and the percentage. Thus, if the base is $40, and the percentage $2, the *amount* is $40 + $2 = $42.

373. ***The Difference*** is the remainder found by subtracting the percentage from the base. Thus, if the base is $40, and the percentage $2, the *difference* is $40 — $2 = $38.

CASE I.

374. The base and rate given to find the percentage.

1. What is 10% of 80?

ANALYSIS.—10% of 80 is $\frac{10}{100}$, or $\frac{1}{10}$ of 80, which is 8. Hence 10% of 80 is 8.

How much is

2. 8% of 50 lb.?
3. 10% of 150 bu.?
4. 4% of 100 mi.?
5. 5% of 250 A.?

How much is

6. 25% of 64 gal.?
7. 30% of 40 sheep?
8. 40% of $60?
9. 50% of $120?

How much is

10. 35 gal. + 20%?
11. 120 A. + 25%?
12. 24 doz. + 12½%?
13. $75 + 4%?

How much is

14. 100 hdd. — 8%?
15. 45 cd. — 33⅓%?
16. 200 ft. — 2½%?
17. $80 — 25%?

18. A farmer having 120 bu. of wheat, sold 25% of it. How many bushels did he sell? What % did he keep?

19. Bought 25 bbl. of apples, and upon opening them found 4% of them worthless. How many were good?

20. A locomotive running 30 mi. an hour, increases its speed 40%. How far does it then run in an hour?

21. A man bought 160 pine-apples and lost 12½% of them by rot. How many had he left?

22. A cask containing 60 gal. of wine sprang a leak, and 5% ran out. How many gallons remained?

375. PRINCIPLE.—*The percentage of any number is the same part of that number as the given rate is of* 100%.

WRITTEN EXERCISES.

1. What is 12½% of $1346?

OPERATION.

$$\begin{array}{r} \$1346 \\ .12\tfrac{1}{2} \\ \hline \$168.25 \end{array}$$

Or,

$$\$1346 \times \tfrac{1}{8} = \$1346 \div 8 = \$168.25$$

ANALYSIS.—Since 12½% is .12½, 12½% of $1346 is $1346 × .12½ = $168.25. Or,

Since 12½% is ⅛ of 100%, 12½% of $1346 is ⅛ of $1346, which is $168.25.

How much is
2. 20% of $86.25?
3. 8⅓% of 1320 bbl.?
4. 14% of 650 miles?

How much is
5. 6¼% of 3212 rd.?
6. 27% of $475.20?
7. 7¼% of 1136 bu.?

RULE.—*Multiply the base by the rate expressed, either as a decimal or as a common fraction.*

It is well to solve examples by both methods, always reducing the common fraction to its lowest terms before multiplying.

How much is
8. 33⅓% of 125.76?
9. 6% of 310.35?
10. 75% of 548 tons?

How much is
11. 1¾% of $243.60?
12. 3⅘% of 2640 lb.?
13. ½% of $5000?

14. A man having 320 sheep, lost 6¼% of them. How many had he then?

15. A builder bought 10 boxes of glass, each box containing 60 panes. On opening them, 8⅓% were found broken. How many panes were broken?

16. A man who owed $6720.50, paid his creditors 30%. What sum did he pay? How much was unpaid?

17. Having \$10720, I invested 25% of it in land, and $12\frac{1}{2}$% of the remainder in fencing it. How much had I remaining?

18. Of 280 barrels of apples, $2\frac{1}{2}$% are spoiled. How many barrels are sound?

19. Two men engaged in trade, each with \$3540. One of them gained $33\frac{1}{3}$% of his capital, and the other gained 60%. How much more did the one gain than the other?

CASE II.

376. The base and percentage given to find the rate.

1. What per cent of 80 is 4?

ANALYSIS.—Since 80 is 100% of itself, 4 is $\frac{4}{80}$, or $\frac{1}{20}$ of 100%, which is 5%. Hence 4 is 5% of 80.

What per cent of
2. 21 lb. are 7 lb.?
3. 25 bbl. are 15 bbl.?
4. \$40 are \$30?
5. $12\frac{1}{2}$ rd. are $2\frac{1}{2}$ rd.?
6. 1 A. are 40 sq. rd.?
7. 1 da. are 8 hr.?
8. 1 yd. are 9 in.?

What per cent
9. Are \$$6\frac{1}{4}$ of \$$12\frac{1}{2}$?
10. Are 36 bu. of 60 bu.?
11. Are $3\frac{1}{4}$ qt. of 13 qt.?
12. Are $8\frac{1}{3}$ cents of \$1?
13. Is $\frac{3}{16}$ of $\frac{3}{4}$?
14. Is $\frac{1}{3}$ of $\frac{5}{6}$?
15. Is $\frac{3}{8}$ of $1\frac{1}{2}$?

16. $\frac{3}{10}$ of 40 is what per cent of 40?

17. $\frac{1}{3}$ of a farm is what per cent of it?

18. $\frac{4}{5}$ of a cargo is what per cent of it?

19. $1\frac{1}{2}$ times a number is what per cent of it?

20. If a miller take 4 qt. of every bushel he grinds, what % of it does he take?

21. If \$5 is paid for the use of \$20 one year, what is the rate per cent?

22. A man pays $6 an acre for the use of land valued at $72 an acre. What % of the value of the land does he pay?

377. PRINCIPLE.—*The rate is the number of hundredths which the percentage is of the base.*

WRITTEN EXERCISES.

1. What per cent of 320 is 80?

OPERATION.

$80 \div 320 = .25 = 25\%$

Or, $\frac{80}{320} = \frac{1}{4}$;

and $\frac{1}{4} \times 100\% = 25\%$

ANALYSIS.—Since the percentage is the *product* of the base and rate, divide the percentage 80 by the base 320, and the quotient will be 25, the rate. Or,

Since 320 is 100% of itself, 80 is $\frac{80}{320}$ or $\frac{1}{4}$ of 100%, which is 25%

What per cent
2. Of 450 is 90?
3. Of 56 gal. are 7 gal.?

What per cent
4. Are 45 cents of $9?
5. Are 40 lb. of 250 lb.?

RULE.—*Divide the percentage by the base.* Or,

Take such part of 100% *as the percentage is of the base.*

What per cent
6. Of $2 are 15 cents?
7. Of 760 lb. are 190 lb.?
8. Of 96 A. are 60 A.?
9. Of $240 are $13.20?

What per cent
10. Are £5⅓ of £32?
11. Are 5 dimes of $100?
12. Is 4.5 of 75?
13. Is .45 of .60?

14. A regiment went into battle with 600 men, and came out with 320. What % were lost?

15. Of 150 pine-apples, 3 of every 5 are bad. What % are bad?

16. Of 4000 A. of land I sell 140 A. What % do I retain?

CASE III.

378. The rate and percentage given to find the base.

1. 4 is 5% of what number?

ANALYSIS.—Since $5\% = \frac{5}{100}$ or $\frac{1}{20}$ of a number is 4, $\frac{20}{20}$ is 20 times 4, which is 80. Hence 4 is 5% of 80.

2. 16 is 4% of what number? 8% of what?
3. 1.2 is 6% of what number? 4% of what?
4. $84 are 12% of what number? 21% of what?
5. 30 lb. are 20% of what number? 5% of what?
6. 56 doz. are $12\frac{1}{2}\%$ of what number? 14% of what?
7. Sold 105 bbl. of potatoes, which was 35% of all I raised. How many did I raise?
8. 10% of 40 is 20% of what number?
9. 5% of 25 is 6% of what number?
10. $33\frac{1}{3}\%$ of 36 is 4% of what number?
11. A farmer sold 7.5 A. of land, which was 15% of all he owned. How many acres did he own

379. PRINCIPLE.—*The base is as many times the percentage as* 100% *is times the rate.*

WRITTEN EXERCISES.

1. 80 is 25% of what number?

OPERATION.

$$80 \div .25 = 320$$

Or,

$$80 \div \frac{1}{4} = 80 \times 4 = 320$$

ANALYSIS.—Since 80, the percentage, is the *product* of the base by the rate, the base is the quotient found by dividing 80 by the rate, .25, which is 320. Or,

Since 80 is $\frac{25}{100}$, or $\frac{1}{4}$, of some number, that number is 4 times 80, or 320.

2. 18.5 tons are 5% of what?
3. $92.44 are 20% of what?
4. $21.60 are 3% of what?
5. 126 A. are 9% of what?

RULE.—*Divide the percentage by the rate expressed, either as a decimal or as a common fraction.*

6. 6.5 gal. are 5% of what?
7. $18.75 are $6\frac{1}{4}\%$ of what?
8. 420 bbl. are $12\frac{1}{2}\%$ of what?
9. $300 are 48% of what?
10. 18 yd. are 36% of what?
11. $975 are 15% of what?
12. 78.5 is 25% of what?
13. 19.8 lb. are $7\frac{1}{2}\%$ of what?
14. 2% of $125.50 is 8% of what sum?
15. $12\frac{1}{2}\%$ of 400 ft. is $2\frac{1}{2}\%$ of how many feet?

16. A man spends $825.60, which is $33\frac{1}{3}\%$ of his salary. How much is his salary?

17. A man drew out 9% of his bank deposit to pay a debt of $243.72. How much had he in bank?

CASE IV.

380. The amount, or difference, and rate given to find the base.

1. What number increased by 20% of itself, is 48?

ANALYSIS.—Since 48 is the number increased by 20%, or $\frac{20}{100}$ of itself, 48 is $\frac{120}{100}$ or $\frac{6}{5}$ of the number; and if $\frac{6}{5}$ of the number is 48, the number itself is 5 times $\frac{1}{6}$ of 48, which is 40.

2. What number increased by 10% of itself is 55?

3. What sum increased by $12\frac{1}{2}\%$ of itself is $18.90?

4. $45 is 25% more than what sum?

5. A tailor sold a coat for $26, which was 30% more than it cost him. How much did he give for it?

6. A coal-dealer sold coal for $7 a ton, which was $16\frac{2}{3}\%$ more than he paid for it? How much did he pay for it?

7. What number diminished by 20% of itself is 36?

Analysis.—Since 36 is the number diminished by $\frac{20}{100}$ of itself, 36 is $\frac{80}{100}$, or $\frac{4}{5}$ of the number; and if $\frac{4}{5}$ of the number is 36, the number itself is 5 times $\frac{1}{4}$ of 36, which is 45.

8. What number diminished by 5% of itself is 57?

9. What sum diminished by 50% of itself is $12.50?

10. 60 cwt. are 40% less than what number?

11. 35 yd. are 12½% less than what number?

12. A boy spent 25% of his money, and had 75 cents left. How much had he at first?

13. A grocer sells coffee for 36 cents a pound, which is 12½% above cost. What did it cost?

14. A farmer sold 33⅓% of his flock of sheep, and kept 100. How many had he at first?

15. In a grammar class there are 18 girls, which is 20% more than the number of boys. How many pupils in the class?

WRITTEN EXERCISES.

1. What number increased by 20% of itself amounts to 360?

OPERATION.

$$1 + .20 = 1.20$$
$$360 \div 1.20 = 300$$

Or,

$$\frac{120}{100} = \frac{6}{5}$$
$$360 \div \frac{6}{5} = 300$$

Analysis.—Since the number is increased by 20%, or .20 of itself, 360 is 120%, or 1.20 times the number. Hence dividing 360 by 1.20 equals 300, the required number. Or,

Since a number increased by 20% or $\frac{1}{5}$ of itself is $\frac{6}{5}$ of the same number, 360 is $\frac{6}{5}$ of the required number, etc.

2. What number increased by 15% of itself is 644?

3. What sum increased by 8% of itself is 1475?

4. What number diminished by 12½% of itself is 175?

OPERATION.

$$1 - .125 = .875$$
$$175 \div .875 = 200$$
$$\text{Or, } \frac{87\frac{1}{2}}{100} = \frac{7}{8}$$
$$175 \div \tfrac{7}{8} = 200$$

ANALYSIS.—Since the number is diminished by 12½%, or .125 of itself, 175 is 87½%, or .875 of the number. Hence dividing 175 by .875 equals 200, the required number. Or,

Since a number diminished by 12½%, or ⅛ of itself, is ⅞ of the same number, 175 is ⅞ of the required number, etc.

5. What number diminished by 30% of itself is 350?

6. What number diminished by 4½% of itself is 1640?

RULE.—*Divide the amount by* 1 *plus, or* 1 *minus the rate, as the case may require.*

The nature of the question will sufficiently indicate whether 1 is to be *increased* or *diminished* by the *rate*, to form the divisor.

What number increased	What number diminished
7. By 15% of itself is 345?	12. By 65% of itself is $2590?
8. By 36% is 238 A.?	13. By 50% is 28.5 ft.?
9. By 100% is 84.6?	14. By 16⅔% is 1035 mi.?
10. By 6% is 1272 bu.?	15. By 4% is $465.60?
11. By 22% is $549?	16. By 37½% is $203.375?

17. Sold a horse for $340, which was 15% less than his value. What was his value?

18. A man having increased his bank deposit 40%, it amounted to $840. How much had he at first?

19. My income this year is $2232, which is 7% less than it was last year. How much was it last year?

20. A man sold 160 A. from his farm, which was 12½% less than the number of acres he retained. How many acres in his farm?

PROFIT AND LOSS.

381. ***Profit and Loss*** are *commercial* terms used to express the gain or loss in business transactions.

382. Profits and losses are usually estimated at some *rate per cent* on the cost, or on the money or capital invested.

383. *Four cases* occur which involve the same *principles* and *operations* as those of Percentage already considered.

The corresponding terms are the following:

The ***Base*** is the *Cost,* or capital invested.

The ***Rate*** is the *per cent* of profit or loss.

The ***Percentage*** is the profit or loss.

The ***Amount*** is the cost *plus* the gain, or the Selling Price.

The ***Difference*** is the cost *minus* the loss, or the Selling Price.

ORAL EXERCISES.

384. 1. A watch that cost $80 sold for 20% more than cost. What was the gain, and the selling price?

ANALYSIS.—Since the gain was 20%, = $\frac{20}{100}$, or $\frac{1}{5}$ of $80, or $16, the selling price was $80 + $16 = $96. Hence, etc. (**374**).

2. Paid $48 for a cow, and sold her for 12½% less than cost. What was the loss, and the selling price?

3. A merchant bought cloth at $4 a yard, and sold it at 25% advance. How much was his profit?

4. A carriage that cost $120 sold at a loss of 10%. How much did it bring?

5. A grocer bought sugar at 12 cents a pound. For how much must he sell it to gain 25%?

6. Bought gloves at $12 a dozen. For how much a pair must they be sold to gain 50%?

7. Butter bought at 32 cents a pound was sold at a loss of 12½%. At what price did it sell?

8. Flour was bought for $8, and sold for $10 a barrel. What was the gain per cent?

Analysis.—The whole gain is the difference between $8 and $10, which is $2. Since $8 gains $2, or ¼ of itself, the gain per cent is ¼ of 100%. Hence, etc. (**376**).

9. What is gained per cent, by selling tea at $.90 that cost $.80?

10. Sold oranges for 2 cents each that cost 36 cents a dozen. What was the loss per cent?

11. A man sold a boat for $20, which was ⅔ of what it cost him. What was his loss per cent?

12. Sold damaged goods at a loss of ⅛ of what they cost. What was the loss per cent?

13. If an article be sold at *double* the cost, what % is gained?

14. What % is lost on articles sold at *one-half* the cost?

15. What per cent does a merchant lose, who sells goods at ⅘ their cost?

16. What % profit does a grocer make who buys coffee at 30 cents and sells it at 35 cents?

17. A fruit dealer sold pears at a profit of $3 a barrel, which was a gain of 20%. What did they cost?

Analysis.—Since the gain was 20%, = $\frac{20}{100}$ or ⅕, $3 is 20%, or ⅕ the cost; $3 is ⅕ of 5 times $3, or $15. Hence, etc. (**378**).

18. Sold a horse for $30 more than cost, and gained 16⅔%. What did he cost?

19. A dealer gained 30% by selling boots at $1.50 a pair above cost. What did they cost, and what was the selling price?

20. A drover sells sheep at a profit of $1½ a head, which is a gain of 25%. What did they cost, and what is the selling price?

21. Sold muslin at 45 cents a yard, and gained 12½%. What did it cost?

ANALYSIS.—Since the gain was 12½%, or $\frac{1}{8}$ the cost, 45 cents, the selling price, is $\frac{9}{8}$ the cost. $\frac{1}{9}$ of 45 cents, or 5 cents, is $\frac{1}{8}$ the cost, and $\frac{8}{8}$ is 8 times 5 cents, or 40 cents. Hence, etc. (**380**).

22. A music-box sold for $72, which was 10% less than cost. How much did it cost?

23. Sold turkeys at 30 cents a pound, and gained 20%. How much did they cost a pound?

24. If I lose 20% by selling flour at $8 a barrel, how much would I receive if I sold it at 20% above cost?

WRITTEN EXERCISES.

385. The *difference* between the *cost* and *selling price* is the gain or loss.

Gain, when the *selling price* is the greater.

Loss, when the *cost* is the greater.

386. To find the selling price,

Add the *gain* to the cost. Or,
Subtract the *loss* from the cost.

Find the *profit,* or *loss,*

1. On tea that cost $135, and sold at 20% gain (**374**).

2. On goods that cost $3½ a yd., and sold at 25% gain.
3. On a house bought for $3270, and sold at 7% gain.
4. On sugar that cost $623.75, and sold at 6¼% loss.

387. Find the *selling price* of goods
1. Bought at $126.50, and sold at 9% gain (**374**).
2. Bought at $3424, and sold at 12½% loss.
3. Bought at $1032.50, and sold at 14% gain.
4. Bought at $3000, and sold at 8¼% loss.

388. Find the *rate per cent* of profit or loss on goods
1. Sold for $330, bought for $275 (**376**).
2. Sold for $.75, bought for $.60.
3. Bought for $640, sold for ¾ as much.
4. Bought for $250, sold for ⅗ as much.
5. Sold for $72.96, at a gain of $9.12.
6. Bought for $4.25, sold for $4.93.
7. Sold for $450.84, at a loss of $225.42.

389. Find the *cost* of goods
1. Sold for $170, being a gain of 25% (**380**).
2. Sold for $15.24, at a loss of 40%.
3. Sold for $4.80, at a profit of 33⅓%.
4. Sold for $.96, and gained 28%.

5. A grocer bought 1,075 lb. of sugar, at 8⅓ cents pound, and sold it at a profit of 18%. What did he gain

6. Bought 825 bu. of wheat, at $1.37½ a bushel, an sold the flour at a loss of 8% on the cost of the wheat What was the whole loss?

7. Bought 4 pieces of silk, each piece containin 56 yd., at $84 a piece, and sold the same at $2 a yard What was the whole gain, and the gain %.

8. Sold cloth at $3.60 a yard, and gained 12½%. What would the selling price have been, if I had gained 20%?

9. Sold a horse for $275 and lost 16⅔%. At what price should he have been sold to gain 12½%?

10. If by selling muslin for 18 cents a yard, a merchant gains 12½%, what will be his per cent of gain, if he sells for 19½ cents a yard?

11. At what price must a grocer mark tea that cost 76 cents a pound, in order that he may abate 5%, and yet make a profit of 25%?

Analysis.—Since he is to gain 25%, he must sell the tea at 76 cents + 19 cents, or 95 cents; and since this is 5% less than the marking price, it is .95 of it. Hence the marking price is $1.00.

12. What must be the asking price for a house that cost $20000, so that 20% may be abated, and yet a gain of 10% be realized?

13. What must a man ask for land that cost him $30 an acre, so that after abating 14²⁄₇%, he may still gain 20%?

14. Two building lots were sold for $420 each; on one there was a gain of 25%, and on the other a loss of 25%. Was there a gain or loss on the whole, and how much?

15. Having used a carriage one year, I sold it for $210, which was 30% below cost. How much would I have received had I sold it for 8% more than cost?

16. If a merchant sells his goods at an average profit of 15%, what amount must he sell to clear $3000?

17. Bought 60 bbl. of flour at $9½ a barrel; 80 bbl. at $8; 100 bbl. at $8½, and sold the whole at $9.75. What was the gain per cent?

18. Sold ⅝ of a piece of land for what the whole cost. What per cent. was gained on the part sold?

COMMISSION.

390. ***Commission*** is the compensation allowed an agent or commission merchant for transacting business. It is computed at a certain *rate per cent* of the money involved in the transaction.

391. ***An Agent,*** or ***Commission Merchant,*** is a person who buys or sells merchandise, or transacts other business for another called the *Principal.*

392. ***The Net Proceeds*** of a sale or other transaction, are the sum of money that remains after all expenses of commission, etc., are paid.

393. ***A Broker*** is a person who buys or sells stocks, bills of exchange, real estate, etc., for a commission, which is called *Brokerage.*

394. The corresponding terms are the following:

The Base is the money invested or collected.

The Rate is the per cent allowed for services.

The Percentage is the commission or brokerage.

The Amount, or Difference, is the net proceeds, or the amount of sales, plus or minus the commission.

WRITTEN EXERCISES.

395. Find the *commission* or *brokerage,*

1. On a sale of merchandise for \$2150 at 2¼%.

OPERATION.—\$2150 × .02¼ = \$48.37½. (**374.**)

2. On the sale of a farm for \$4650, at 2½%.

3. On the purchase of 375 bbl. of flour, at \$6.25 a barrel, commission at 3¼%.

4. A broker buys 125 shares of stock, at $87½ a share. What is his brokerage at ⅛%?

5. A commission merchant sells 150 bales of cotton, averaging 530 pounds, at 14½ cents a pound. What is his commission at 1⅛%?

6. What will be the commission at 1¼%, on the sale of a house and lot for $22500.

396. Find the *rate* of commission or brokerage,

1. If $52.20 commission is paid for selling goods for $2320.

OPERATION.—$52.20 ÷ $2320 = .0225 = 2¼%. (**376.**)

2. If $78 is paid for selling goods for $5200.

3. If a real estate agent charges $189 for selling a farm for $7560.

4. If a broker charges $10.72 for selling $8576 worth of stock.

5. If an agent charges $143.22 for buying 13200 lb. of wool, at $.46½ a pound.

6. An agent in New Orleans charged me $18.75 for buying 75 bbl. of sugar, averaging 200 lb., at 8⅓ cents a pound. At what rate was his commission?

397. Find the *amount of sales*,

1. When a commission of $105 is charged at 2½%.

OPERATION.—$105 ÷ .025 = $4200. (**378.**)

2. When $378 commission is paid at 2¼%.

3. When $190 brokerage is charged at 3⅓%.

4. If an agent charges $59.50 commission at 1¾%.

5. Paid an agent $394 for selling a piece of property, at 2½%. What price did the property bring?

6. If the rate of brokerage is ⅛%, what amount of stock must be sold that a broker may realize $184?

7. Paid $157½ for collecting a note, which was a commission of 4½%. What was the face of the note?

398. Find the *amount to be invested,*

1. When a commission merchant receives $3726, out of which he is to take his commission at 3½%.

OPERATION.—$3726 ÷ 1.035 = $3600. (**380.**)

2. When $4455 is received, commission at 1¼%.

3. When $6617.70 is received, commission 2⅛%.

4. When $9909.30 is received, commission ½%.

5. A broker receives $9612, to invest in stock at $96 a share, deducting his commission at ⅛%. How many shares can he buy?

6. Sent to my agent in Chicago $1508.80, to invest in flour at $5¾ a barrel, after deducting his commission at 2½%. How many barrels can he buy?

7. What quantity of coffee, at 16½ cents a pound, can be bought for $614.79, after deducting a commission of 3½%?

REVIEW QUESTIONS.

399. 1. Define Percentage. Per Cent. The Sign. What are the principal elements of Percentage? Define Rate. Base. Percentage. Amount. Difference. What is Case I? Principle? Rule? Case II? Principle? Rule? Case III? Principle? Rule? Case IV? Rule?

2. Define Profit and Loss. How estimated? In what respects the same as the first four cases of Percentage? Name the corresponding terms. How is gain or loss found? How is the selling price found?

3. Define Commission. An Agent. Net Proceeds. A Broker. Name the corresponding terms in commission.

INTEREST.

400. 1. When 5% is charged for the use of money, how many cents are paid for the use of 100 cents? How many dollars for the use of $100?

2. When 7% is charged? 4%? 8%? 10%? 15%?

3. When 5 cents are paid for the use of $1, what is the rate %? When 6 cents are paid? 7 cents? 9 cents? 10 cents? 12½ cents?

4. At 6%, what decimal part of the money borrowed is equal to the money paid for its use? At 5%? 7%? 8%?

5. When $7 is paid for the use of $100 one year, what is the yearly, or annual rate per cent?

6. When $5½ is paid for the use of $100 1 year, what is the annual rate per cent.

7. What is the annual rate per cent when 5 cents is paid for 100 cents? $4 for $100? 10 cents for $1? $8 for $100?

DEFINITIONS.

401. ***Interest*** is a sum paid for the use of money, or its equivalent.

402. ***The Principal*** is the sum for the use of which interest is paid.

403. ***The Rate of Interest*** is the rate per cent, or number of hundredths of the principal paid for its use *one year.*

404. ***The Amount*** is the sum of the principal and the interest.

405. ***Legal Interest*** is the interest according to the rate per cent *fixed by law.* It varies in different States, as follows:

Name of State.	Rate.		Name of State.	Rate.	
Alabama	8%	....	Mississippi	6%	10%
Arkansas*	6%	Any.	Missouri	6%	10%
California*	10%	Any.	New Hampshire.	6%	....
Connecticut*	6%	Any.	New Jersey	7%	....
Colorado*	10%	Any.	New York	7%	....
Delaware	6%	....	North Carolina	6%	8%
Dist. Columbia	6%	10%	Nebraska	10%	12%
Florida*	8%	Any.	Nevada*	10%	Any.
Georgia	7%	10%	Ohio	6%	8%
Illinois	6%	10%	Oregon	10%	12%
Indiana	6%	10%	Pennsylvania	6%	7%
Iowa	6%	10%	Rhode Island*	6%	Any.
Kansas	7%	12%	South Carolina*.	7%	Any.
Kentucky	6%	10%	Tennessee	6%	10%
Louisiana	5%	8%	Texas*	8%	Any.
Maine*	6%	Any.	Utah*	10%	Any.
Maryland	6%	....	Vermont	6%	....
Massachusetts*	6%	Any.	Virginia	6%	12%
Michigan	7%	10%	West Virginia	6%	....
Minnesota	7%	12%	Wisconsin	7%	10%

In the States marked thus (*), the rate per cent is unlimited if agreed upon by the parties in writing.

ORAL EXERCISES.

406. 1. At 5% for 1 yr., what decimal part of the principal equals the interest? At 6%? At 7%? At 8%? 10%? 12½%?

407. 2. Find the interest of $4 for 1 year at 6%.

Analysis.—Since the interest of any sum at 6% for 1 yr. is .06 of the principal, the interest of $4 for 1 year at 6% is .06 of $4, equal to $.24, or 24 cents.

3. What is the interest of $8 for 1 yr. at 7%? 5%? 8%?

4. What is the interest of $12 for 1 yr. at 4%? 6%? 8%?

408. 5. At 5% for 3 yr., what decimal part of the principal equals the interest?

Analysis.—Since the interest of any sum at 5% for 1 yr. is .05 of the principal, the interest for 3 yr. is 3 times .05, or .15 of the principal. Or, it is 3 times the interest for 1 year.

6. At 4% for 4 yr., what decimal part of the principal equals the interest? At 6% for 3 yr.? 7% for 4 yr.?

409. 7. Find the interest of $5 for 3 yr. at 4%.

Analysis.—Since the interest of any sum at 4% for 1 yr. is .04 of the principal, the interest of $5 for 1 yr. at 4% is .04 of $5, or $.20; and for 3 yr. it is 3 times $.20, or $.60.

8. Find the int. at 6% of $10 for 2 yr.; $5 for 4 yr.
9. Find the int. at 7% of $4 for 3 yr.; $6 for 2 yr.
10. Find the int. at 8% of $10 for 4 yr.; $20 for 3 yr.

410. 11. At 6% for 2 yr. 6 mo., what decimal part of the principal equals the interest?

Analysis.—Since the interest of any sum for 1 yr. at 6% is .06 of the principal, the interest on the same for 2 yr. 6 mo. is 2½ times .06, or .15 of the principal. Or, it is 2½ times the interest for 1 yr.

12. At 4% for 3 yr. 3 mo., what decimal part of the principal equals the interest? At 6% for 2 yr. 8 mo.?
13. Find the interest of $6 at 5% for 2 yr. 8 mo.
14. Find the interest of $9 for 1 yr. 8 mo. at 7%. 8%.
15. Find the int. of $20 for 3 yr. 4 mo. at 5%. 6%.
16. Find the int. of $15 at 6% for 1 yr. 6 mo. 2 yr. 4 mo. 6 mo. 8 mo. 10 mo.

411. Principle.—*The interest is the product of three factors, namely: the principal, rate per annum, and time.*

WRITTEN EXERCISES.

Case I.

412. To find the interest or amount of any sum, at any rate per cent, for years and months.

1. Find the amount of $86.50 at 6% for 3 yr. 4 mo.

OPERATION.

$86.50	
.06	
$5.1900	Int. for 1 yr.
$3\frac{1}{3}$	
$17.30	Int. for 3 yr. 4 mo.
86.50	Principal.
$103.80	Amount.

Analysis.—Since the interest of any sum at 6% for 1 year, is .06 of the principal, the interest of $86.50 is $86.50 × .06 = $5.19; the *interest* for 3 yr. 4 mo. is $3\frac{1}{3}$ times the interest for 1 yr., or $17.30. And $17.30 + $86.50 = $103.80, the *amount*.

2. Find the interest of $48.75 for 2 yr. 6 mo. at 8%.
3. Find the interest of $276.60 for 1 yr. 9 mo. at 7%.
4. Find the amount of $750 for 4 yr. 4 mo. at 5%.

Rule.—I. *Multiply the principal by the rate per cent, and the product will be the interest for 1 year.*

II. *Multiply the interest for 1 year by the time in years and the fraction of a year, and the product will be the required interest.*

III. *Add the principal to the interest for the amount.*

Find the interest

5. Of $276.12 for 1 yr. 6 mo. at 7%. At 5%. 12%.
6. Of $600 at 6% for 2 yr. 8 mo. For 3 yr. 9 mo.
7. What is the amount of $324 for 1 yr. 9 mo. at 6%?
8. If a person borrow $540.75 at 8%, how much will be due the lender at the end of 2 yr. 1 mo.?

CASE II.

413. To find the interest on any sum, for any time, at any rate per cent.

OBVIOUS RELATIONS BETWEEN TIME AND INTEREST.

414. I. The interest on any sum for 1 yr. at 1% is .01 of that sum, and is therefore equal to the principal with the decimal point removed two places to the left.

II. The interest for 1 mo. is $\frac{1}{12}$ of the interest for 1 yr.

III. The interest for 3 da. is $\frac{3}{30}$, or $\frac{1}{10} = .1$ of the interest for 1 mo.; hence any number of days may readily be reduced to *tenths* of a month by dividing by 3.

IV. The interest on any sum for 1 mo. multiplied by the number of *months* and *tenths* of a month in the given time, and the product by the number expressing the *rate*, will be the required interest.

WRITTEN EXERCISES.

1. Find the int. of $267.60 for 1 yr. 4 mo. 18 da. at 5%.

OPERATION.

$2.676	(.01 of the Prin.) Int. for 1 yr. at 1% (**414**, I).
.223	Int. for 1 mo. at 1% (**414**, II).
16.6	Number of months and tenths (**414**, III).
$3.7018	Int. for 1 yr. 4 mo. 18 da. at 1%.
5	Rate of interest.
$18.5090	Int. for 1 yr. 4 mo. 18 da. at 5% (**414**, IV).

What is the interest

2. Of $35.16 for 2 yr. 9 mo. 15 da. at 7%?
3. Of $217.68 for 3 yr. 1 mo. 21 da. at 8%?
4. Of $664.20 for 1 yr. 11 mo. 3 da. at 6½%?

Find the amount

5. Of $841.44 for 11 mo. 27 da. at 12%.
6. Of $1200 for 1 yr. 9 da. at 4½%.
7. Of $370.80 for 1 yr. 10 mo. 4 da. at 6%.
8. Of $2400 for 2 yr. 1 mo. 20 da. at 7%.
9. Of $5000 for 5 mo. 16 da. at 8¼%.

RULE.—I. To find the interest for 1 yr. at 1%.

Remove the decimal point in the given principal two places to the left.

II. To find the interest for 1 mo. at 1%.

Divide the interest for 1 *year by* 12.

III. To find the interest for *any time* at 1%.

Multiply the interest for 1 *month by the number of months and tenths of a month in the given time.*

IV. To find the interest at any rate %.

Multiply the interest at 1% *for the given time by the number expressing the given rate.*

Find the interest

10. Of $540 for 2 yr. 3 mo. 21 da. at 8%?
11. Of $95.50 for 3 mo. 15 da. at 5½%?
12. Of $105.48 for 2 yr. 1 mo. 14 da. at 7%?
13. Find the amount of $960 for 10 mo. 24 da. at 7%.
14. Find the amount of $116.46 for 9 mo. 16 da. at 10%.
15. Find the interest of $325.28 at 8% from June 20th, 1873, to Sept. 4th, 1875.
16. What is the value of a note of $65.75 due with interest for 3 yr. 2 mo. at 8%?

415. SIX PER CENT METHOD.

At 6% per annum, the interest of $1

For 12 mo. . . . . is 6 cents, or .06 of the principal.
" 2 " or $\frac{1}{6}$ of 12 mo., " 1 " " .01 " "
" 1 " " $\frac{1}{12}$ " " " $\frac{1}{2}$ " " .005 " "
" 6 da. " $\frac{1}{5}$ of 1 " " $\frac{1}{10}$ " " .001 " "
" 1 " " $\frac{1}{6}$ of 6 da. " .000$\frac{1}{6}$ " "

Hence the following

416. PRINCIPLES.—1. *The interest of any sum at 6%, is* ONE-HALF *as many hundredths of the principal, as there are* MONTHS *in the given time.*

2. *The interest of any sum at 6% is* ONE-SIXTH *as many thousandths of the principal as there are* DAYS *in the given time.*

Thus, the interest on any sum at 6% for 1 yr. 5 mo., equal to 17 mo., is $\frac{1}{2}$ of .17, or .085, of the principal; and for 24 da. it is $\frac{1}{6}$ of .024, or .004, of the principal. Hence, for 1 yr. 5 mo. 24 da., it is .085 + .004 = .089 of the principal.

It is evident that an odd month is $\frac{1}{2}$ of .01, or .005; and that any number of days less than 6 is such a fractional part of .001 as the days are of 6 days.

What is the interest at 6%,

1. On $1 for 2 yr.? 3 yr.? 4 yr.? 5 yr.? 7 yr.? 10 yr.?
2. On $5 for 1 yr.? 2 yr.? 3 yr.? 6 yr.? 8 yr.? 7 yr.?
3. On $1 for 1 mo.? 2 mo.? 3 mo.? 4 mo.? 6 mo.?
4. On $10 for 1 mo.? 4 mo.? 6 mo.? 8 mo.? 15 mo.?
5. On $1 for 1 yr. 2 mo.? 2 yr. 1 mo.? 3 yr. 3 mo.?
6. On $100 for 1 yr. 1 mo.? 2 yr. 2 mo.? 1 yr. 10 mo.?
7. On $1 for 2 da.? 4 da.? 5 da.? 6 da.? 9 da.?
8. On $1 for 18 da.? 33 da.? 63 da.? 93 da.?

9. On \$1 for 1 mo. 18 da.? 2 mo. 25 da.? 6 mo. 9 da.?

Find the interest at 6%,

10. On \$2 for 1 yr. 9 mo. On \$3. \$5. \$8. \$10. \$40.

11. On \$5 for 2 yr. 6 mo. On \$10 for 1 yr. 7 mo. 18 da.

12. On \$400 for 33 da. On \$50 for 63 da.

13. If the interest on a sum of money at 6% is \$24, what would the interest be at 5%? 7%? 8%? 9%? 10%?

5% is $\frac{1}{6}$ less than 6%; 7% is $\frac{1}{6}$ more than 6%; 8% is $\frac{1}{3}$ more, etc.

14. If the interest of a certain principal at 6% is \$60, what would it be at 4%? 3%? 7%? 9%? 12%? 10%?

WRITTEN EXERCISES.

1. What is the int. of \$362.40 at 6% for 11 mo. 25 da.?

OPERATION.

$$\frac{1}{2} \text{ of } .11 = .05\tfrac{1}{2} = .055$$
$$\frac{1}{6} \text{ of } .025 = \underline{.004\tfrac{1}{6}}$$
$$\text{Int.} = .059\tfrac{1}{6} \text{ of the Prin.}$$
$$\$362.40 \times .059\tfrac{1}{6} = \$21.5024$$

ANALYSIS. — Since the interest on any sum for 11 mo. 25 da. is $.059\frac{1}{6}$ of the principal (**416**), \$362.40 × $.059\frac{1}{6}$ = \$21.50+, the required interest.

Find the interest at 6% of

2. \$267.27 for 6 mo. 24 da.
3. \$146.18 for 1 yr. 21 da.
4. \$256.84 for 2 yr. 4 mo. 12 da.
5. \$38.90 for 1 yr. 1 mo. 6 da.
6. \$146.48 for 9 mo. 10 da.
7. \$275.50 for 11 mo. 27 da.

RULE.—*Multiply the given principal by the decimal expressing one-half as many hundredths as there are months, or one-sixths as many thousandths as there are days, in the given time, and the product will be the required interest.*

To find the interest at any other rate %, *increase* or *diminish* the interest at 6% by such part of itself as the given rate is greater or less than 6%. Thus, for 7% add $\frac{1}{6}$; for 4% subtract $\frac{1}{3}$; for 10% add $\frac{2}{3}$, etc.

Find the interest by either method,

8. Of $227.10 for 3 yr. at 7%.
9. Of $1320 for 1 yr. 5 mo. at 6%.
10. Of $62.84 for 10 mo. 12 da. at 6%.
11. Of $413.75 for 9 mo. 25 da. at 6%.
12. Of $2162.94 for 1 yr. 18 da. at 10%.
13. Of $56.80 for 4 yr. 1 mo. at 4½%.
14. Of $137.14 for 6 mo. 15 da. at 12%.
15. Of $2000 for 33 da. at 6%.
16. Of $1552.73 for 63 da. at 6%.

Find the amount,

17. Of $63.70 for 1 yr. 3 mo: 12 da. at 7%.
18. Of $475.57 for 1 yr. 7 mo. 17 da. at 6%.
19. Of $525.35 for 3 yr. 6 mo. 21 da. at 7½%.
20. Of $1140.63 for 9 mo. 16 da. at 10%.
21. Of $2500 for 93 days at 8%.

22. What is the interest of $154.75 from Apr. 10th to Nov. 24th at 6%?

23. Required, the interest of $861.50 from June 3, 1875, to March 25, 1876, at 6%?

24. Required, the amount of $1500 from May 1 to Sept. 20, at 7%?

25. If a note for $175, given Jan. 12, 1874, on interest at 10%, be paid May 16th, 1876, what amount will then be due?

26. A man borrowed $450.80 March 6th, and paid it with 6% interest, Dec. 20th. What was the amount?

27. Bought a house for $5400. Paid $2000 cash on delivery, $1500 in 9 mo. and the remainder in 1 yr. 4 mo. with 7% interest. What was the whole amount paid?

PROBLEMS IN INTEREST.

PROBLEM I.

417. Interest, time, and rate given to find the principal.

1. What sum of money will gain $20 in 2 yr. at 5%?

ANALYSIS.—The interest of $1 for 2 yr. at 5%, is $.10; hence $20 is the interest of as many dollars as $.10 are contained times in $20, or $200. Hence, etc.

What sum of money will gain

2. $49 int. in 1 yr. at 7%?
3. $50 int. in 2 yr. at 5%?
4. $75 int. in 5 yr. at 5%?
5. $200 int. in 10 yr. at 10%?
6. $80 int. in 6 mo. at 8%?
7. $60 int. in 3 mo. at 8%?

WRITTEN EXERCISES.

1. What sum of money put at interest 2½ yr. at 7%, will gain $612.50?

OPERATION.

Interest of $1 for 2½ yr. at 7% = $.175.

$612.50 ÷ $.175 = 3500 times. $1 × 3500 = $3500, Prin.

What *principal*

2. Will gain $75.60 in 2 yr. 3 mo. at 6%? At 8%?
3. Will gain $350 in 2 yr. 6 mo. at 7%? At 10%?

RULE.—*Divide the given interest by the interest of* $1 *for the given time, at the given rate.*

4. What sum invested at 7% will yield an annual income of $406?
5. What sum will gain $507 in 2 yr. 9 mo. 24 da. at 6%?
6. What sum will gain $154 int. in 4 mo. 24 da. at 7%?
7. What sum must be invested at 8%, to produce an annual income of $1560?

PROBLEM II.

418. Amount, rate, and time given to find the principal.

1. What sum of money will amount to $242 in 3 yr. at 7%?

ANALYSIS.—The interest of $1 for 3 yr. at 7%, is $.21, and the amount is $1.21; hence $242 is the amount of as many dollars as $1.21 are contained times in $242, or $200. Hence, etc.

What sum of money will amount to

2. $44 in 2 yr. at 5%?	5. $36 in 10 yr. at 10%?
3. $128 in 4 yr. at 7%?	6. $50 in 5 yr. at 5%?
4. $206 in 6 mo. at 6%?	7. $204 in 3 mo. at 8%?

WRITTEN EXERCISES.

1. What principal will amount to $476.70 in 2 yr. 3 mo. at 6%?

OPERATION.

Amount of $1 for 2 yr. 3 mo. at 6% = $1.135.
$476.70 ÷ $1.135 = 420 times. $1 × 420 = $420, Principal.

What *Principal*

2. Will amount to $544.50 in 3 yr. 6 mo. at 6%?

3. Will amount to $435.70 in 1 yr. 3 mo. 2 da. at 7%?

RULE.—*Divide the given amount by the amount of* $1, *for the given time, at the given rate.*

4. What principal will amount to $360 in 5 yr. 6 mo., at 8%?

5. What is the interest of that sum, which in 3 yr. 7 mo. 18 da., at 6%, amounts to $791.70?

6. A note which had run 4 yr. 6 mo. amounted to $2000, at 7%. What was the principal?

PROBLEM III.

419. Principal, interest, and time given, to find the rate.

1. At what rate will \$300 gain \$42 in 2 yr.?

ANALYSIS.—Since the interest of \$300 for 2 yr. at 1%, is \$6, \$42 is as many per cent as \$6 are contained times in \$42, or 7%.

At what *rate* will

2. \$200 gain \$28 in 2 yr.?
3. \$600 gain \$90 in 3 yr.?
4. \$500 gain \$200 in 4 yr.?
5. \$10 gain \$1.50 in 3 yr.?
6. \$80 gain \$40 in 5 yr.?
7. \$25 double itself in 10 yr.?

8. At what rate per annum will any sum double itself in 4, 6, 8, and 10 years, respectively?

ANALYSIS.—At 1%, any sum will double itself in 100 years; hence to double itself in 4 yr., the rate must be as many times 1% as 4 yr. are contained times in 100 yr., or 25%.

WRITTEN EXERCISES.

1. At what rate will \$1500 gain \$350 in 3 yr. 4 mo.?

OPERATION.

Interest of \$1500 at 1% for $3\frac{1}{3}$ yr. = \$50.
\$350 ÷ \$50 = 7 times. 1% × 7 = 7%, required rate.

2. At what *rate* will \$350 gain \$70 in 2 yr. 6 mo.?

3. At what *rate* will \$550 gain \$220 in 5 yr. 4 mo.?

RULE.—*Divide the given interest by the interest of the given principal, for the given time, at 1%.*

4. At what rate will \$1000 amount to \$1250 in $2\frac{1}{2}$ yr.?

5. If I receive \$206.25 each quarter for rent of a house that cost me \$15000, what rate do I receive on the investment?

6. At what rate will any sum double itself in 16⅔ yr.? In 12½ yr.? In 8 yr. 4 mo.? In 6 yr. 3 mo.?

7. An investment doubled itself in 7½ years. What per cent did it pay annually?

PROBLEM IV.

420. Principal, interest, and rate given, to find the time.

1. In what time will $500 gain $100 at 5%?

ANALYSIS.—The interest of $500 at 5% for 1 yr. is $25; hence, the time is as many years as $25 is contained times in $100, or 4 yr.

In what *time* will

2. $60 gain $36 at 6%?
3. $200 gain $50 at 5%?
4. $500 gain $150 at 10%?
5. $10 gain $4 at 8%?
6. $50 gain $25 at 6%?
7. $70 gain $9.80 at 7%?

8. How long will it take any sum of money to double itself at 4%, 5%, 6%, 8%, and 10%, respectively?

ANALYSIS.—At 100%, any sum of money will double itself in 1 yr.; hence, to double itself at 4%, it will require as many years as 4% is contained times in 100%, or 25 yr.

WRITTEN EXERCISES.

1. In what time will $450 gain $78.75 at 7%?

OPERATION.

Interest of $450 at 7% for 1 yr. = $31.50.

$78.75 ÷ $31.50 = 2½ times. 1 yr. × 2½ = 2 yr. 6 mo., the time.

2. In what time will $350 gain $85.75 at 7%?

3. In what time will $245 gain $2.94 at 6%?

4. In what time will $600 gain $132 at 8%?

Rule.—*Divide the given interest by the interest of the given principal, at the given rate, for* 1 *year.*

5. In what time will $550 amount to $576.95 at 7%?

6. In what time will $1320 amount to $1578.94 at 5½%?

7. In what time will $1500 amount to $1718.75 at 6¼%?

8. In what time will the interest of $240 at 8%, equal the principal? Equal twice the principal? Equal one-half the principal?

COMPOUND INTEREST.

421. ***Compound Interest*** is interest not only on the principal, but on the interest added to the principal when it becomes due.

Interest is generally compounded annually, semi-annually, or quarterly, according to the time at which the interest becomes due.

1. What is the compound interest of $600 in 3 yr. at 6%?

OPERATION.

$600	Prin. for 1st yr.
1.06	
$636	Prin. for 2d yr.
1.06	
$674.16	Prin. for 3d yr.
1.06	
$714.61	Total amount.
600	
$114.61	Compound int.

Analysis.—Since the amount is 1.06 of the principal, the amount for the 1st year is 1.06 of $600, or $636, which is the principal for the second year; 1.06 of this principal, or $674.16, is the principal for the 3d year, and 1.06 of this, or $714.61, is the total amount. Hence, by subtracting the given principal, $600, the result is the compound interest, $114.61.

What is the *compound interest* of

2. $500 for 2 yr. at 5%?
3. $750 for 3 yr. at 6%?
4. $800 for 4 yr. at 7%?
5. $640 for 4 yr. at 5%?

RULE.—I. *Find the amount of the given principal at the given rate, for the first period of time at the end of which interest is due, and make that amount the principal for the second period.*

II. *Find the amount of this principal for the next period; and so continue till the end of the given time.*

III. *Subtract the given principal from the last amount, and the remainder will be the compound interest.*

When the time contains months and days, less than a single period, find the amount up to the end of the last period, and compute the simple interest upon that amount for the remaining months and days, which add to find the total amount.

6. What will $1000 amount to in 5 years at 7%?

7. What is the compound interest of $400 for 1 yr. 6 mo., at 7%, interest being payable semi-annually?

8. Find the compound interest of $2000 for 1 yr. at 8%, interest payable quarterly.

9. What will $900 amount to in 2 yr. 6 mo. 12 da. at 6%, interest payable annually?

PARTIAL PAYMENTS.

422. ***Partial Payments*** are payments in part of a note, bond, or other obligation.

423. ***Indorsements*** are the acknowledgment of such payments, written on the back of the note, bond, etc., stating the time and amount of the same.

424. ***A Promissory Note*** is a written promise to pay a certain sum of money, on demand, or at a specified time.

425. ***The Face*** of a note is the sum of money made payable by the note.

426. ***A Negotiable Note*** is one made payable to bearer, or to the order of any person.

It is said to be *negotiable* because it can be transferred from one person to another, the person who makes or signs a note is called the *Maker* or *Drawer;* the person to whom, or to whose order, it is payable is called the *Payee;* and the person who endorses it, and thus makes himself responsible for its payment, is called the *Indorser*.

The ***Form*** of a negotiable note is as follows :

$600. NEW YORK, July 5, 1874.

Nine months after date, I promise to pay Alfred Tobias, or order, six hundred dollars, for value received, with interest.

WILLIAM CARSWELL.

1. On the above note, the following payments were endorsed : Aug. 5, 1874, $25 ; Dec. 25, 1874, $10 ; Jan. 10, 1875, $50. What was due April 5, 1875 ?

ANALYSIS.—The interest of $600 for 1 mo., from July 5 to Aug. 5, at 7%, is $3.50 ; amount, $603.50, from which deduct the first payment, $25, and there will remain $578.50, a new principal. The interest of $578.50 for 4 mo. 20 da., is $15.748 ; now, since the payment, $10, is less than the interest, it cannot be deducted from the amount without compounding the interest, which is not allowed by law ; hence find the interest of $578.50 to the time of the next payment, that is, from Dec. 25, 1874, to Jan. 10, 1875,—16 days. This is $1.683, and the whole amount to that time, $595.93, from which deduct the sum of the payments, or $60, leaving $535.93, a new principal, the amount of which to April 5, 1875, or for 2 mo. 25 da., is $544.787, the sum due at that time.

427. PRINCIPLE.—*The principal must not be increased by the addition of interest due at the time of any payment, so as to compound the interest.*

Upon this is based the rule prescribed by the Supreme Court of the U. S., and adopted by most of the States.

UNITED STATES RULE.

I. *Find the amount of the given principal to the time of the first payment, and if this payment equals or exceeds the interest then due, subtract it from the amount obtained, and treat the remainder as a new principal.*

II. *If the interest exceed the payment, find the amount of the same principal to a time when the sum of the payments equals or exceeds the interest then due, and subtract the sum of the payments from that amount.*

III. *Proceed in the same manner with the remaining payments.*

$1000. RICHMOND, Oct. 1, 1874.

2. Two years after date, for value received, I promise to pay Smith & Blackwell, or order, one thousand dollars, with interest.

JOHN CROMWELL.

Indorsed as follows: Dec. 1, 1874, $75; April 1, 1875, $10; Nov. 15, 1875, $150; Jan. 1, 1876, $80. What was due Oct. 1, 1876?

3. A note for $2500, dated March 1, 1873, bore the following indorsements: Received, April 15, 1873, $150; June 1, 1873, $25; Dec. 10, 1873, $50; Feb. 1, 1874, $275. What was due May 18, 1874, interest being computed at 7%?

4. On a note for $1800, dated July 1, 1874, the following payments were indorsed: Sept. 18, 1874, $500; April 1, 1875, $56; Oct. 1, 1875, $15. What was due Dec. 13, 1875, interest at 7%?

DISCOUNT.

428. ***Discount*** is an allowance, or abatement, on any sum of money to be paid, such as the amount of a bill, a debt, or promissory note.

429. ***Commercial Discount*** is a certain per cent deducted from the price of goods, or from the amount of a bill or debt, for prompt payment.

Thus, a *wholesale* purchaser obtains goods at a discount of a certain per cent from the retail price; and a discount is allowed to a *cash* purchaser on the price of goods usually sold on credit.

430. Discount in which time is considered, is of two kinds: *True Discount* and *Bank Discount.*

TRUE DISCOUNT.

431. ***The Present Worth*** of a debt payable at a future time, without interest, is such a sum as, being put at legal interest, will amount to the debt when it becomes due.

432. ***True Discount*** is the difference between the whole debt and the present worth. It is, therefore, the interest on the present worth for the given time.

Hence, the Present Worth is a *principal,* of which the True Discount is the *interest,* and the Debt the *amount.*

WRITTEN EXERCISES.

Find the *Present Worth* and *Discount*,

1. Of \$465.75, due in 6 mo., at 7%.

OPERATION.

Amount of \$1 for 6 mo. at 7% = \$1.035.

\$465.75 ÷ \$1.035 = 450. Hence, \$450 is the *Present Worth*, and \$465.75—\$450=\$15.75, the *True Discount.*

2. Of \$614.40, due in 4 mo. 24 da., at 6%.
3. Of \$2640, due in 9 mo. 18 da., at 7%.
4. Of \$475, due in 7 mo. 6 da., at 5%.

RULE.—I. *Divide the whole debt by the amount of* \$1 *for the given time, at the given rate, and the quotient will give the present worth.*

II. *Subtract the present worth from the debt, and the remainder will be the true discount.*

5. Bought goods, amounting to \$1252.35, on 6 mo. credit. What should be the cash price, if money is worth 7%?

6. A person bought a farm of 125 A. 20 P., at \$37.50 an acre. One-half of the purchase money he paid in cash, and the balance in 9 mo. What was the cost of the farm in ready money, reckoning discount at 8%?

7. A merchant bought goods for \$1500, on 6 months' credit, and sold the whole at an advance of 20%, one-half on 3 months' credit, and the remainder on 9 months' credit. What was his gain, computing discount at 6%?

8. If I sell goods for \$1250 on 6 months' credit, which cost me \$1195 cash, what is my gain, money being worth to me 10%?

BANK DISCOUNT.

433. *Bank Discount* is the deduction made by a bank for the payment of a note before it is due.

This deduction is the interest computed on the *face of the note,* for the time specified in the note, and *three days* more, called the *Days of Grace.*

These three days are allowed by law for the payment of a note after the expiration of the time specified in the note. At the end of the days of grace, a note is said to be at *maturity,* and if not paid may be *protested;* that is, its non-payment formally stated by an officer, called a *Notary Public.*

434. *The Proceeds* of a note is the sum received for it when discounted.

435. The computation of bank discount is the same as that of *simple interest;* but since it is the same as interest paid in advance, it is in fact *compound interest.*

Thus, in discounting a note of $500 payable in 1 yr. at 7%, the bank deducts $35, 1 year's interest, and pays as the present worth of the note, $465. Hence, it receives not only the interest of $465 for 1 year, which is $32.55, but the interest of the interest, $35, which is $2.45. Therefore $35, equal to $32.55 + $2.45, is really *compound interest.*

WRITTEN EXERCISES.

Find the *Bank Discount,*

1. On a note at 90 days for $517, at 7%.

OPERATION.

90 da. + 3 da. (*of grace*) = 93 da.

Int. of $517 for 93 da. at 7% = $9.35, *Bank Discount.*

$517 − $9.35 = $507.65, *Proceeds of the Note.*

2. Find the bank discount on a note at 6 mo., for $672.50, at 6%.

3. What will be the proceeds of a note at 3 months for $1500, discounted at a bank at 7%?

4. A note for $785, dated June 1, 1875, at 90 da., was discounted by a bank in Philadelphia at 6%, July 5, 1875. Find its proceeds.

5. Find the difference between the true discount and the bank discount of $1000, due in 5 mo. 24 da., at 7%?

436. Find the *Face of a Note*,

1. Payable in 90 da., that when discounted at 6%, the proceeds may be $236.28.

OPERATION.

Int. of $1 for 93 da. at 6% = $.0155.

Proceeds of $1 = $1 − $.0155 = $.9845.

$236.28 ÷ $.9845 = 240. Hence $240 is the *Face* of the *Note*.

ANALYSIS.—Since $.984½ is the proceeds of $1, $236.28 is the proceeds of as many dollars as $.984½ is contained times in that sum, or $240.

2. What must be the face of a note payable in 8 mo. 9 da., to realize $2844, discounted at 7%?

3. Find the face of a 30-day note, which, when discounted at 8%, will yield $357.36?

4. If I owe a man $1500, and I pay him by a 60-day note, what must be the face of the note, to be discounted at 7%?

5. For what sum must a merchant draw his note at 3 mo. to pay for 175¾ yd. of cloth at $4.37½ a yard; discount at 7%?

INSURANCE.

437. ***Insurance*** is a contract of indemnity against loss or damage.

438. ***Fire Insurance*** is indemnity for loss of property by fire.

439. ***Marine*** and ***Inland Insurance*** is indemnity for loss of vessel or cargo by casualties of navigation on the ocean, or on inland waters.

440. ***The Policy*** is the written contract between the parties.

441. ***The Premium*** is the sum paid for insurance, and is a certain per cent of the sum insured.

442. The corresponding terms are the following:

The Base is the amount of insurance.

The Rate is the per cent of premium.

The Percentage is the premium.

WRITTEN EXERCISES.

443. Find the *Premium,*

1. For insuring a house for $6500 at 1½%.

OPERATION.—$3500 × .015 = $97.50. **(371.)**

Find the premium for insuring

2. A house and furniture valued at $9700, at ¾%.

3. Dry goods valued at $3840, at ⅞%.

4. A cargo of 5000 bu. of grain valued at $1.20 a bushel, at 2¼% on ⅔ of its value.

444. Find the *rate of Insurance,*

1. If $49.25 is paid for an insurance of $3250.

OPERATION.—$49.25 ÷ $3250 = .015, or 1½%. **(373.)**

What is the rate of insurance,

2. If $172.20 is paid for an insurance of $9840?

3. If $38.25 is paid for an insurance of $6120?

4. If $28.40 is paid for an insurance of $3550?

5. A house valued at $6800 was insured for ½ its value, at a cost of $46.75. What was the rate of insurance?

445. Find the *amount of insurance,*

1. If it cost $38.25 to insure a certain house and furniture, at ⅝%.

OPERATION.—$38.25 ÷ .625 = $6120. (**375.**)

2. A merchant paid $1500 premium on a cargo of cotton from New York to Belgium, which was 2½% on its valuation. What was the amount insured?

3. A steam-mill was insured at 4½%, to cover ¾ of its value. The premium paid was $526.50: what was the value of the mill?

QUESTIONS FOR REVIEW.

446. 1. Define Interest. The Principal. The Rate of interest. The Amount. Legal Interest. Principle. Rule for finding the interest and amount of any sum for years and months.

2. To find the interest for any time, at any rate per cent. Obvious relations. Rule. Six Per Cent Method. Principles 1, 2. Illustrate. Rule. How to find the interest at any other rate.

3. State Problem I in interest. Rule. Prob. II. Rule. Prob. III. Rule. Prob. IV. Rule. Define Compound Interest. Rule. Partial Payments. Rule. Indorsements. A Promissory Note. The Face of a Note. A Negotiable Note. Form. Principle. U. S. Rule.

4. Define Discount. Commercial Discount. Present Worth. True Discount. Rule. Bank Discount. Days of Grace. The Proceeds.

5. Define Insurance. Fire. Marine and Inland. Policy. Premium. State the corresponding terms.

REVIEW OF PERCENTAGE.

ORAL EXAMPLES.

447. 1. What is $\frac{3}{4}$% of 28 bbl. of flour? $\frac{4}{5}$% of $60?

2. If I lose $8\frac{1}{3}$% of $120, what part of it do I lose?

3. If a man save $\frac{3}{8}$ of his income, what % does he spend?

4. From a box of tea containing 45 lb., 15 lb. were sold at one time, and 12 lb. at another. What per cent of the whole remained?

5. What % of 24 is 4? Of 50 is $12\frac{1}{2}$? Of 100 is 40?

6. Bought 10 bbl. of flour for $80, and sold 8 bbl. for what they all cost. What was the gain per cent?

7. A man sold a watch for $24, thereby gaining $\frac{1}{5}$ of the cost. How much would he have gained per cent had he sold it for $30?

8. Thirty is 25% more, and 25% less, than what numbers?

9. $\frac{2}{3}$ of $12 is what per cent of $\frac{2}{5}$ of $60?

10. $\frac{1}{2}$ of an hour is what per cent of $\frac{2}{3}$ of an hour?

11. An auctioneer received $25 for selling a pair of horses and a carriage, which was 5% of their value. What was their value?

12. What per cent does a grocer make, by selling coffee at $\frac{4}{3}$ of its cost?

13. What per cent does a merchant lose by selling cloth at $\frac{4}{5}$ of its cost?

14. If a house bring $\frac{1}{2}$ of its value every 4 years in rent, what is the gain per cent each year?

15. A broker exchanged $240 at $\frac{1}{4}$%. What brokerage did he receive?

16. What part of any sum equals the interest of it for 5 yr. 4 mo. at 9%?

17. A harness was sold for ¾ of ⅘ of what it cost. What was the loss per cent?

18. If a milkman adds 1 qt. of water to every gallon of milk he sells at cost, what is his gain %?

19. A jeweler sold two gold pencils for $12 each. On one he gained 20%, on the other he lost 20%. Did he gain or lose on both, and how much?

20. If a yearly rent of $300 is 10% of the cost of a store, what did it cost?

21. How much wheat must be taken to the mill, that 12 bu. may be retained after giving toll of 6 lb. per bushel, or 10%, for grinding?

22. At what per cent will $90 gain $27 in 6 yr.?

23. In what time will $300 gain $84 at 7%?

24. What sum in 1 yr. 8 mo. will gain $36 int., at 9%?

25. Gave a note of $200 for 2 yr.: what was the amount at 5½%?

26. An article bought at ⅘ the market price, is sold for 5% more than the market price. What % is gained?

27. What is the annual premium of insurance, at ¾%, on a house valued at $4000?

28. How many barrels of apples at $2 a barrel, can an agent buy for $1250, and retain 2½% commission?

29. At 2%, what amount of insurance can I obtain on a shipment of goods, for $36 premium?

30. What per cent is gained in buying corn at $.60, and selling it at $.72 a bushel?

31. If ⅔ of the price received for an article is equal to ¾ of its cost, what is the gain per cent?

32. The interest on a certain sum for 4 yr. was ½ the principal. What was the per cent?

GENERAL REVIEW.

WRITTEN EXAMPLES.

448. 1. If ¾ of a hundred pounds of sugar cost $7⅛, how much can be bought for $49.87½ ?

2. How many bushels of oats worth $⅗ a bushel, will pay for ¾ of a barrel of flour at $8.40 a barrel ?

3. What cost 5 T. 14 cwt. 75 lb. of hay, at $16½ per ton ?

4. Bought a hhd. of sugar, containing 9 cwt. 44 lb., for $84.96 ; paid $4.72 for freight. At what price per pound must it be sold to gain 20% on the cost ?

5. A piece of property valued at $3768, rents for 1 yr. 10 mo. for $656.26. What per cent does it pay ?

6. What must be paid to enclose a farm ½ mile square, with a fence, at a cost of $⅞ a rod ?

7. A farmer sold 125 bu. of corn at $.62½ a bushel, and 2½ tons of hay at $18½ a ton. He received in payment 96 lb. of sugar at 8⅓ cents a pound, 28 lb. of tea at $⅝ a pound, 3 bbl. of flour at $7.87½, and the remainder in cash. What amount of cash did he receive ?

8. What is the value of a piece of land 50 rd. long and 40 rd. wide, at $112½ an acre ?

9. A man bought 20 head of young cattle at $16.50 a head, and gave his note payable in 8 mo. at 8%. At the end of that time he sold the cattle at $18.75 a head. How much did he get for keeping them ?

10. A grocer purchases 9 bbl. of sugar, each weighing 1 cwt. 84 lb., at $8.36 per cwt., and retails it at 9½ cents a pound. What was his whole gain ?

11. How many yards of cloth ¾ of a yard wide, will line 20½ yd., 1¼ yd. wide ?

12. Bought 2500 bu. of wheat in Chicago, at $1.40 a bushel; paid 4½% insurance, 3% freight, and 2½% commission for selling. At what price per bushel must it be sold in New York to realize 40% net profit on the purchase?

13. How many square feet of boards will make a box and cover, 14 ft. long, 6 ft. wide, and 4 ft. high?

14. Which is the more profitable, and how much, to borrow $320 at 7% interest to pay the rent of a store in advance, or to pay $350 at the end of the year?

15. What will it cost to plaster the walls and ceiling of a room 18 ft. long, 15½ ft. wide, and 9½ ft. high, at 6¼ cents a square foot, allowing 147 sq. ft. for doors, windows, and base?

16. What will be the cost of a pile of wood 30 ft. long, 10 ft. wide, and 7 ft. 6 in. high, at $3.60 a cord?

17. What amount of money must I invest at 6% to receive an annual income of $1290?

18. Bought tea at $.90 a pound. What must I ask per pound to abate 10% and still make 20%?

19. How many yards of carpeting 27 in. wide will cover a floor 48 ft. long by 33 ft. 9 in. wide; and what will be the cost at $1.62½ a yard?

20. How many pounds of coffee at 24 cents a pound, can be bought with 12 bu. 54 lb. of wheat, worth $1.60 a bu.?

21. A shoe dealer in 9 mo. gained $2640. What amount of goods did he sell, allowing 20% profit

22. How many cubic feet of stone in the walls of a cellar 37 ft. long, 18 ft. wide, 6 ft. high, and 1½ ft. thick?

23. Which is the more advantageous, to buy flour at $6½ a barrel on 6 mo., or at $6.75 a barrel on 9 mo., money being worth 8%?

24. What is the present value of a debt of $248.80, due in 4 yr. 2 mo., at 8%?

25. Bought cloth at $5 a yard; what must be the asking price, that 25% may be abated, and a profit of 20% still be made on the cost?

26. A bin 20 ft. long, 8 ft. wide, and 3 ft. 6 in. deep, is full of wheat. What is its value at $1.28 a bushel?

27. A tank 4 ft. long, 3¾ ft. wide, and 1⅔ ft. deep, is full of water. Find the number of gallons, and its weight?

28. May 10, 1874, I borrowed $6840, with which I purchased flour at $5.70 a barrel. June 21, 1875, I sold the flour for $6.62½ a barrel, cash. How much did I gain by the transaction, interest being reckoned at 6%?

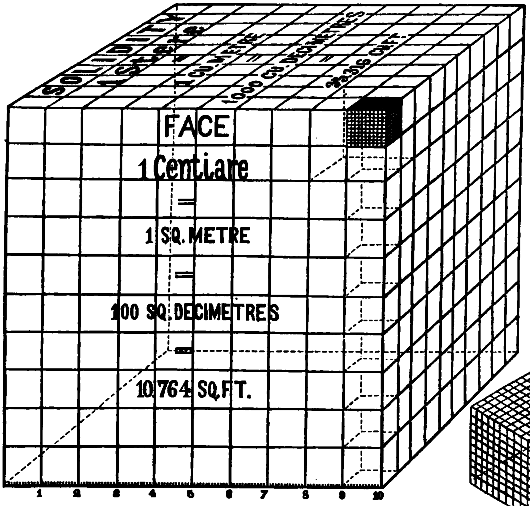
29. How much should be discounted for the present payment of a note for $430, due in 1 yr. 6 mo. 1 da., money being worth 5½%?

30. A floor 52 ft. long by 35 ft. wide is to be covered with carpeting 2 ft. 4 in. wide. Find the length, and the cost, at $1¾ a yard?

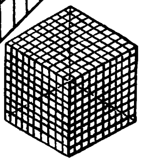
31. What will be the cost of digging a cellar 41 ft. 3 in. long, 33 ft. wide, and 8 ft. deep, at $.48 a cu. yard?

32. Bought goods for $1370 on 3 months' credit; sold the same at an advance of 25% of the cash value, and took in payment $350 cash, and the balance in two notes of equal amount, one at 3 mo., and the other at 6 mo. Reckoning discount at 6%, what is my net gain?

☞ Pupils desiring to take up the other subjects of Percentage, and of Higher Arithmetic, will find them fully treated in the "SECOND PART" of the Complete Arithmetic of the "*Shorter Course*," which is bound up in a separate volume, and furnishes a very thorough and complete *sequel* to this book.



The three dimensions of this cube are, 1 Meter, or 10 Decimeters, or 100 Centimeters, in length.
SCALE $\frac{1}{10}$ OF THE *Exact size.*



1 Cu. Centimeter.
Exact size.

METRIC SYSTEM.

449. ***The Metric System*** of weights and measures is based upon the *decimal scale.*

450. ***The Meter*** is the *base* of the system, and is the *one ten-millionth* part of the distance on the earth's surface from the equator to either pole, or 39.37079 inches.

451. From the *Meter* are made the *Are* (air), the *Stere* (stair), the *Liter* (leeter), and the *Gram ;* these constitute the *primary* or *principal* units of the system from which all the others are derived.

452. ***The Multiple Units,*** or higher denominations, are named by prefixing to the name of the *primary* units the Greek numerals, *Deka* (10), *Hecto* (100), *Kilo* (1000), and *Myra* (10000).

453. ***The Sub-multiple Units,*** or lower denominations, are named by prefixing to the names of the *primary* units the Latin numerals, *Deci* ($\frac{1}{10}$), *Centi* ($\frac{1}{100}$), *Mille* ($\frac{1}{1000}$).

Hence, it is apparent from the *name* of a unit, whether it is *greater* or *less* than the standard unit, and also *how many times.*

MEASURES OF EXTENSION.

454. ***The Meter*** is the *unit of length,* and is equal to 39.37 inches nearly.

TABLE.

Metric Denominations.					U. S. Value.
			1 Millimeter	=	.03937079 in.
10 Millimeters,	*mm.*	=	1 Centimeter	=	.3937079 in.
10 Centimeters,	*cm.*	=	1 Decimeter	=	3.937079 in.
10 Decimeters,	*dm.*	=	1 ***Meter***	=	39.37079 in.
10 METERS	*M.*	=	1 Dekameter	=	32.808992 ft.
10 Dekameters,	*Dm.*	=	1 Hectometer	=	19.927817 rd.
10 Hectometers,	*Hm.*	=	1 Kilometer	=	.6213824 mi.
10 Kilometers,	*Km.*	=	1 Myriameter (*Mm.*)	=	6.213824 mi.

The ***Meter,*** like our yard, is used in measuring cloths and short distances.

The ***Kilometer*** is commonly used for measuring long distances, and is about $\frac{5}{8}$ of a common mile.

455. *The Are* is the *unit of land measure,* and is a square whose side is 10 meters, equal to a *square dekameter,* or 119.6 sq. yards.

TABLE.

1 Centiare, *ca.* = (1 Sq. Meter) = 1.196034 sq. yd.
100 Centiares, " = 1 ***Are*** = 119.6034 sq. yd.
100 ARES *A.* = 1 Hectare (*Ha.*) = 2.47114 acres.

456. *The Square Meter* is the *unit* for measuring ordinary surfaces; as flooring, ceilings, etc.

TABLE.

100 Sq. Millimeters, *sq.mm.* = 1 Sq. Centimeter = .155 + sq.in.
100 Sq. Centimeters, *sq. cm.* = 1 Sq. Decimeter = 15.5 + sq. in.
100 Sq. Decimeters, *sq. dm.* = 1 ***Sq. Meter*** (*sq.M.*) = 1.196 + sq.yd.

457. *The Stere* is the *unit* of wood or solid measure, and is equal to a *cubic meter,* or .2759 cord.

TABLE.

1 Decistere = 3.531 + cu. ft.
10 Decisteres, *dst.* = 1 ***Stere*** = 35.316 + cu. ft.
10 STERES *St.* = 1 Dekastere (*DSt.*) = 13.079 + cu. yd.

458. *The Cubic Meter* is the *unit* for measuring ordinary solids; as excavations, embankments, etc.

TABLE.

1000 Cu. Millimeters, *cu. mm.* = 1 Cu. Centimeter = .061 + cu. in.
1000 Cu. Centimeters, *cu. cm.* = 1 Cu. Decimeter = 61.026 + " "
1000 Cu. Decimeters, *cu. dm.* = 1 ***Cu. Meter*** = 35.316 + cu. ft.

MEASURES OF CAPACITY.

459. ***The Liter*** is the *unit of capacity,* both of Liquid and of Dry Measures, and is a vessel whose volume is equal to a cube whose edge is *one-tenth* of a *meter,* equal to 1.05673 qt. Liquid Measure, and .9081 qt. Dry Measure.

TABLE.

10 Milliliters, *ml.* . . . . = 1 Centiliter.
10 Centiliters, *cl.* . . . . = 1 Deciliter.
10 Deciliters, *dl.* . . . . = 1 ***Liter.***
10 LITERS *L.* . . . . = 1 Dekaliter.
10 Dekaliters, *Dl.* . . . . = 1 Hectoliter.
10 Hectoliters, *Hl.* . . . . = 1 Kiloliter, or Stere.
10 Kiloliters, *Kl.* . . . . = 1 Myrialiter (*Ml.*).

The Hectoliter is the *unit* in measuring liquids, grain, fruit and roots in large quantities.

460. EQUIVALENTS IN UNITED STATES MEASURES.

Metric Denominations.	Cubic Measure.	Dry Measure.	Wine Measure.
1 Myrialiter	= 10 cubic meters	= 283.72 + bu.	= 2641.4 + gal.
1 Kiloliter	= 1 cubic meter	= 28.372 + bu.	= 264.17 gal.
1 Hectoliter	= $\frac{1}{10}$ cubic meter	= 2.8372 + bu.	= 26.417 gal.
1 Dekaliter	= 10 cu. decimeters	= 9.08 quarts	= 2.6417 gal.
1 ***Liter***	= 1 cu. decimeter	= .908 quart	= 1.0567 qt.
1 Deciliter	= $\frac{1}{10}$ cu. decimeter	= 6.1022 cu. in.	= .845 gill.
1 Centileter	= 10 cu. centimeters	= .6102 cu. in.	= .338 fluid oz.
1 Milliliter	= 1 cu. centimeter	= .061 cu. in.	= .27 fluid dr.

MEASURES OF WEIGHT.

461. ***The Gram*** is the *unit of weight,* and equal to the weight of a cube of distilled water, the edge of which is *one-hundredth* of a *meter,* equal to 15.432 Troy grains.

TABLE.

10 Milligrams,	*mg.*	=1 Centigram	= .15432+	oz.	Troy.
10 Centigrams,	*cg.*	=1 Decigram	= 1.54324+	"	"
10 Decigrams,	*dg.*	=1 ***Gram***	= 15.43248+	"	"
10 GRAMS	*G.*	=1 Dekagram	= .35273+	oz.	Avoir.
10 Dekagram	*Dg.*	=1 Hectogram	= 3.52739+	"	"
10 Hectograms,	*Hg.*	=1 Kilogram, or, ***Kilo***	= 2.20462+	lb.	"
10 Kilograms,	*Kg.*	=1 Myriagram	= 22.04621+	"	"
10 Myriagrams, or, 100 Kilogram	*Mg.*	=1 Quintal	= 220.46212+	"	"
10 Quintals, or 1000 KILOS		=1 Tonneau, or ***Ton***	=2204.62125	"	"

The Kilogram, or ***Kilo,*** is the *unit* of common weight in trade and is a trifle less than 2½ lb. Avoirdupois.

The Tonneau is used for weighing very heavy articles, and is about 204 lb. more than a common ton.

462. ***Units*** of the *Common* System may be readily changed to *units* of the *Metric* System by the aid of the following

TABLE.

1 Inch	= 2.54 centimeters.	1 Cu. inch	= 16.39 cu. centimet.
1 foot	= 30.48 centimeters.	1 Cu. foot	= 28320 cu. centimet.
1 yard	= .9144 meter.	1 Cu. yard	= .7646 cu. meter.
1 Rod	= 5.029 meters.	1 Cord	= 3.625 steres.
1 Mile	= 1.6093 kilometers.	1 Fl. ounce	= 2.958 centiliters.
1 Sq. inch	= 6.4528 sq. centimet.	1 Gallon	= 3.786 liters.
1 Sq. foot	= 929 sq. centimeters.	1 Bushel	= .3524 hectoliter.
1 Sq. yard	= .8361 sq. meter.	1 Troy gr.	= 64.8 milligrams.
1 Sq. rod	= 25.29 centiares.	1 Troy lb.	= .373 kilo.
1 Acre	= 40.47 ares.	1 Av. lb.	= .4536 kilo.
1 Sq. mile	= 259 hectares.	1 Ton	= .907 tonneau.

ROMAN NOTATION.

463. This method employs seven capital letters to represent numbers.

LETTERS.	I,	V,	X,	L,	C,	D,	M.
VALUES.	1,	5,	10,	50,	100,	500,	1000.

This system embraces the following:

464. PRINCIPLES.—1. Repeating a letter repeats its value.

Thus, XX represents 20; CCC, 300; DD, 1000.

2. When a letter is placed *after* one of greater value, its value is to be *added* to that of the greater.

Thus, VI represents 6; XV, 15; LXX, 70; DC, 600.

3. When a letter is placed *before* one of greater value, its value is *taken from* that of the greater.

Thus, IV represents 4; XL, 40; CD, 400.

4. A bar (—) placed over a letter increases its value a thousand times.

Thus, $\overline{\text{V}}$ represents 5000; $\overline{\text{L}}$, 50,000; $\overline{\text{C}}$, 100,000, etc.

465. TABLE OF ROMAN NOTATION.

I	is	1.	XI	is	11.	LX	is	60.
II	“	2.	XII	“	12.	LXXX	“	80.
III	“	3.	XIII	“	13.	XC	“	90.
IV	“	4.	XIV	“	14.	C	“	100.
V	“	5.	XV	“	15.	CC	“	200.
VI	“	6.	XIX	“	19.	D	“	500.
VII	“	7.	XX	“	20.	DC	“	600.
VIII	“	8.	XXX	“	30.	M	“	1,000.
IX	“	9.	XL	“	40.	$\overline{\text{X}}$	“	10,000.
X	“	10.	L	“	50.	$\overline{\text{C}}$	“	100,000.

MULTIPLICATION TABLE.

1	13	14	15	16	17	18	19	20	21	22	23	24
2	26	28	30	32	34	36	38	40	42	44	46	48
3	39	42	45	48	51	54	57	60	63	66	69	72
4	52	56	60	64	68	72	76	80	84	88	92	96
5	65	70	75	80	85	90	95	100	105	110	115	120
6	78	84	90	96	102	108	114	120	126	132	138	144
7	91	98	105	112	119	126	133	140	147	154	161	168
8	104	112	120	128	136	144	152	160	168	176	184	192
9	117	126	135	144	153	162	171	180	189	198	207	216
10	130	140	150	160	170	180	190	200	210	220	230	240
11	143	154	165	176	187	198	209	220	231	242	253	264
12	156	168	180	192	204	216	228	240	252	264	276	288

	13	14	15	16	17	18	19	20	21	22	23	24
13	169	182	195	208	221	234	247	260	273	286	299	312
14	182	196	210	224	238	252	266	280	294	308	322	336
15	195	210	225	240	255	270	285	300	315	330	345	360
16	208	224	240	256	272	288	304	320	336	352	368	384
17	221	238	255	272	289	306	323	340	357	374	391	408
18	234	252	270	288	306	324	342	360	378	396	414	432
19	247	266	285	304	323	342	361	380	399	418	437	456
20	260	280	300	320	340	360	380	400	420	440	460	480
21	273	294	315	336	357	378	399	420	441	462	483	504
22	286	308	330	352	374	396	418	440	462	484	506	528
23	299	322	345	368	391	414	437	460	483	506	529	552
24	312	336	360	384	408	432	456	480	504	528	552	576
25	325	350	375	400	425	450	475	500	525	550	575	600

ANSWERS.*

MULTIPLICATION.

Art. 84.

12. 22764; 13008; 26016; 19512; 16260.
13. 37092; 61820; 86548; 98912.
14. 921840; 1382760; 1843680; 1152300; 2074140.
15. $2079.
16. $17120.
17. $4350.
18. 2274 cents.
19. 16848 cents.
20. $784.
21. 47520 feet.
22. $16600.
23. $24920.

Art. 86.

2. 5850.
3. 11088.
4. 21852.
5. 25696.
6. 133336; 176194.
7. 142002; 89523.
8. 495560; 548928.
9. 465381; 538272.
10. 109300; 209856; 292924; 559616.
11. 220680; 257460; 285045; 625260.
12. 14988456; 11078424; 20093220; 3873828.
13. 11155248.
14. 182151828.
15. 148644288.
16. 1724573025.
17. 8828716566.
18. 601344 bu.
19. 29784450 feet.
20. $7364100944.
21. 1045982730 rd.
22. 1186058580 lb.
23. 12236298750; 49189432500; 273240703125.
24. 25091683200; 541358873300; 801772210400.
25. $7640136.
26. 2026402 lb.
27. $204768.
28. $44975.
29. $121552.
30. 104160 sheets.
31. $299720.
32. 203322 yards.
33. 73152.
34. 1031875.
35. 000.
36. 14357520.
37. 974191104.
38. 18899264.
39. 40805480.
40. 000.
41. 538788276.
42. 17169241664.
43. $10904.
44. $5040.
45. 65208 yards.
46. $40032.

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* The answers to all the *oral* exercises in this book, and to all the *written*, in Addition and Subtraction, with a few others of the most simple, have been omitted.

47. 500832 yards.
48. 228672 lb.
49. 616896 cents.
50. $2209788.
51. $1400.

Art. 88.

2. 136332.
3. 340144.
4. 222480.
5. 2642832.
6. 3203008.
7. 8103672.
8. $14775.
9. $14760.
10. 40986 pounds.

11. $77472.

Art. 89.

3. 4360; 43600; 436000; 4360000.
4. 140400; 1170000; 7020000; 5850000.
5. 44160000.
6. 10125000.
7. 292400000.
8. 6134280000.
9. 904500000.
10. 20163111000.

11. $9000.
12. $80000.
13. 1431000 shin.

Art. 91.

1. $480.
2. 3132 pounds.
3. 163473.
4. 145611.
5. 5314.
6. 42274.
7. $163 gain.
8. $466.
9. 994 miles.
10. $2154.
11. $7242.

DIVISION.

Art. 109.

8. 6171$\frac{3}{4}$; 6093$\frac{1}{5}$; 7126$\frac{4}{6}$.
9. 10421; 9118$\frac{3}{8}$; 14589$\frac{2}{5}$; 12157$\frac{5}{6}$; 24315$\frac{2}{3}$.
10. 53266; 23673$\frac{7}{9}$; 26633; 42612$\frac{4}{5}$; 35510$\frac{4}{6}$.
11. 36842; 20467$\frac{7}{9}$; 30701$\frac{4}{6}$; 61403$\frac{1}{3}$.
12. 108790$\frac{4}{8}$; 217581; 96702$\frac{6}{9}$; 1451054.1
13. 29066; 40692$\frac{2}{5}$; 25432$\frac{6}{8}$; 50865$\frac{2}{4}$.
14. 40053$\frac{1}{9}$; 51496$\frac{6}{7}$; 120159$\frac{1}{3}$; 45059$\frac{6}{8}$.
15. 50384$\frac{4}{8}$; 17150; 13338$\frac{8}{9}$.
16. 417911$\frac{2}{9}$; 149510$\frac{3}{6}$; 128151$\frac{6}{7}$.
17. 2089; 622; 1035.
18. 4856$\frac{2}{9}$; 934; 1020$\frac{7}{9}$.
19. 2546$\frac{3}{8}$; 704$\frac{5}{8}$; 1684$\frac{4}{18}$.
20. 14061$\frac{4}{6}$; 1118; 3909$\frac{2}{6}$.
21. $1484.
22. 205$\frac{4}{6}$ pounds
23. 3413 days.
24. 1014 feet.
25. $1460.
26. 1577 men.
27. 2096 miles.
28. 5011 acres.
29. 12515 tons.
30. 6052$\frac{2}{6}$ bushels.
31. 7106\frac{5}{6}$.

Art. 112.

3. 13; 21; 350.
4. 14; 114; 236.
5. 175$\frac{5}{15}$; 497$\frac{10}{15}$; 1220$\frac{6}{15}$.
6. 14; 145; 2974$\frac{13}{17}$.

7. 409; 578; 614.
8. 472 barrels.
9. 465 acres.
10. 1607 barrels.
11. $4732.
12. 2093 pieces.
13. 460 pounds.
14. 784 ; 720 ; 735 ; $641\frac{25}{55}$.
15. 35464; 38688; 32736 ; $30397\frac{30}{42}$.
16. $840\frac{2}{114}$; $630\frac{2}{152}$. $560\frac{2}{171}$; $469\frac{86}{204}$.
17. $5099\frac{45}{125}$; $4488\frac{124}{142}$; $2405\frac{95}{265}$; $3464\frac{44}{184}$.
18. $135.
19. 456 pounds.
20. $60.
21. 61 days.
22. $537\frac{3}{75}$.
23. $7973\frac{23}{96}$.
24. $927\frac{80}{324}$.
25. $2774\frac{193}{562}$.
26. $2946\frac{453}{703}$.
27. $34959\frac{716}{1236}$.
28. $27959\frac{333}{2004}$.
29. $1326\frac{441}{605}$.
30. $988\frac{1254}{2431}$.
31. $20282\frac{2660}{3042}$.
32. $1495\frac{6676}{7315}$.
33. $3759\frac{29556}{63041}$.
34. $285340\frac{4322}{15003}$.
35. $76360\frac{33498}{407053}$.

Art. 113.

2. $472\frac{7}{10}$; $47\frac{27}{100}$.
3. $30\frac{62}{100}$; $3\frac{62}{1000}$.
4. $137\frac{426}{1000}$; $13\frac{7426}{10000}$.
5. $2031\frac{684}{10000}$; $203\frac{10684}{100000}$.
6. 9\frac{4}{10}$.
7. 65 watches.
8. 9\frac{750}{1000}$.

Art. 114.

2. 12.
3. $113\frac{160}{420}$.
4. 315.
5. $11\frac{17670}{23000}$.
6. $104\frac{600}{1400}$.
7. $180\frac{460}{4000}$.
8. $370\frac{500}{2500}$.
9. $435\frac{180}{1800}$.
10. $21\frac{42200}{75000}$.
11. $99\frac{6034}{64000}$.
12. 150 bales.
13. 75 horses.

Art. 116.

1. 15.
2. 15.
3. 15.
4. 27.
5. 17976.
6. 149.
7. 1385.
8. 96.
9. 12933.
10. 28089.
11. 50496.
12. 93636.
13. 348.
14. $271\frac{5}{13}$.
15. $2425.
16. $31.
17. 127 days.
18. 48 tons.
19. 56 casks.
20. $1800.
21. 7172.
22. $622\frac{18}{40}$ hours.
23. $2100.
24. 296.
25. 122108.
26. $28\frac{99}{3000}$.
27. 5182.
28. $64.
29. 21124.
30. 30 tons.
31. 24904.
32. 40 cows.
33. $2.
34. 8897.
35. 33099.
36. 50017.
37. 3414.

PROPERTIES OF NUMBERS.

Art. 132.

2. 2, 3, 3, 7 ;
3, 3, 5, 5 ;
2, 7, 11.
3. 2, 2, 3, 7.
4. 2, 2, 2, 5, 5.
5. 2, 2, 2, 7, 7.
6. 2, 2, 2, 5, 11.
7. 5, 5, 5, 5.
8. 2, 2, 2, 3, 3, 7.
9. 2, 2, 2, 2, 2, 2,
11.
10. 7, 7, 11.
11. 3, 3, 3, 5, 7.
12. 3, 5, 7, 11.
13. 2, 2, 2, 2, 2, 2,
3, 3, 3.
14. 3, 3, 5, 7, 7.

Art. 138.

2. 24.
3. 18.
4. 7.
5. 13.
6. 96.
7. 22.
8. 4.
9. 9.
10. 12.
11. 3.
12. 4.
13. 8.
14. 6 cents.
15. 14 feet.

Art. 139.

2. 14.
3. 4.
4. 63.
5. 45.
6. 216.
7. 45.
8. 182.
9. 63.
10. 28.
11. 72.
12. 42.
13. 47.
14. 28.

Art. 145.

2. 36.
3. 32.
4. 1008.
5. 1188.
6. 432.
7. 126.
8. 720.
9. 450.

Art. 146.

2. 200.
3. 126.
4. 216.
5. 144.
6. 156.
7. 720.
8. 3240.
9. 1125.
10. 420.
11. 840.
12. 480.
13. 16800.
14. 720.
15. 720.
16. 216.
17. 72 marbles.
18. $2310.
19. $60.
20. 2800 acres.

Art. 149.

4. 4.
5. 13.
6. 21.
7. 6
8. 4.
9. 12.
10. $1\frac{12}{45}$.
11. 6.
12. 7.
13. 14.
14. 11.
15. 80.
16. 48.
17. 33.
18. 5.
19. $9\frac{1}{7}$.
20. 12 barrels.
21. 34 bushels.
22. 100 cents.
23. 20 suits.
24. $4\frac{2}{3}$ tons.
25. $4500.
26. 56 cents.
27. 49 men.
28. $6\frac{2}{3}$.
29. 24 gallons.
30. 3 bales.
31. 12 barrels.

FRACTIONS.

Art. 172.

2. $\frac{25}{30}$.
3. $\frac{35}{45}$.
4. $\frac{15}{40}$.
5. $\frac{28}{35}$.
6. $\frac{45}{63}$.
7. $\frac{49}{84}$.
8. $\frac{55}{75}$.
9. $\frac{90}{108}$.
10. $\frac{168}{256}$.

Art. 174.

2. $\frac{5}{6}$.
3. $\frac{3}{5}$.
4. $\frac{8}{9}$.
5. $\frac{24}{25}$.
6. $\frac{7}{9}$.
7. $\frac{2}{3}$.
8. $\frac{12}{13}$.
9. $\frac{1}{7}$.
10. $\frac{7}{8}$.
11. $\frac{1}{8}$; $\frac{7}{9}$.
12. $\frac{2}{3}$; $\frac{4}{9}$.
13. $\frac{2}{11}$; $\frac{3}{4}$.
14. $\frac{7}{8}$.

Art. 175.

3. $\frac{176}{4}$ bushels.
4. $\frac{47}{3}$ years.
5. $\frac{208}{7}$ weeks.
6. $\frac{384}{8}$.
7. $\frac{941}{6}$.
8. $\frac{1116}{9}$.
9. $\frac{953}{10}$.
10. $\frac{1008}{14}$.
11. $\frac{5409}{25}$.
12. $\frac{984}{13}$.
13. $\frac{4037}{18}$.
14. $\frac{7693}{25}$.
15. $\frac{15854}{90}$.
16. $\frac{11647}{120}$.
17. $\frac{1467}{17}$.
18. $\frac{9012}{20}$.
19. $\frac{20887}{80}$.
20. $\frac{7947}{45}$.
21. $\frac{35032}{115}$.
22. $\frac{186517}{152}$.
23. $\frac{74274}{30}$.
24. $\frac{97959}{75}$.
25. $\frac{160003}{64}$.
26. $\frac{961561}{240}$.

Art. 176.

2. 12.
3. $6\frac{11}{16}$ pounds.
4. $23\frac{1}{7}$.
5. 93.
6. $20\frac{7}{12}$.
7. 28.
8. $56\frac{13}{22}$.
9. $56\frac{1}{24}$.
10. 39.
11. $128\frac{3}{35}$.
12. $100\frac{19}{59}$.
13. 52.
14. $68\frac{1}{2}$.
15. $515\frac{4}{11}$.
16. $676\frac{1}{4}$.

Art. 178.

2. $\frac{14}{21}$, $\frac{18}{21}$.
3. $\frac{36}{96}$, $\frac{40}{96}$.
4. $\frac{10}{60}$, $\frac{42}{60}$.
5. $\frac{240}{360}$, $\frac{165}{360}$.
6. $\frac{18}{36}$, $\frac{24}{36}$, $\frac{30}{36}$.
7. $\frac{35}{105}$, $\frac{63}{105}$, $\frac{60}{105}$.
8. $\frac{192}{240}$, $\frac{40}{240}$, $\frac{150}{240}$.
9. $\frac{2464}{2688}$, $\frac{576}{2688}$, $\frac{840}{2688}$.
10. $\frac{108}{72}$, $\frac{54}{72}$, $\frac{56}{72}$.
11. $\frac{168}{240}$, $\frac{300}{240}$, $\frac{200}{240}$.
12. $\frac{210}{360}$, $\frac{180}{360}$, $\frac{288}{360}$, $\frac{240}{360}$.
13. $\frac{252}{378}$, $\frac{189}{378}$, $\frac{270}{378}$, $\frac{336}{378}$.
15. $\frac{12}{16}$, $\frac{10}{16}$, $\frac{9}{16}$.
16. $\frac{32}{40}$, $\frac{12}{40}$, $\frac{35}{40}$.
17. $\frac{24}{168}$, $\frac{210}{168}$, $\frac{84}{168}$.
18. $\frac{15}{20}$, $\frac{14}{20}$, $\frac{16}{20}$.
19. $\frac{35}{63}$, $\frac{54}{63}$, $\frac{42}{63}$.
20. $\frac{54}{36}$, $\frac{27}{36}$, $\frac{28}{36}$.
21. $\frac{30}{42}$, $\frac{9}{42}$, $\frac{20}{42}$.
22. $\frac{36}{60}$, $\frac{28}{60}$, $\frac{40}{60}$.
23. $\frac{52}{78}$, $\frac{24}{78}$, $\frac{75}{78}$, $\frac{8}{78}$.
24. $\frac{144}{120}$, $\frac{56}{120}$, $\frac{25}{120}$. $\frac{74}{120}$.
25. $\frac{84}{96}$, $\frac{40}{96}$, $\frac{78}{96}$, $\frac{29}{96}$.

Art. 180.

2. $1\frac{8}{15}$.
3. $1\frac{31}{36}$.
4. $1\frac{23}{24}$.
5. $18\frac{1}{12}$.
6. $38\frac{11}{20}$.
7. $33\frac{1}{24}$.
8. $66\frac{49}{72}$.
9. $2\frac{1}{40}$.
10. $1\frac{8}{9}$.
11. $2\frac{5}{21}$.
12. $2\frac{7}{8}$.
13. $7\frac{11}{12}$.
14. $45\frac{23}{24}$.
15. $6\frac{7}{16}$.
16. $103\frac{13}{15}$.
17. \$$95\frac{21}{40}$.
18. \$$622\frac{9}{14}$.
19. \$$189\frac{1}{12}$.

Art. 182.

2. $\frac{1}{12}$.
3. $\frac{7}{24}$.
4. $\frac{27}{56}$.
5. $\frac{23}{60}$.
8. $7\frac{1}{4}$.
9. $8\frac{1}{24}$.
10. $13\frac{1}{7}$.
11. $15\frac{11}{30}$.
12. $235\frac{13}{15}$.
13. $213\frac{14}{25}$.
14. $\frac{1}{2}$.
15. $2\frac{7}{12}$.
16. $\frac{1}{33}$.
17. $2\frac{28}{39}$.
18. $\frac{7}{76}$.
19. $\frac{11}{40}$.
20. $\frac{7}{30}$.
21. $\frac{7}{18}$.
22. $\frac{6}{35}$.
23. $\frac{27}{77}$.
24. $17\frac{19}{48}$.
25. $11\frac{23}{72}$.
26. $16\frac{7}{25}$.
27. $50\frac{9}{14}$.
28. $83\frac{13}{20}$.
29. $122\frac{7}{11}$.
30. $108\frac{4}{11}$.
31. $106\frac{5}{17}$.
32. $175\frac{41}{45}$.
33. $5\frac{11}{28}$.
34. \$$\frac{19}{80}$.
35. $38\frac{17}{18}$ gallons.

Art. 184.

1. $1\frac{17}{56}$.
2. $\frac{67}{80}$.
3. $\frac{4}{27}$.
4. $\frac{7}{20}$.
5. $32\frac{7}{30}$.
6. $14\frac{5}{24}$.
7. $1\frac{31}{63}$.
8. $11\frac{1}{6}$.
9. $\frac{7}{48}$.
10. \$$11\frac{15}{16}$.
11. $\frac{5}{36}$.
12. $27\frac{31}{36}$.
13. $10\frac{27}{32}$.
14. $16\frac{11}{40}$.
15. $\frac{7}{8}$.
16. $1\frac{23}{24}$.
17. $15\frac{1}{4}$.
18. $10\frac{4}{7}$.
19. $5\frac{5}{32}$.
20. $33\frac{35}{36}$.
21. $27\frac{2}{5}$.
22. $6\frac{17}{18}$.

Art. 186.

2. $1\frac{1}{2}$.
3. $2\frac{2}{5}$.
4. $4\frac{4}{15}$.
5. $1\frac{1}{6}$.
6. $4\frac{19}{64}$.
7. $4\frac{1}{4}$.
9. 54.
10. $138\frac{3}{5}$.
11. 465.
12. $399\frac{3}{8}$.
13. $298\frac{2}{7}$.
14. $1565\frac{3}{10}$.
15. 3 ; 7 ; 6 ; 15.
16. 903 ; $3264\frac{1}{4}$;
    $1881\frac{3}{5}$; $5282\frac{6}{7}$.
17. \$$21\frac{1}{4}$.
18. \$$356\frac{1}{2}$.
19. \$33.
20. $128\frac{1}{2}$; 257 ;
    $411\frac{1}{5}$ miles.
21. \$$1896\frac{7}{8}$;
    \$$3186\frac{3}{4}$;
    \$$4704\frac{1}{4}$;
    \$$5311\frac{1}{4}$.
22. \$$15850\frac{4}{5}$.

Art. 188.

2. 15.
3. 42.
4. 54.
5. $181\frac{8}{25}$.
6. $253\frac{5}{7}$.
7. $116\frac{2}{3}$.
9. $156\frac{2}{3}$.
10. 279.
11. 658.
12. 1022.
13. 658.
14. 2147.
15. $96\frac{3}{13}$.
16. $241\frac{2}{3}$.
17. $357\frac{21}{25}$.

18. $166\frac{2}{13}$.
19. $116\frac{2}{3}$.
20. $41\frac{13}{22}$.
21. $7263\frac{1}{2}$.
22. $22387\frac{1}{5}$.
23. $40762\frac{2}{7}$.
24. 147; 330; $496\frac{1}{2}$; $195\frac{3}{4}$; 675.
25. 1484; $1169\frac{7}{9}$; 1888; $3266\frac{2}{3}$; $5397\frac{21}{31}$.
26. \$112.
27. 606 miles.
28. $\$87\frac{1}{9}$.
29. $\$6\frac{3}{4}$.
30. $808\frac{1}{2}$ cents.
31. 1386 cents.

Art. 189.

2. $\frac{28}{135}$.
3. $\frac{33}{56}$.
4. $\frac{3}{8}$.
5. $\frac{1}{6}$.
6. $\frac{51}{105}$.
7. $\frac{2}{7}$.
9. $3\frac{1}{2}$.
10. 2.
11. $9\frac{5}{8}$.
12. 54.
13. $10\frac{1}{2}$.
14. $3\frac{3}{4}$.
15. $\$\frac{3}{8}$.
16. $\$1\frac{1}{2}$.
17. $\frac{4}{15}$.
18. $\frac{5}{72}$.
19. $\frac{1}{9}$.
20. $\frac{4}{21}$.
21. $\frac{34}{35}$.
22. 3.

23. $29\frac{2}{5}$.
24. $\frac{5}{11}$.
25. $\frac{1}{3}$.
26. $\frac{20}{33}$.
27. $\frac{4}{7}$.
28. $\frac{1}{21}$.
29. $5\frac{1}{4}$.
30. $\frac{2}{3}$.
31. $\frac{2}{7}$.
32. $1\frac{2}{3}$.
33. $2\frac{1}{2}$.
34. 90.
35. $\$\frac{7}{24}$.
36. $40\frac{5}{8}$ cents.
37. $328\frac{1}{8}$ cents.
38. $\$2\frac{29}{40}$.
39. $\$114\frac{3}{8}$.
40. $273\frac{3}{5}$ cents.
41. 1050 cents.
42. $172\frac{1}{3}$.
43. $67\frac{1}{2}$ cents.
44. $87\frac{1}{2}$ cents.

Art. 191.

2. $\frac{3}{7}$.
3. $\frac{4}{35}$.
4. $\frac{7}{37}$.
5. $\frac{7}{125}$.
6. $\frac{8}{165}$.
7. $\frac{37}{900}$.
9. $\frac{4}{5}$.
10. $2\frac{19}{40}$.
11. $4\frac{9}{42}$.
12. $5\frac{13}{70}$.
13. $7\frac{5}{36}$.
14. $5\frac{11}{116}$.
15. $\frac{2}{45}$.
16. $\frac{9}{40}$.
17. $\frac{1}{210}$.
18. $\frac{2}{97}$.

19. $7\frac{3}{8}$.
20. $6\frac{2}{7}$.
21. $14\frac{27}{128}$.
22. $18\frac{11}{45}$.
23. $\frac{5}{8}$.
24. $7\frac{43}{75}$.
25. $\frac{1}{3}$.
26. $3\frac{9}{22}$.
27. $2\frac{3}{32}$ rods.
28. $\$5\frac{47}{64}$.
29. $\$4\frac{3}{8}$.

Art. 192.

2. $13\frac{1}{2}$.
3. $24\frac{1}{2}$.
4. 36.
5. $51\frac{1}{3}$.
6. 100.
7. $76\frac{1}{2}$.
9. $7\frac{1}{9}$.
10. 12.
11. $8\frac{8}{25}$.
12. $17\frac{1}{2}$.
13. 6.
14. $15\frac{3}{67}$.
15. $7\frac{28}{77}$.
16. $23\frac{71}{73}$.
17. $27\frac{4}{23}$.
18. $102\frac{2}{3}$.
19. 108.
20. $302\frac{5}{7}$.
21. 220.
22. $117\frac{9}{11}$.
23. 126.
24. $342\frac{6}{7}$.
25. $430\frac{5}{9}$.
26. $45\frac{3}{13}$.
27. 288.
28. 315.
29. $16\frac{13}{29}$.

30. 30 laborers.
31. $11\frac{13}{17}$; $41\frac{2}{3}$ cd.
32. $25\frac{3}{5}$; $32\frac{1}{2}$ bu.
33. $13\frac{1}{11}$ hours.
34. 24 days.
35. $30\frac{2}{3}$ bushels.
36. $73\frac{9}{17}$ acres.
37. $21\frac{3}{7}$ days.
38. $16.
39. 24 ; $37\frac{1}{5}$ tons.
40. $9\frac{9}{11}$; $26\frac{2}{11}$; $69\frac{9}{11}$ yards.
41. $293\frac{1}{3}$ pounds.
42. 100 baskets.

Art. 193.

2. $\frac{5}{6}$.
3. $1\frac{1}{2}$.
4. $\frac{7}{10}$.
5. $\frac{20}{21}$.
6. $\frac{1}{3}$.
7. $\frac{8}{15}$.
8. $\frac{10}{27}$.
9. $\frac{14}{15}$.
10. 1.
12. $1\frac{1}{2}$.
13. $2\frac{17}{20}$.
14. $1\frac{21}{29}$.
15. $3\frac{39}{112}$.
16. $17\frac{19}{21}$.
17. $\frac{24}{35}$.
18. $\frac{3}{8}$.
19. $2\frac{8}{21}$.
20. $\frac{18}{49}$.
21. $\frac{3}{7}$.
22. $1\frac{3}{7}$.
23. $7\frac{7}{10}$.
24. $3\frac{2}{3}$.
25. $1\frac{5}{7}$.
26. $5\frac{11}{15}$.
27. 7.
28. $13\frac{17}{27}$ pounds.
29. $7\frac{13}{16}$ cords.
30. $17\frac{11}{12}$ days.
31. $\$\frac{2}{3}$.
33. $\frac{5}{8}$.
34. $\frac{2}{9}$.
35. $\frac{2}{3}$.
36. $1\frac{1}{5}$.
37. $\frac{5}{7}$.
38. $4\frac{1}{2}$; $\frac{11}{28}$; $1\frac{1}{2}$; $3\frac{1}{9}$.
39. $\frac{40}{63}$; $\frac{3}{20}$; 12 ; $5\frac{5}{11}$.
40. $17\frac{11}{15}$ days.
41. $6\frac{32}{49}$ bushels.
42. $1\frac{5}{7}$ yards.

Art. 194.

1. 64.
2. 135.
3. 289.
4. $321\frac{1}{9}$.
5. 1.
6. $1\frac{1}{9}$.
7. $193\frac{7}{9}$.
8. 385.
9. 224 sheep.
10. $23925.
11. $10\frac{10}{13}$.
12. $250\frac{5}{7}$.
13. $450.
14. $17\frac{1}{2}$ barrels.

Art. 196.

1. $\frac{5}{9}$; $\frac{110}{253}$; $\frac{5}{6}$;
2. $236\frac{7}{25}$; $2195\frac{12}{13}$; $45\frac{5}{91}$; 385.
3. $\frac{2301}{14}$; $\frac{2909}{61}$; $\frac{11657}{112}$.
4. $\frac{525}{25}$; $\frac{1700}{50}$; $\frac{1350}{18}$.
5. $\frac{30}{80}$, $\frac{48}{80}$, $\frac{130}{80}$.
6. $10\frac{5}{28}$.
7. $4\frac{8}{15}$.
8. $\frac{3}{14}$.
9. $\frac{7}{12}$.
10. 120 bushels.
11. W. $\$1\frac{2}{3}$ more.
12. $47\frac{1}{2}$ bushels.
13. $8\frac{7}{24}$ bushels.
14. $\$4\frac{1}{2}$.
15. $4200.
16. 18 gallons.
17. $11\frac{1}{4}$ yards.
18. $1\frac{1}{35}$ pounds.
19. 36 years.
20. $\$263\frac{8}{9}$.
21. $17\frac{1}{3}$ pounds.
22. $57\frac{3}{5}$.
23. $1248.
24. 60 feet.
25. $120.
26. $\$2\frac{5}{12}$.
27. $23\frac{9}{35}$.
28. $3\frac{18}{37}$.
29. $\frac{28}{55}$.
30. $33\frac{7}{27}$.
31. $16\frac{59}{80}$.
32. 189.
33. $\frac{215}{901}$.
34. $21\frac{7}{8}$.
35. 36.

DECIMALS.

Art. 217.

2. .12.
3. .009.
4. .46.
5. .025.
6. .092.
7. .0008.
8. .058.
9. .0023.
10. .0126.
11. .00032.
12. .00320.
13. .001308.
14. 45.045.
15. 10.02037.
16. 1200.0120.
17. 1000.00500.
18. .0127.
19. .4056.
20. .00132.
21. .00208.
22. 48.0175.
23. 218.003046.

Art. 228.

2. .0600, .0340.
3. .90000, .24000
4. .004100,
.070000.
5. .3260000.
6. .32016000.
7. .00600.
8. .8000, .1040,
.0031.
9. 24.0000,
2.7300,
.0062, .9000.

Art. 229.

11. 4700 cents.
12. 16300 cents.
13. 35000 mills.
14. 104000 mills.
15. 670 mills.
16. 870 mills.
17. 5600 ; 20000 ;
10700 ;
32100 cents.
18. 27000; 96000;
72000 ;
87000 mills.

Art. 230.

22. .23.
23. .102
24. .9.
25. .07.
26. 4.040.
27. 3.1204.
28. .75030.
29. 2.008000.

Art. 232.

2. $\frac{2}{5}$.
3. $\frac{4}{25}$.
4. $\frac{3}{4}$.
5. $\frac{1}{125}$.
6. $\frac{2}{25}$.
7. $\frac{9}{40}$.
8. $\frac{3}{125}$.
9. $\frac{3}{400}$.
10. $\frac{3}{16}$.
11. $\frac{1}{32}$.
12. $\frac{1}{800}$.
13. $\frac{17}{2500}$.
14. $\frac{3}{8}$.
15. $\frac{19}{40}$.
16. $\frac{7}{8}$.
17. $\frac{3}{1250}$.
18. $\frac{17}{250}$.
19. $\frac{5}{16}$.
20. $\frac{9}{16}$.
21. $\frac{1}{625}$.
22. $\frac{3}{5000}$.
23. $\frac{1}{8}$.
24. $\frac{5}{8}$.
25. $\frac{5}{16}$.
26. $8\frac{1}{4}$.
27. \$$12\frac{3}{4}$; \$$25\frac{1}{200}$;
\$$36\frac{1}{8}$.
28. $37\frac{3}{80}$; $9\frac{5}{8}$;
$42\frac{3}{16}$.

Art. 233.

3. .25.
4. .625.
5. .08.
6. .8.
7. .85.
8. .1875.
9. .7.
10. .0375.
11. .032.
12. .375.
13. .75.
14. .875.
15. .666+.
16. .12.
17. .45.
18. .5625.
19. .475.
20. .392.

21. $5.6.
22. 3.75.
23. $12.2.

Art. 235.

8. .59985.
9. $80.42.
10. 354.45 acres.
11. $358.43.
12. $115.2375.
13. $808.80.
14. 1516.65.
15. 154.6350 A.
16. $57.6475.
17. 25.6148.
18. $196.788.
19. 6.769005.

Art. 236.

10. 8.99472.
11. 5000.99998.
12. 1.9998.
13. 9.99.
14. 3.999976.
15. .0003767.
16. 29999.999997
17. 50.44.
18. .001.
19. 29.92366.
20. 24762.13.
21. 2499.75.
22. $594.
23. .426573.
24. $549.38.
25. $234.75.
26. $706.63.
27. 31.625 yards.
28. $2.82.
29. 254.785 tons.
30. $4149.125.
31. 2.6387.
32. $30.89.
33. 23.177.
34. $9.40.
35. 19.5828.
36. $62.325.

Art. 238.

2. 14.914.
3. 240.37086.
4. .0273238.
5. 5.4008.
6. $14.70.
7. 2.934.
8. 104.182.
9. .0570.
10. $3.125.
11. .8448.
12. .84217.
13. .021252.
14. .1620.
15. 4.36692.
16. .8323.
17. .74772.
18. 6.25.
19. 171.
20. 176.4.
21. .000392.
22. .05391828.
23. $303.125.
24. 1.2.
25. $203.2.
26. .077.
27. .002116.
28. $181.875.
29. 3.9375.
30. 42.
31. .0006076.
32. $1.50.
33. $4.08.
34. 380.
35. 133.56.
36. .00143.
37. 182.9625 bu.
38. $8.09375.
39. $3467.25.
40. $962.50.
41. $63.5625.
42. $78.125.
43. $8.50.
44. $83.6875.
45. $1125.
46. 495.0825 ft.
47. $547.50.
48. $7.75.
49. 47.97544.
50. 218.5605.
51. 739.280625.
52. 1409.98.
53. 40.841.
54. 1009.6.

Art. 240.

3. 1.168.
4. 4.5.
5. $3.33.
6. $7.60.
7. 4.7.
8. .4.
9. 100.
10. .01.
11. .1.
12. .2.
13. .5.
14. 1.76.
15. .31.
16. $.25.
17. .0105.
18. $2.125.
19. 110.

20. $12.48.
21. .139 ; 1.39 ;
13.9 ; 139.
22. 1.2 ; .6 ; .9 ; .8.
23. .9 ; 1.04 ;
.136875 ;
.00435 ;
.25091.
24. 1.2 ; .12 ; .012 ;
.0012.
25. 12 lots.
26. $.50.
27. $.08.
28. $1.17.
29. $31.565.
30. $.875.
31. $.113.
32. $.25.
33. $.625.
34. $.09.
35. 25.63 bu.
36. 8.125 rods.
37. 14 cocoa-nuts.
38. 1.25 tons.
39. $5.

Art. 241.

1. $\frac{1}{80}$.
2. .0023.
3. .6428 +.
4. $142.875.
5. $7\frac{625}{1000}$.
6. $7.8125.
7. 19999.980.

8. 36.
9. .00015625.
10. 10.025.
11. 9.84.
12. .23.
13. 3050.
14. .00016.
15. $5.85.
16. 28.8 bushels.
17. $10.875 ;
$14.625 ;
$23.0625
18. $8.25 ; $33 ;
$82.50.
19. $65.28.
20. .4.
21. 193.75.
22. 30.
23. 44.32.
24. .26.
25. 2.52.
26. $125.
27. 1.6432.
28. $9.25.

Art. 243.

2. $107.
3. $\$24.66\frac{2}{3}$.
4. $210.50.
5. $29.
6. $\$195.33\frac{1}{3}$;
$293 ;
$\$390.66\frac{2}{3}$.
7. $\$76.66\frac{2}{3}$.

Art. 244.

2. 204 ; 170 ;
136 pounds.
3. 726 ;
484 baskets.
4. 3072 ; 2024 ;
1536 pounds.

Art. 245.

2. $200.88.
3. $104.9325.
4. $18.4875.
5. $34.95.
6. $29.725.
7. $41.875.
8. $8.82.
9. $329.398.

Art. 246.

2. $2.86875.
3. $4.76.
4. $2.9196.
5. $14.615.
6. $10.74564.
7. $152.118.

Art. 252.

1. $197.36.
2. $973.38.
3. $728.601.
4. $124.005.
5. $117.25.
6. $102.86.

DENOMINATE NUMBERS.

Art. 339.

2. 1832d.
3. 444 pt.
4. 1152 pwt.
5. 5152d.
6. 11895 far.
7. 1590d.
8. 675 far.
9. $87.597.

10. $27.9818+.
11. 960 far.
12. 320s.
13. 4500 ct.
14. 280 fr.
15. $81.06.
16. $3.348.
17. 9464 yd.
18. 562 in.
19. 336 in.
20. 15840 ft.
21. 18000 l.
22. 56 fourths yd.
23. 84 eighths yd.
24. 148 sixteenths yd.
25. 20530¾ sq. ft.
26. 24200 sq. yd.
27. 18948 sq. in.
28. 200000 sq. l.
29. 1984 sq. rd.
30. 144 qr. sec.
31. 2128 cu. ft.
32. 216000 cu. in.
33. 693 cu. ft.
34. 42768 cu. in.
35. 603 qt.
36. 352 ft.; 584 ft.
37. 474 f ℥.
38. 1036 pt.
39. 224 pt.
40. 1412 pt.
41. 4704 gr.
42. 107 gr.
43. 9260 lb.
44. 2368 oz.
45. 1400 lb.
46. 7448 lb.
47. 3800 lb.
48. 3200 lb.
49. 1260 lb.
50. 2725 lb.
51. 1590 lb.
52. 1344 lb.
53. 1450 lb.
54. 712½ lb.
55. 465 lb.
56. 3640 lb.
57. 44640 min.
58. 8784 hr.
59. 2505600 sec.
60. 2184 hr.
61. 51600″.

Art. 341.

3. £26 4s. 4d.
4. 23 lb. 5 oz. 10 pwt.
5. 26 gal. 1 qt.
6. $.65.
7. $12.40.
8. 5 fr.
9. 5 Nap.
10. £15.
11. 11s. 7d.
12. 22 sov. 5s. 2d.
13. 21 cr.
14. 24 fl. 1s. 11d.
15. 34 yd. 1 ft. 6 in.
16. 4 mi.
17. 239 fath. 1 ft.
18. 9 rd. 13 ft. 6 in.
19. 40⅝ yd.
20. 1 mi. 4 ch. 7 l.
21. 160 A.
22. 8 sq. rd. 4 sq. ft. 112 sq. in.
23. 3 sections.
24. 65 A.
25. 29 cu. yd. 3 cu. ft.
26. 342 cd. 84 cu. ft.
27. 6 cd. ft.
28. 4 qr. sec.
29. 293 gal. 1 qt. 1 pt.
30. 6 bbl.
31. 16 gal.
32. 6 Cong. 4 O. 11f℥ 4 f ʒ.
33. 500 bu.
34. 77 pk. 2 qt.
35. 4 lb. 1 oz. 3 pwt. 5 gr.
36. 51 oz. 7 dr. 1 sc.
37. 30 lb. 7 oz. 2 pwt.
38. 5 cwt. 26 lb. 9 oz.
39. 75⅜ bu.
40. 216 bu.
41. 4½ bbl.
42. 250 bu.
43. 8 wk. 4 da.
44. 7 hr. 57 min. 15 sec.
45. 15 mo.
46. 2° 46′ 40″.
47. 19 gro.
48. 144 doz.
49. 203 score.
50. 36 rm. 15 Qr. 7 sheets.
51. 24 bun.

Art. 342.

1. $318.24.
2. $469.44.

3. $1900.80.
4. $201.60.
5. $10.92.
6. $73.50.
7. $220.50.
8. $5425.
9. $7.
10. $135.
11. $30.
12. $28.80.
13. $64.
14. $90.
15. $57.60.
16. 8 gal. 3 qt. 1 pt.
17. 2 oz. 18 pwt.
18. 112 farms.
19. \$.08$\frac{1}{2}$.
20. 2880 min.
21. 24$\frac{11}{16}$ A.
22. $2.10.
23. $40.50.
24. 12 spoons.
25. 75 bbl.
26. 220 cu. yd.
27. $2700.
28. 20 sq. yd.
29. $12 gain.
30. 1280 rd.
31. $224.
32. 288 bricks.
33. $26.
34. $101.25.
35. $54.
36. $22.70.

Art. 344.

2. 14 hr. 24 min.
3. 7 oz. 4 pwt.
4. 3$\frac{3}{5}$ qr. yd.
5. 3 gal. 3 qt. 1 pt. 2 gi.
6. 22 hr. 4 min. 48 sec.
7. 41 lb. 8.96 oz.
8. 3 wk. 1 da. 9 hr. 36 min.
9. 133 sq. rd. 10 sq. yd. 108 sq. in.
10. 1 da. 9 hr. 36 min.
11. 14 ft. 5$\frac{1}{4}$ in.
12. 3.6 qr. yd.
13. 39 gal. 1 qt. 1 pt.
14. 987$\frac{3}{7}$.
15. 2 pk. 4 qt.
16. 7s. 5d. 3.6 + far.
17. 42′ 54″.
18. 2 rd. 9 ft. 2.88 in.
19. 4 da. 14 hr. 42 min. 43.2 sec.
20. 14.4 oz.
21. 3 bun. 1 rm. 10 Qr.

Art. 345.

2. $\frac{1}{9}$ lb.
3. .4375 cd.
4. $\frac{4}{9}$ wk.
5. $\frac{3}{10}$ lb.
6. $\frac{7}{64}$ bu.
7. $\frac{1}{2}$ bbl.
8. $\frac{3}{8}$ yd.
9. 1$\frac{1}{4}$ pk.
10. $\frac{3}{10}$ da.
11. $\frac{2}{3}$ lb.
12. £$\frac{3}{40}$.
13. $\frac{9}{32}$ cd.
14. $\frac{1}{3}$ ft.
15. $\frac{27}{32}$ rd.
16. £.60625.
17. .2616 + bu.
18. .629$\frac{1}{6}$ fath.
19. .01 ton.
20. .14583 + gal.
21. .4375 wk.
22. .5 yd.
23. .3548 + lb.
24. .8333 + lb.
25. .217°.
26. .3125 rm.
27. $\frac{1}{4}$ bbl.
28. $\frac{211}{1521}$ cwt.
29. $\frac{13}{76}$ gal.
30. .27083 + lb.
31. $\frac{71}{100}$, or .71 o
32. .7 f ℥.
33. .09 pk.

Art. 346.

2. 503 A. 78 P 13 sq. yd. sq. ft. 13 sq. i
3. 64 lb. 2 oz. 1 pwt.
4. 6 ℔ 11 ℥ 1 2 ℈ 10 gr.
5. 22 cd. 3 cd. f 15 cu. ft.
6. 6 s. 12° 36′ 37
7. 101 A. 64 P.
8. 5 T. 1 cwt. 8 lb. 3 oz.
9. 9 bun. 1 rm. Qr. 18 sheet
10. 190 bu. 45 lb

Art. 347.

2. 4 hhd. 13 gal. 3 qt. 1 pt.
3. 6 wk. 6 da. 5 hr. 55 min. 14 sec.
4. 58 yd. 2 ft. 9 in.
5. 84 A. 134 P.
6. 4 T. 16 cwt. 5 lb. 12 oz.
7. 3 ℔ 9 ℥ 3 ʒ 1 ℈ 3 gr.
8. 36 cd. 3 cd. ft. 13 cu. ft.
9. 13 sq. yd. 3 sq. ft. 132 sq. in.
10. 7 rm. 15 Qr. 19 sheets.
11. 14 gal. 1 qt. 1 pt.
12. 71 lb. 8 oz.
13. 319 A. 75 P.
14. £149 16s. $2\frac{1}{4}$d.
15. 223 A. 71 P.

Art. 348.

2. 6 y. 5 mo. 17 da.
3. 3 yr. 2 mo. 24 da.
4. 57 y. 1 mo. 25 da.
6. 208 da.
7. 1 y. 7 mo. 19 da. 19 hr. 40 min.

Art. 349.

2. 67 bu. 3 pk. 5 qt. 1 pt.
3. 214 lb. 4 oz. 8 pwt. 12 gr.
4. £57 4s. 3d.
£66 14s. $11\frac{1}{2}$d.
£85 16s. $4\frac{1}{2}$d.
5. 31 hhd. 40 gal. 3 qt.
50 hhd. 40 gal.
94 hhd. 59 gal.
6. 17 rd. 1 yd. 1 ft. 6 in.
51 rd. 4 yd. 1 ft. 6 in.
7. 66 cu. yd. 26 cu. ft. 184 cu. in.
133 cu. yd. 25 cu. ft. 368 cu. in.
8. 69 ℔ 6 ℥ 2 ʒ 0 ℈ 18 gr.
1931 ℔ 0 ℥ 5 ʒ 2 ℈ 20 gr.
9. 2 da. 20 hr. 34 min.
8 da. 13 hr. 42 min.
10. 376 cd. 3 cd. ft. 8 cu. ft.
635 cd. 2 cd. ft.
11. 336 A. 144 P.
505 A. 52 P.
589 A. 88 P.
12. 403 rm. 15 Qr.
13. 27 bu.
14. $101\frac{1}{2}$ bu.
15. 1 lb. 7 oz. 12 pwt.
16. 42 T. 1750 lb.
17. 396 lb.
18. 423 A. 120 P.
19. £101.
20. 212 hhd. 24 gal.

Art. 350.

2. 10 lb. 4 oz. 15 pwt. 8 gr.
3. 4 wk. 5 da. 20 hr. 32 min. 10 sec.
5. $11\frac{3}{17}$ times.
6. 24 times.
7. 41 gal. 3 qt. 1 pt.
8. 54 yd. 2 ft. $2\frac{1}{2}$ in.
9. 20 A. 17 P. 28 sq. yd., $115\frac{4}{7}$ sq. in.
10. 49 gal. 2 qt. 1 pt.
11. 17° 1′ 48″.
12. £4 2s. 4d.
13. $10\frac{398}{559}$.
14. 1 cd. 2 cd. ft. 5 cu. ft.
15. 1 oz. 17 pwt. 4 gr.

16. 15 da.
17. 880 rails.
18. 8 cwt. 64 lb. $12\frac{4}{5}$ oz.
19. 48 farms.

Art. 351.

1. 486 sq. ft.
2. 7 ft.
3. 11 A. 7 sq. ch.
4. 54 ft.
5. $150.
6. $22\frac{1}{2}$ A.
7. $312\frac{1}{2}$ sq. ft.
8. $2568.75.
9. 32 rd.
10. \$$43.87\frac{1}{2}$.
11. $122.50.
12. 216.
13. $122\frac{1}{2}$ A.
14. $11.70.
15. 72.
16. $270.40.
17. 100 sq. ft.
18. $87.50.
19. 1800 sq. rd.
20. $487.50.
21. $28\frac{4}{5}$ yd.
22. $52.
23. 40 yd.
24. $57.
25. $14400.

Art. 352.

1. 1430 cu. ft.
2. $3\frac{3}{8}$ cu. ft.
4. $15\frac{7}{8}$ Cd.
5. 310 cu. yd.
6. $23.75.
7. 40 ft.
8. 1728.
9. 357 cu. ft.
10. $112.50.
11. 108.
12. $4085\frac{3}{8}$ cu. ft.
13. 2 Cd.
14. $998.40.
15. $356.25.

Art. 353.

1. 320 bu.
2. 192 bu.
3. 400 cu. ft.
4. $397.80.
6. $11\frac{7}{8}$ ft.
7. 20 ft.

Art. 354.

2. $374\frac{2}{77}$ gal.
3. 8 ft. $1\frac{23}{72}$ in.
4. $1640\frac{5}{8}$ lb.
5. $14.95+.
6. $149\frac{47}{77}$ gal.; 1250 lb.
7. $38\frac{29}{63}$ hhd.
8. $218\frac{31}{32}$.
9. 3383.1 cu. in
10. $20.68+.

PERCENTAGE.

Art. 375.

2. $17.25.
3. 110 bbl.
4. 91 mi.
5. 200.75 rd.
6. $128.304.
7. 82.36 bu.
8. 41.92.
9. 18.621.
10. 411 tons.
11. $4.263.
12. 100.2 lb.
13. $25.
14. 300 sheep.
15. 50 panes.
16. $2016.15 paid. $470.35 unp'd.
17. $7035.
18. 273 bbl.
19. $944.

Art. 377.

2. 20%.
3. $12\frac{1}{2}$%.
4. 5%.
5. 16%.
6. $7\frac{1}{2}$%.
7. 25%.
8. $62\frac{1}{2}$%.
9. $5\frac{1}{2}$%.
10. $16\frac{2}{3}$%.
11. $\frac{1}{2}$%.
12. 6%.
13. 75%.
14. $46\frac{2}{3}$%.
15. 60%.
16. $96\frac{1}{2}$%.

Art. 379.

2. 370 tons.
3. 462.2.
4. $720.

5. 1400 A.
6. 130 yd.
7. $300.
8. 3360 bbl.
9. 627.08\frac{1}{4}$.
10. 50 yd.
11. $6500.
12. 314.
13. 264 lb.
14. $31.375.
15. 2000 ft.
16. $2476.80.
17. $2706.

Art. 380.

2. 560.
3. 1365.74+.
5. $500.
6. $1717.27+.
7. $300.
8. $175 A.
9. 42.3.
10. 1200 bu.
11. $450.
12. $7400.
13. 57 ft.
14. 1242 in.
15. $485.
16. $325.72.
17. $400.
18. $600.
19. $2400.
20. 342$\frac{6}{7}$ A.

Art. 386.

1. $.27 gain.
2. $.875 gain.
3. $228.90.
4. $38.084+.

Art. 387.

1. $137.885.
2. $3852.
3. $1177.05.
4. $3250.

Art. 388.

1. 20% gain.
2. 25% gain.
3. 25% loss.
4. 40% loss.
5. 14$\frac{38}{133}$% gain.
6. 16%.
7. 33$\frac{1}{3}$%.

Art. 389.

1. $136.
2. $25.40.
3. $3.60.
4. $75.
5. $16.125.
6. $90.75.
7. $112 whole gain.
33$\frac{1}{3}$% gain.
8. $3.84.
9. $371.25.
10. 21$\frac{7}{8}$%.
12. $27500.
13. $42.
14. $21 loss.
15. $324.
16. $20000.
17. 13$\frac{61}{103}$%.
18. 60%.

Art. 395.

2. $116.25.
3. $76.17+.
4. $13.67+.
5. $129.68+.
6. $281.25.

Art. 396.

2. 1$\frac{1}{2}$%.
3. 2$\frac{1}{2}$%.
4. $\frac{1}{8}$%.
5. 2$\frac{1}{3}$%.
6. 1$\frac{1}{2}$%.

Art. 397.

2. $16800.
3. $5700.
4. $3400.
5. $15760.
6. $147200.
7. $3500.

Art. 398.

2. $4400.
3. $6480.
4. $9860.
5. 100 shares.
6. 256 bbl.
7. 3600 lb.

Art. 412.

2. $9.75.
3. $33.8835.
4. $912.50.
5. $28.9926; $20.709; $49.7016.
6. $96; $135.
7. $24.02.
8. $90.125.

Art. 414.

2. $6.87+.
3. $54.71+.
4. $83.108+.

5. $941.571+.
6. $1255.35.
7. $412.206.
8. $2759.33⅓.
9. $5190.208⅓.
10. $99.72.
11. $1.531+.
12. $15.669+.
13. $1020.48.
14. $125.712+.
15. $60.922+.
16. $82.406+.

Art. 416.

2. $9.081+.
3. $9.282+.
4. $36.471+.
5. $2.568+.
6. $6.835+.
7. $16.392+.
8. $47.691.
9. $112.20.
10. $3.267+.
11. $20.273+.
12. $227.108+.
13. $10.436.
14. $8.914+.
15. $11.
16. $16.303+.
17. $69.422+.
18. $522.096+.
19. $665.552+.
20. $1255.08.
21. $51.66⅔.
22. $5.88.
23. $50.54+.
24. $35.50.
25. $216.02+.
26. $472.137.
27. $5656.08⅓.

Art. 417.

2. $560; $420.
3. $2000; $1400.
4. $5800.
5. $3000.
6. $5500.
7. $19500.

Art. 418.

2. $450.
3. $400.54.
4. $250.
5. $141.70.
6. $1520.91+.

Art. 419.

2. 8%.
3. 7½%.
4. 10%.
5. 5½%.
6. 6%, 8%, 12%, 16%.
7. 13⅓%.

Art. 420.

2. 3 yr. 6 mo.
3. 2 mo. 12 da.
4. 2 yr. 9 mo.
5. 8 mo. 12 da.
6. 3 yr. 6 mo. 24 da.
7. 2 yr. 4 mo.
8. 12½ yr., 25 yr., 7 mo. 15 da.

Art. 421.

2. $51.25.
3. $143.26+.
4. $248.63+.
5. $137.92+.
6. $1402.55+.
7. $43.487.
8. $164.86+.
9. $1043.599+.

Art. 427.

2. $789.588+.
3. $2198.33+.
4. $1370.845.

Art. 432.

2. $600 P. W.
   $14.40 T. D.
3. $2500 P. W.
   $140 T. D.
4. $461.165 P.W
   $13.835 T. D
5. $1210.
6. $4559.38.
7. $291.63+.
8. $4.52+ loss.

Art. 435.

2. $20.51.
3. $1472.875.
4. $777.276+.
5. $1.69+.

Art. 436.

2. $2990.53+.
3. $360.
4. $1518.60.
5. $783.08.

Art. 443.

2. $72.75.
3. $33.60.
4. $90.

Art. 444.

2. 1¾%.
3. ⅝%.
4. ⅛%.
5. 1⅜%.

Art. 445.

2. $60000.
3. $15600.

Art. 448.

1. 525 lb..
2. $10\frac{1}{2}$ bu.
3. $94.67.
4. $\$.11\frac{2}{3}$.
5. $9\frac{1}{2}\%$.
6. $3560.
7. $75.25.
8. $1406.25.
9. $27.40.
10. $18.88.
11. $34\frac{1}{6}$ yd.
12. $2.156.
13. 328 sq. ft.
14. $7.60. 1st.
15. $43.03+.
16. $63.28+.
17. $21500.
18. $1.20.
19. 240 yd.; $390.
20. 86 lb.
21. $13200.
22. 5994 ft.
23. The latter, $.12 nearly.
24. $186.60.
25. $8.
26. $573.44.
27. $187\frac{1}{77}$ yd.; $1562\frac{1}{2}$ lb.
28. $652.86.
29. $33.44+.
30. 260 yd.; $455.
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